Scientific
Research
Publishing

Cosmological Constant and Energy Density of Random Electromagnetic Field

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Abstract

It is shown that the non-equilibrium electrically neutral and relativistically invariant vacuum-like state with the negative energy density and positive pressure may exist at the non-zero temperature in the system of spinor particles, antiparticles, and random electromagnetic field generated by particle-particle, particle-antiparticle, and antiparticle-antiparticle transitions. At the temperature of the order of 10^{-5} K, the energy density of its state corresponds to the dark energy density in absolute magnitude. The cosmological constant for such material medium turns out to be negative.

1. Introduction

Article [1] shows that the quantum theory allows building up the relativistically invariant state of matter, for which pressure p and energy density ε are related as follows

$$p = -\varepsilon \tag{1}$$

Interest in the investigation of such states is related to search for the physical interpretation of the cosmological constant in the Einstein's equations [1]-[3] and dark energy nature [4]-[6].

If a material medium exists, whose energy-momentum density tensor is expressible in the following form

$$T^{(\Lambda)}_{\ \mu\nu} = g_{\mu\nu}\varepsilon^{(\Lambda)}, \tag{2}$$

then the cosmological constant Λ in the Einstein's equations [1] can be related to the energy density of such medium $\varepsilon^{(\Lambda)}$

$$\Lambda = \varepsilon^{(\Lambda)}\left(8\pi G/c^4\right). \tag{3}$$

Here G is the gravitation constant, c is the velocity of light. In case of the empty flat space, when the Minkovsky's tensor $\eta^{\mu\nu}$ with $(1,-1,-1,-1)$ diagonal is the metric tensor $g_{\mu\nu}$, energy density $\varepsilon^{(\Lambda)}$ and pressure $p^{(\Lambda)}$ obviously satisfy the relationship (1).

If $\varepsilon^{(\Lambda)} > 0$, then negative pressure $p^{(\Lambda)}$ can be interpreted [4] [6] as the cause of the antigravity. The opposite possibility, where $\varepsilon^{(\Lambda)} < 0$ [1] [2], is not also excluded. In such a case, the gravitational attraction between massive objects starts increasing at the distances exceeding some threshold one.

The cosmological constant problem is particularly relevant in the context of the experimental confirmation of the accelerated expansion of the Universe [6]. However, this phenomenon can be explained not only with the help of the General Relativity, but also by the different alternative versions of the Gravity Theory (see [7] and the articles cited there), where dark energy concept isn't introduced. In paper [7] it is shown that the analysis of the interference of gravitational waves will allow making a choice for that or another Gravity Theory.

Experimental discovery of gravitational waves [8] allows hoping for increasing of our understanding of the nature of gravitation. Data obtained now confirm justice of the General Relativity [8]. But it is only the first experiments and it is not enough for final conclusions.

In any case, up to date, the material medium model-building problem, for which relationship (1) is true and density

$$\rho^{(\Lambda)} = \varepsilon^{(\Lambda)}/c^2 \tag{4}$$

is close, at least in the order-of-magnitude, to the value of

$$\rho^{(\Lambda)} \approx \left(7.21 \pm 0.25\right) \times 10^{-30}\ \mathrm{g\cdot cm^{-3}}$$

resulted from the interpretation of astrophysical data [6] that has not been solved. Solving this problem seems important for the understanding of potential physical phenomena leading to the formation of such material medium.

2. Energy-Momentum Tensor of Random Electromagnetic Field

Let us consider the model where the electromagnetic field tensor

$$F^{\mu\nu}(x) = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x) \tag{5}$$

is governed by the Maxwell's equations

$$\partial_\mu F^{\mu\nu}(x) = \left(4\pi e/c\right)q^\nu(x), \tag{6}$$

with random source $q^\nu(x)$ on the right side. Here $A^\mu(x)$ is the vector potential of electromagnetic field, and e is the electric charge.

