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Weakly Guiding Fibers and LP Modes in Circular and Elliptical Waveguides

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ABSTRACT

This paper gives the simple and logical approach of LP modes in circular and elliptical waveguides. Earlier the basic approach of modes in circular and elliptical fibers was studied by the authors. In this paper, the role of radial antinode in circular and elliptical waveguides is given clearly. Splitting of modes in circular and elliptical fiber has been discussed.

Keywords: Linearly Polarized Modes; Weakly Guiding Fibers; Circular Waveguides; Elliptical Waveguides

1. Introduction

Optical fibers (or waveguides) are important components of optical communication systems and information technology [1-7]. The aim of this paper is to describe LP modes in circular and elliptical waveguides. These LP modes consist of certain patterns of electromagnetic waves formed within the fiber due to structurally imposed (transverse) boundaries on the propagation fields. Each mode is a pattern of electric and magnetic field distributions that is repeated along the fiber at equal intervals. In modern communication systems we use as few modes as possible so that the interaction among several modes is minimized. Because optical fibers are used as basic mediums for transmission of optical signals, it is useful to make a detailed study of mode designation in optical waveguides.

2. Circular Waveguides

In weakly guiding fibers where In weakly guiding fibers where $(n_1 - n_2)/n_1$ is much less than 1 (n_1, n_2 being the core and the cladding refractive index) it is found convenient to describe the modes in terms of linearly polarised modes or LP modes.

These LP modes are due to the superposition of HE or EH modes. However, the LP modes are not exact modes of the step-index fiber and each LP-mode has many degenerate modes.

Apart from degeneracy, there is also an instability of the lobe orientations of the fields.

In weakly guiding fibers one can construct modes whose transverse fields are polarised in one direction. In elliptical fibers the fiber can suport two types of mode, one polarised predominantly in the x-direction and the other polarised predominantly in the y-direction.

In actual practice, in fibers for telecommunication purposes the relative core-cladding index difference $(n_1 - n_2)/n_1$ is usually kept less than 0.02.

This practical requirement permits us to simplify the mathematical analysis by considering what is known as the scalar wave equation in terms of a field variable ψ which may represent any of the cartesian components of the E and H fields. The boundary conditions also become simpler so that ψ and its radial derivative may be treated as continuous across the core-cladding boundary. The modes now are designated as $LP_{\ell m}$ modes [8] the letters L and P standing for the phrase "**Linearly polarised**" The suffix ℓ stands for the ℓth order Bessel function which corresponds to the cutoff condition for the mode and the other suffix m enumerates the successive zeroes of the corresponding Bessel function. If we show the positions of the field antinodes of a particular mode on the cross-section of the fiber, the mode $LP_{\ell m}$ will have 2ℓ antinodes in a ring of a certain radius and there will be n such rings on the cross-section thus in **Figure 1** we show the antinodes of LP_{31} and in **Figure 2** we show the antinodes of LP_{52}. In a similar manner we can show the antinodes of LP_{42} and LP_{62} in **Figure 3** and **Figure 4** respectively.

The lowest order mode (fundamental mode) which is never cutoff is represented by LP_{01} and this corresponds to the HE_{11} mode of the previous section. The corre-
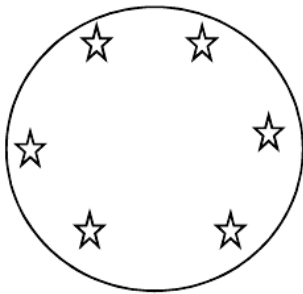
Figure 1. Antinodes for linearly polarized modes LP$_{31}$.

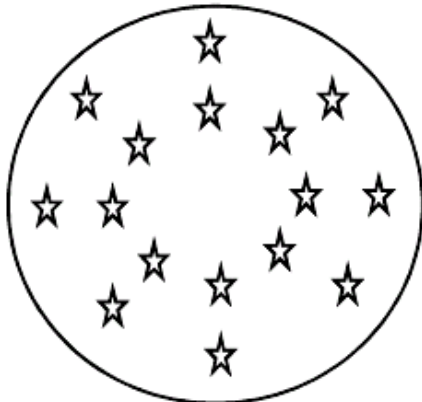**Figure 2. Antinodes for linearly polarized modes LP$_{52}$.**

Figure 3. Antinodes for linearly polarized modes LP$_{42}$.

Figure 4. Antinodes for linearly polarized modes LP$_{62}$.

spondence between the mode description such as HE$_{mn}$ etc. and the LP-modes are as follows:

$$HE_{11} = LP_{01} \text{ mode}$$

$$TE_{01}, TM_{01}, HE_{21} = LP_{11} \text{ mode etc.}$$

As a precaution we must note that for LP$_{0m}$ modes there will be no radial antinode at the center of the cross section.

3. Elliptical Waveguides

So far we have considered only fibers of circular cross-section. Now we turn to elliptical cross-sections. Since the ellipse is less symmetrical than the circle there can be two orientations for the field configuration in elliptical fibers. The fields in an elliptical fiber can be described in terms of Mathieu functions which are rather complicated functions. Mathieu functions are generally grouped in to two classes:

1) The even Mathieu functions; and
2) The odd Mathieu function.