We consider that for flow density $q^\mu(x)$ the conservation law is true

$$\partial_\mu q^\mu(x) = 0, \tag{7}$$

and for potential $A^\mu(x)$ the Lorentz' gage condition is true

$$\partial_\mu A^\mu(x) = 0. \tag{8}$$

Suppose, devices available enable measuring only some time interval and/or volume mean values, which will be indicated with brackets $\langle ...\rangle$. Suppose that

$$\left\langle q^\mu(x)\right\rangle = 0, \tag{9}$$

but at the same time for decomposition components $q^\mu(x)$ on some set of functions $q^\mu(x,\lambda,\lambda')$,

$$q^\mu(x) = \sum_\lambda \sum_{\lambda'} q^\mu(x,\lambda,\lambda') \tag{10}$$

the following relationship is true:

$$\left\langle \int \mathrm{d}^4 x'' \int \mathrm{d}^4 x' q^{*\mu}(x'',\sigma,\sigma')q^\nu(x',\lambda,\lambda')\exp\left\{-ik_\alpha x''^\alpha + i\chi_\alpha x''^\alpha\right\}\right\rangle$$
$$= \delta_{\lambda\sigma}\delta_{\lambda'\sigma'}n^{\mu\nu}(k,\sigma,\sigma';\chi,\lambda,\lambda') \tag{11}$$

where $n^{\mu\nu}(k,\sigma,\sigma';\chi,\lambda,\lambda')$ is the tensor, whose exact form will be defined later on.

The formal solution of Equations (6) for $A^{\mu}(x)$ meeting the initial condition

$$A^{\mu}(x)=0 \text{ at } t<0, \tag{12}$$

taking into account Decomposition (10), can be written down in the following form

$$A^{\mu}(x)=-(4\pi e/c)(2\pi)^{-4}\sum_{\lambda}\sum_{\lambda'}\int d^4\chi\int d^4x' q^{\mu}(x,\lambda,\lambda')\exp\{-i\chi_{\nu}(x^{\nu}-x'^{\nu})\}/(\chi^k\chi_k+2i\alpha\chi_0), \tag{13}$$

where $\alpha>0, \alpha\to 0$. According to Equations (5), (9), and (13)

$$\langle A^{\mu}(x)\rangle=0, \langle F^{\mu\nu}(x)\rangle=0. \tag{14}$$

Let us determine energy-momentum density tensors of electromagnetic field $F^{\mu\nu}(x)$ and its interaction with charged flow $q^{\nu}(x)$

$$t^{\mu\nu}(A(x))=-(4\pi c)^{-1}(F^{\mu k}(x)F^{\nu}{}_k(x)-(1/4)\eta^{\mu\nu}F^{kl}(x)F_{kl}(x)), \tag{15.1}$$

$$t^{\mu\nu}(q(x),A(x))=-(e/c^2)q^{\mu}(x)A^{\nu}(x). \tag{15.2}$$

These tensors can be presented using quadratic combinations $q^{\nu}(x)$, and therefore their average values may be non-zero. Using Formula (11), for the average value of the sum of tensors $t^{\mu\nu}(A(x))$ and $t^{\mu\nu}(q(x),A(x))$, we obtain

$$\begin{aligned} T^{(s)\mu\nu}(x)&=\langle t^{\mu\nu}(A(x))\rangle+\langle t^{\mu\nu}(q(x),A(x))\rangle\\ &=\sum_{\lambda}\sum_{\lambda'}\int d^4\chi\int d^4k T^{(s)\mu\nu}(k,\chi;\lambda,\lambda')\exp\{-i(\chi_l-k_l)x^l\}, \end{aligned} \tag{16.1}$$

$$\begin{aligned} T^{(s)\mu\nu}(k,\chi;\lambda,\lambda')=&-(4\pi e^2/c^3)(2\pi)^{-8}((k^{\alpha}k_{\alpha}-2i\alpha k_0)(\chi^{\alpha}\chi_{\alpha}+2i\alpha\chi_0))^{-1}\\ &\times\{(k^{\mu}\chi^{\nu}-(1/2)\eta^{\mu\nu}k^{\beta}\chi_{\beta})n^{\alpha}{}_{\alpha}(k,\lambda,\lambda';\chi,\lambda,\lambda')\\ &+(k^{\alpha}\chi_{\alpha}-(k^{\alpha}k_{\alpha}-2i\alpha k_0))n^{\mu\nu}(k,\lambda,\lambda';\chi,\lambda,\lambda')-k^{\alpha}\chi^{\nu}n^{\mu}{}_{\alpha}(k,\lambda,\lambda';\chi,\lambda,\lambda')\\ &-k^{\mu}\chi_{\alpha}n^{\alpha\nu}(k,\lambda,\lambda';\chi,\lambda,\lambda')+(1/2)\eta^{\mu\nu}k^{\alpha}\chi_{\beta}n^{\beta}{}_{\alpha}(k,\lambda,\lambda';\chi,\lambda,\lambda')\}. \end{aligned} \tag{16.2}$$

Suppose, tensor $n^{\mu\nu}(k,\lambda,\lambda';\chi,\lambda,\lambda')$ describes transitions between acceptable states of spinor particles and antiparticles with mass m and charges e and $-e$ in the binary mixture of their gases. Then it can be defined using the following relationships

$$\begin{aligned} n^{\mu\nu}(k,\lambda,\lambda';\chi,\lambda,\lambda')=&c^2(mc/2\pi\hbar)^6(2\pi)^8[n^{(e,e)}{}_{(r,r')}{}^{\mu\nu}(k,\chi;p,p')\\ &+n^{(h,h)}{}_{(r,r')}{}^{\mu\nu}(k,\chi;p,p')+n^{(h,e)}{}_{(r,r')}{}^{\mu\nu}(k,\chi;p,p')], \end{aligned} \tag{17.1}$$

$$\begin{aligned} n^{(e,e)}{}_{(r,r')}{}^{\mu\nu}(k,\chi;p,p')=&Q^{(e,e)}(p,p',F^{(e)},\Theta)\delta^4(k-(p-p')/\hbar)\delta^4(\chi-(p-p')/\hbar)\\ &\times w^{*(e,e)}{}_{(r,r')}{}^{\mu}(p,p')w^{(e,e)}{}_{(r,r')}{}^{\nu}(p,p'), \end{aligned} \tag{17.2}$$

$$\begin{aligned} n^{(h,h)}{}_{(r,r')}{}^{\mu\nu}(k,\chi;p,p')=&Q^{(h,h)}(p,p',F^{(h)},\Theta)\delta^4(k-(p-p')/\hbar)\delta^4(\chi-(p-p')/\hbar)\\ &\times w^{*(h,h)}{}_{(r,r')}{}^{\mu}(p,p')w^{(h,h)}{}_{(r,r')}{}^{\nu}(p,p'), \end{aligned} \tag{17.3}$$

$$\begin{aligned} n^{(h,e)}{}_{(r,r')}{}^{\mu\nu}(k,\chi;p,p')=&Q^{(h,e)}(p,p',F^{(e)},F^{(h)},\Theta)\delta^4(k-(p-p')/\hbar)\delta^4(\chi-(p-p')/\hbar)\\ &\times w^{*(h,e)}{}_{(r,r')}{}^{\mu}(p,p')w^{(h,e)}{}_{(r,r')}{}^{\nu}(p,p'), \end{aligned} \tag{17.4}$$

$$Q^{(e,e)}(p,p',F^{(e)},\Theta)=\theta((E^{(e)}(p')-E^{(e)}(p))/mc^2)f(s_e(p',F^{(e)},\Theta))(1-f(s_e(p,F^{(e)},\Theta))), \tag{17.5}$$

$$Q^{(h,h)}(p,p',F^{(h)},\Theta)=\theta((E^{(h)}(p)-E^{(h)}(p'))/mc^2)f(s_h(p,F^{(h)},\Theta))(1-f(s_h(p',F^{(h)},\Theta))), \tag{17.6}$$

$$Q^{(h,e)}\left(p, p', F^{(e)}, F^{(h)}, \Theta\right) = f\left(s_e\left(p', F^{(e)}, \Theta\right)\right) f\left(s_h\left(p, F^{(h)}, \Theta\right)\right). \tag{17.7}$$