A hybrid mode in an elliptical fiber is designated by a prescript, e or o, where e and o stand for the even mode and the odd mode respectively. The axial magnetic field of an even mode is represented by even Mathieu functions whereas the axial electric field of an even mode is represented by an odd Mathieu functions. This mode is symbolically represented as the **e**HE$_{mn}$ mode. In the case of the odd hybrid mode **o**HE$_{mn}$ the axial magnetic field is represented by an odd Mathieu function and the axial electric field is represented by an even Mathieu function.

In a similar manner, one can describe the EH modes. The axial electric field of an even mode is represented by even Mathieu functions whereas the axial magnetic field of an even mode is represented by an odd Mathieu functions. This mode is symbolically represented as the eEHmn mode. In the case of the odd hybrid mode oEHmn the axial electric field is represented by an odd Mathieu function and the axial magnetic field is represented by an even Mathieu function. Now we turn to LP$_{lm}$ modes in the case of elliptical waveguides. Mode which is designated by a prescript, e or o, where e and o stand for the even mode and the odd mode respectively. So in this case the mode will be symbolically represented by eLP$_{lm}$ mode and oLP$_{lm}$ mode. In eLP$_{lm}$ mode and oLP$_{lm}$ mode the letters L and P standing for the phrase **Linearly polarised** the suffix l stands for the lth order Mathieu function which corresponds to the cut off condition for the even mode and odd mode and the other suffix m indicates the successive zeros of the corresponding Mathieu functions.

For linearly polarized modes in elliptical fibers the hybrid LP$_{11}$ mode is split in to even LP$_{11}$ and odd LP$_{11}$ modes with well-defined mode intensity patterns. The

even LP_{11} and odd LP_{11} modes have significantly different cutoff wavelengths, which allow the existence of a wavelength range within which only even LP_{01} and LP_{11} modes are supported by the fiber. The elliptical core fibers that support two stable special modes, the LP_{01} and LP_{11} even/odd modes, are called elliptical core two-mode fibers. One important application of these fibers is to interferometric model/polarimetric sensors, which is useful to measure strain and temperature. The antinodes of even LP_{11} and odd LP_{11} are shown in **Figure 5** and **Figure 6** respectively.

4. Splitting of Modes in Circular and Elliptical Fibers

All guided optical modes in a circular symmetric optical fiber are fundamentally a transverse electric (TE_{om}), transverse magnetic (TM_{om}) or a hybrid mode (EH_{nm} or HE_{nm}). This is described in the weekly guiding approximation, where the solution is the well known LP_{nm} modes. The LP_{nm} modes are due to superpositions of the transverse and hybrid modes.

Modes splitting of higher order is due to the fact that the degeneracy of the linearly polarized modes in circular case is lifted as the fiber core is made elliptical. Thus the TE_{om} and TM_{om} modes become the hybrid modes oEH_{0m} and eHE_{0m} when it's made elliptical. The prefix e and o indicates even or odd function. Each hybrid mode from the from the circular symmetric case splits in the elliptical case into two hybrid modes, as symmetry dictates only two field configuration in an elliptical core fiber. This can be seen by EH_{nm} that splits into eEH_{nm} and oEH_{nm}. In weekly guiding approximation, there is no observable splitting in cutoff measurements between the even and odd versions of a given hybrid mode, as b/a is changed from 1 to 0.1. As a consequence only the $LP1m$ and $LPnm$ (n > 2) split as fibre becomes elliptical and each of these LP modes will only split into two other modes called eLP and oLP, where e and o here denotes if the LP mode is even or odd. A summary is further given in **Table 1**.

5. Conclusion

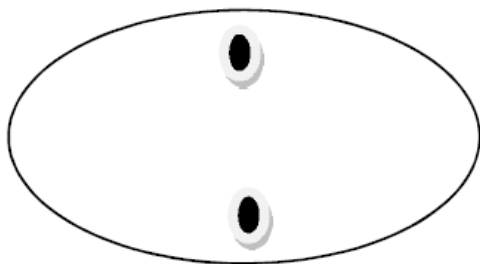This paper gives the basic and logical concept of LP

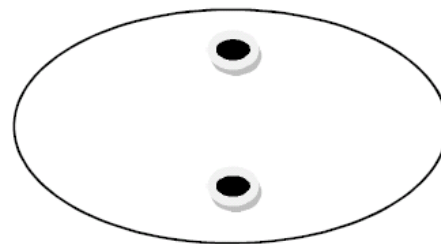**Figure 5. Antinodes for even linearly polarized modes.**

Figure 6. Antinodes for odd linearly polarized modes LP_{11}.

Table 1. Mode splitting in weakly guiding approximation.

Circular fiber		Elliptical fiber	
LP_{0m}	HE_{1m}	eLP_{0m}	e/oHE_{1m}
$LP1_m$	TE_{om} TM_{om} HE_{2m}	eLP_{1m}	oEH_{om}, eHE_{om}
		oLP_{1m}	e/oHE_{2m}
LP_{nm} $n \geq 2$	$HE_{n+1,m}$ $EH_{n-1,m}$	eLP_{nm} $n \geq 2$	$e/oEH_{n-1,m}$
		oLP_{nm} $n \geq 2$	$e/oEH_{n+1,m}$

modes in circular waveguides and even and odd LP modes in elliptical waveguide. Some basic figures are introduced when necessary without derivation. The application of modes is useful in illumination engineering. This helps our attention on the physical description of the problem.

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