Here indices e and h pertain to particles and antiparticles, respectively; $\hbar$ is the Plank's constant divided by 2π; p^μ, and p'^μ are four-dimensional momenta of particles and antiparticles, for which the following dispersion relationship is true

$$p_0^2 = \left(mc^2 + p^2\right)^{1/2} > 0, \quad p^2 = (\boldsymbol{p}, \boldsymbol{p}), \tag{18}$$

indices r and r' take on the values of ± 1 and correspond to the two possible spin states of particles and antiparticles; $F^{(e)}$ and $F^{(h)}$ are chemical potentials of particles and antiparticles, which we will consider as constant;

$$\Theta = k_B T, \tag{19}$$

k_B is the Boltzmann's constant, T is the temperature;

$$f(s) = \left(\exp\{s\} + 1\right)^{-1} \tag{20}$$

the Fermi-Dirac distribution function;

$$s_e = \left(p_\mu U^\mu - F^{(e)}\right)\!\Big/\Theta, \quad s_h = \left(p_\mu U^\mu + F^{(h)}\right)\!\Big/\Theta, \tag{21}$$

U^μ is the hydrodynamic velocity [9] that meets the relationship

$$U_\mu U^\mu = c^2, \quad U_0 > 0. \tag{22}$$

Vectors $w^{(a,b)}{}_{(r,r')}{}^\mu(p, p')$ correspond to flows of transitions between the states of particles and antiparticles

$$w^{(e,e)}{}_{(r,s)}{}^\mu(p, p') = \bar{u}^{(e)}{}_r(p)\gamma^\mu u^{(e)}{}_s(p'), \quad w^{(h,h)}{}_{(r,s)}{}^\mu(p, p') = \bar{u}^{(h)}{}_r(p)\gamma^\mu u^{(h)}{}_s(p'),$$
$$w^{(h,e)}{}_{(r,s)}{}^\mu(p, p') = \bar{u}^{(h)}{}_r(p)\gamma^\mu u^{(e)}{}_s(p'), \tag{23}$$

where γ^μ is the Dirac gamma-matrices, $u^{(e)}{}_r(p)$ and $u^{(h)}{}_r(p)$ are spinors that satisfy Equations

$$\left(\gamma^\mu p_\mu - mc\right) u^{(e)}{}_r(p) = 0, \quad \left(\gamma^\mu p_\mu + mc\right) u^{(h)}{}_r(p) = 0, \tag{24.1}$$

and normalization conditions

$$\bar{u}^{(e)}{}_r(p) u^{(e)}{}_s(p) = \delta_{rs}, \quad \bar{u}^{(h)}{}_r(p) u^{(h)}{}_s(p) = -\delta_{rs} \tag{24.2}$$

It is easy to show that the relations of orthogonality are true

$$\left(p'_\mu - p_\mu\right) w^{(e,e)}{}_{(r,s)}{}^\mu(p, p') = 0, \quad \left(p'_\mu - p_\mu\right) w^{(h,h)}{}_{(r,s)}{}^\mu(p, p') = 0, \quad \left(p'_\mu + p_\mu\right) w^{(h,e)}{}_{(r,s)}{}^\mu(p, p') = 0. \tag{25}$$

With account of these definitions, Expressions (25), and the formula for the transition from the summation over composite index λ to the integration over momentum p

$$\sum_\lambda(\ldots) \to (mc)^{-2} \sum_r \int d^4 p\, \theta\left(p_0/mc\right) \delta\left(p_\mu p^\mu - m^2 c^2\right)(\ldots) = (mc)^{-2} \sum_r \int \left(d^3 p / p_0\right)(\ldots),$$

we obtain from Relationships (16.1) and (16.2)

$$T^{(s)\mu\nu}(x) = T^{(e,e)\mu\nu}(x) + T^{(h,h)\mu\nu}(x) + T^{(h,e)\mu\nu}(x), \tag{26.1}$$

$$T^{(a,b)\mu\nu}(x) = -\left(\alpha/2\pi^2\right)\left(2\pi\hbar\right)^{-3}(mc)^2 \sum_r \sum_{r'} \int \left(d^3 p'/p'_0\right) \int \left(d^3 p/p_0\right) T^{(a,b)\mu\nu}(p', r', p, r), a, b = e, h, \tag{26.2}$$

$$T^{(e,e)\mu\nu}(p', r', p, r)$$
$$= \theta\left(\left(E^{(e)}(p') - E^{(e)}(p)\right)\Big/mc^2\right) f\left(s_e\left(p', F^{(e)}, \Theta\right)\right)\left(1 - f\left(s_e\left(p, F^{(e)}, \Theta\right)\right)\right) \tag{26.3}$$
$$\times w^{*(e,e)}{}_{(r,r')}{}^\beta(p, p') w^{(e,e)}{}_{(r,r')\beta}(p, p')\left(n^\mu n^\nu - (1/2)\eta^{\mu\nu} n^\beta n_\beta\right)\left(n^\beta n_\beta\right)^{-2}, n^\mu = p'^\mu - p^\mu,$$

$$T^{(h,h)\mu\nu}\left(p',r',p,r\right)$$

$$= \theta\left(\left(E^{(h)}(p)-E^{(h)}(p')\right)\Big/mc^2\right)f\left(s_h\left(p,F^{(h)},\Theta\right)\right)\left(1-f\left(s_h\left(p',F^{(h)},\Theta\right)\right)\right) \tag{26.4}$$

$$\times w^{*(h,h)}{}_{(r,r')}{}^{\beta}\left(p,p'\right)w^{(h,h)}{}_{(r,r')\beta}\left(p,p'\right)\left(n^{\mu}n^{\nu}-(1/2)\eta^{\mu\nu}n^{\beta}n_{\beta}\right)\left(n^{\beta}n_{\beta}\right)^{-2}, n^{\mu}=p^{\mu}-p'^{\mu},$$

$$T^{(h,e)\mu\nu}\left(p',r',p,r\right)=f\left(s_e\left(p',F^{(e)},\Theta\right)\right)f\left(s_h\left(p,F^{(h)},\Theta\right)\right)w^{*(h,e)}{}_{(r,r')}{}^{\beta}\left(p,p'\right)$$

$$\times w^{(h,e)}{}_{(r,r')\beta}\left(p,p'\right)\left(n^{\mu}n^{\nu}-(1/2)\eta^{\mu\nu}n^{\beta}n_{\beta}\right)\left(n^{\beta}n_{\beta}\right)^{-2}, n^{\mu}=p^{\mu}+p'^{\mu}, \tag{26.5}$$

where $\alpha = e^2/\hbar c$ is the fine-structure constant approximately equal to 1/137, if e is the electron charge.

3. Energy Density and Pressure of Interacting Mixture

Assume, the inequations are true

$$F^{(e)} > mc^2, F^{(h)} < -mc^2, \tag{27}$$

i.e. particle and antiparticle gases are degenerating ones. Let us consider the range of temperature and chemical potential values that satisfy the following conditions:

$$p^{(e)}{}_F{}^2/m \ll \Theta \ll mc^2, \quad p^{(h)}{}_F{}^2/m \ll \Theta \ll mc^2. \tag{28}$$

Here $p^{(e)}{}_F$ and $p^{(h)}{}_F$ are Fermi momenta defined in the following relationships:

$$p^{(e)}{}_F = c^{-1}\left(F^{(e)2}-m^2c^4\right)^{1/2}, \quad p^{(h)}{}_F = c^{-1}\left(F^{(h)2}-m^2c^4\right)^{1/2}. \tag{29}$$

In the frame of reference where $U=0$, we obtain the expressions for energy density and pressure

$$\varepsilon^{(s)} \approx -(8\pi)(4/\pi-1)\alpha c(2\pi\hbar)^{-3}(m\Theta)^2 = -\pi^{-2}(4/\pi-1)\left(emk_BT/\hbar^2\right)^2, \tag{30.1}$$

$$p^{(s)} \approx -\varepsilon^{(s)}/3 \approx (8\pi/3)(4/\pi-1)\alpha c(2\pi\hbar)^{-3}(m\Theta)^2 = \left(3\pi^2\right)^{-1}(4/\pi-1)\left(emk_BT/\hbar^2\right)^2 \tag{30.2}$$

related to the components of the energy-momentum tensor by simple relationships

$$\varepsilon^{(s)} = cT^{(s)00}, \quad p^{(s)} = -(c/3)\left(T^{(s)\mu}{}_{\mu}-T^{(s)0}{}_0\right). \tag{31}$$

The following formulas are true under Conditions (28) in the same frame of reference for concentrations of particles and antiparticles $n^{(e)}$ and $n^{(h)}$, their energy densities and pressures

$$n^{(e)} \approx (8\pi/3)\left(p^{(e)}{}_F/2\pi\hbar\right)^3, n^{(h)} \approx (8\pi/3)\left(p^{(h)}{}_F/2\pi\hbar\right)^3, \tag{32.1}$$

$$\varepsilon^{(e)} \approx (8\pi/3)mc^2\left(p^{(e)}{}_F/2\pi\hbar\right)^3 = mc^2n^{(e)}, \varepsilon^{(h)} \approx (8\pi/3)mc^2\left(p^{(h)}{}_F/2\pi\hbar\right)^3 = mc^2n^{(h)}, \tag{32.2}$$

$$p^{(e)} \approx (16\pi/15)\left(p^{(e)}{}_F{}^2/2m\right)\left(p^{(e)}{}_F/2\pi\hbar\right)^3 = (2/5)\left(p^{(e)}{}_F{}^2/2m\right)n^{(e)},$$

$$p^{(h)} \approx (16\pi/15)\left(p^{(h)}{}_F{}^2/2m\right)\left(p^{(h)}{}_F/2\pi\hbar\right)^3 = (2/5)\left(p^{(h)}{}_F{}^2/2m\right)n^{(h)}. \tag{32.3}$$

The condition of electroneutrality of the particle and antiparticle gases mixture has the following form

$$F^{(e)} + F^{(h)} = 0, \tag{33}$$

whence it follows that

$$p^{(e)}{}_F = p^{(h)}{}_F = p_F. \tag{34}$$

In such a case, we obtain for the total energy density and pressure

$$\varepsilon = \varepsilon^{(e)} + \varepsilon^{(h)} + \varepsilon^{(s)} \approx (16\pi/3)mc^2\left(p^{(e)}{}_F/2\pi\hbar\right)^3 - (8\pi)(4/\pi-1)\alpha c(2\pi\hbar)^{-3}(m\Theta)^2, \tag{35.1}$$

$$p = p^{(e)} + p^{(h)} + p^{(s)} \approx (8\pi/3)(4/\pi - 1)\alpha c (2\pi\hbar)^{-3} (m\Theta)^2 . \tag{35.2}$$

The energy density and pressure of electrically neutral particle and antiparticle gases mixture will be relativistically invariant in two cases [1] where

$$\varepsilon = 0, \quad p = 0$$

and where condition (1) is met. In the first case, we obtain from Equations (35.1) and (35.2)

$$T = 0, \quad p_F = 0 . \tag{36}$$

Furthermore, according to Formulas (32.1), concentrations of particles and antiparticles are zero.

In the second case, we obtain the condition of implementing the electrically neutral and relativistically invariant state for the mixture under consideration from Relationships (1), (35.1) and (35.2)

$$p_\Lambda{}^3 = (4/\pi - 1)\alpha (mc)^3 \left(\Theta_\Lambda/mc^2\right)^2 = (4/\pi - 1)\left(e^2 k_B{}^2 T_\Lambda{}^2 m/\hbar c^2\right) . \tag{37}$$

Here the Fermi momentum and temperature, at which conditions (1) is met, are indicated using p_Λ and T_Λ. It is easy to show that momentum p_Λ satisfying Equation (37) satisfies also Inequations (28)

$$p_\Lambda{}^2 \ll m\Theta_\Lambda \Rightarrow \alpha (mc)^3 \left(\Theta_\Lambda/mc^2\right)^2 \ll (m\Theta_\Lambda)^{3/2} \Rightarrow \alpha \left(\Theta_\Lambda/mc^2\right)^{1/2} \ll 1$$

The following expressions are true for the concentrations of particles and antiparticles, energy density and pressure of the mixture

$$n^{(e)} = n^{(h)} = n^{(\Lambda)} \approx (8\pi/3)\left(p_\Lambda/2\pi\hbar\right)^3 = \left(3\pi^2\right)^{-1}(4/\pi - 1)\alpha \left(\Theta_\Lambda{}^2 m/c\hbar^3\right)$$
$$= \left(3\pi^2\right)^{-1}(4/\pi - 1)m\left(ek_B T_\Lambda/\hbar^2 c\right)^2 , \tag{38.1}$$

$$\varepsilon^{(\Lambda)} \approx -p^{(\Lambda)} \approx -(8\pi/3)\alpha c (2\pi\hbar)^{-3}(4/\pi - 1)(m\Theta_\Lambda)^2$$
$$= -\left(3\pi^2\right)^{-1}(4/\pi - 1)\left(emk_B T_\Lambda/\hbar^2\right)^2 = -mc^2 n^{(\Lambda)} . \tag{38.2}$$

We receive the following from Formulas (3), (4), and (38.2) for the cosmological constant and dark energy density

$$\Lambda \approx -\left(8\pi Gm/c^2\right)n^{(\Lambda)} = -(8/3\pi)(4/\pi - 1)\left(Gm^2 e^2 k_B{}^2 T_\Lambda{}^2\right)/(\hbar c)^4$$
$$= -(8/3\pi)(4/\pi - 1)\alpha\alpha_G \left(k_B T_\Lambda/\hbar c\right)^2 = -(8/3\pi)(4/\pi - 1)\alpha\alpha_G R_T{}^{-2} , \tag{39.1}$$

$$\rho^{(\Lambda)} = -(8\pi/3)(4/\pi - 1)(\alpha/c)(2\pi\hbar)^{-3}(m\Theta_\Lambda)^2 = -\left(3\pi^2\right)^{-1}(4/\pi - 1)\alpha m R_e{}^{-1} R_T{}^{-2}$$
$$= -\left(3\pi^2\right)^{-1}(4/\pi - 1)\left(emk_B T_\Lambda/\hbar^2 c\right)^2 = -mn^{(\Lambda)} . \tag{39.2}$$

Here the following is specified

$$\alpha_G = Gm^2/\hbar c ,$$

$-a$ dimensionless value representing a gravitational analogue of the fine structure constant in terms of the build-up method, if m is the mass of electron, then $\alpha_G \approx 1.75 \times 10^{-45}$;

$$R_T = \hbar c/k_B T_\Lambda$$

is the analogue of the Compton wave-length

$$R_e = \hbar/mc$$

for an object with the energy equal to Θ_Λ .

If e and m are the charge and mass of electron, and dark energy density

$$\rho^{(\Lambda)} = -7.21 \times 10^{-30} \text{ g} \cdot \text{cm}^{-3}, \quad \text{then } \Lambda \approx 1.34 \times 10^{-56} \text{ cm}^{-2}, T_\Lambda \approx 1.54 \times 10^{-5} \text{ K} ,$$

$$R_T \approx 1.49 \times 10^4 \text{ cm} = 149 \text{ m}, p_\Lambda \approx 6.5 \times 10^{-28} \text{ g} \cdot \text{cm/s}, v_\Lambda = p_\Lambda/m \approx 0.71 \text{ cm/s} .$$

The second of Formulas (39.2) for density $\rho^{(\Lambda)}$ has the strongly non-isotropic form. As if the dark energy is distributed in a parallelepiped with the very small height R_e as compared with the typical dimensions of edges of bases R_T

$$R_e / R_T \approx 2.5 \times 10^{-15} .$$

Note that the contribution of the random electromagnetic field, including the contribution of interaction of this field with particles and antiparticles, to the total energy density in absolute magnitude is thrice as much as the contributions of particles and antiparticles equal to each other

$$\varepsilon^{(e)} = \varepsilon^{(h)} = mc^2 n^{(\Lambda)}, \ \ \varepsilon^{(s)} = -3mc^2 n^{(\Lambda)} .$$

Just due to the random electromagnetic field generated by transitions between particles and antiparticles, the state that satisfies Condition (1) is possible in the system studied.

In the approximation considered, energy density $\varepsilon^{(s)}$ and pressure $p^{(s)}$ are resulting mainly from particle-particle and antiparticle-antiparticle transitions. The energy density and pressure related to particle-antiparticle transitions are much less than $\varepsilon^{(e,e)}, \varepsilon^{(h,h)},$ and $p^{(e,e)}, p^{(h,h)}$

$$\varepsilon^{(h,e)} \approx p^{(h,e)} \approx -\left(32/9\right)\alpha c \left(p_F{}^3 / m^2 c^2 \right) \left(p_F / 2\pi\hbar \right)^3 , \left| \varepsilon^{(h,e)} / \varepsilon^{(s)} \right| \sim \left| p^{(h,e)} / p^{(s)} \right| \ll 1 .$$

Furthermore, the energy dominance [2] is disrupted for $\varepsilon^{(s)}$ and $p^{(s)}$, and their components.

4. Conclusions

The model of the material medium, for which the electrically neutral relativistically invariant state with non-zero energy density, pressure, concentrations of particles and antiparticles is possible, has been considered. At the same time particles and antiparticles are in the thermal equilibrium, but far from the chemical equilibrium state governed by the equality of their chemical potentials. In the considered case, the expression is true

$$F^{(e)} - F^{(h)} = 2c \left(m^2 c^2 + p_F{}^2 \right)^{1/2} \approx 2mc^2 \left[1 + \left(1/2\right) \left(\left(4/\pi - 1\right) \alpha \left(k_B T_\Lambda / mc^2 \right)^2 \right)^{2/3} \right] .$$

This non-equilibrium electrically neutral state exists due to the random electromagnetic field generated by spontaneous transitions between particles and antiparticles being in different quantum states. The average vector potential and intensity of this field are zero. But the average components of the energy-momentum density tensor in the random process of transitions between the states of particles and antiparticles are non-zero.

The energy density of the above vacuum-like state can be expressed in terms of its temperature T_Λ. The state electroneutrality requirement leads to the equality of Fermi momentum of particles and antiparticles, and the relativistic invariance requirement to the equation relating the Fermi momentum with the temperature.

If the energy density of the system considered is identified with the dark energy density, then temperature T_Λ turns to be of the order of 10^{-5} K and Fermi velocity of particles and antiparticles $v_\Lambda = p_\Lambda / m$ to be approximately 1 cm/s (m is the mass of electron). Furthermore, the dark energy density and cosmological constant are negative.

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