ISSN: 1942-0730 Volume 9, Number 8, August 2017

Scientific Research Publishing

Journal of Electromagnetic Analysis and Applications

ISSN: 1942-0730

www.scirp.org/journal/jemaa

JOURNAL EDITORIAL BOARD

ISSN: 1942-0730 (Print) ISSN: 1942-0749 (Online)

http://www.scirp.org/journal/jemaa

Journal of Electromagnetic Analysis and Applications, 2017, 9, 113-121
http://www.scirp.org/journal/jemaa
ISSN Online: 1942-0749
ISSN Print: 1942-0730

Table of Contents

Volume 9 Number 8 August 2017

Journal of Electromagnetic Analysis and Applications (JEMAA)
Journal Information

SUBSCRIPTIONS

The *Journal of Electromagnetic Analysis and Applications* (Online at Scientific Research Publishing, www.SciRP.org) is published monthly by Scientific Research Publishing, Inc., USA.

Subscription rates:
Print: $89 per issue.
To subscribe, please contact Journals Subscriptions Department, E-mail: sub@scirp.org

SERVICES

Advertisements
Advertisement Sales Department, E-mail: service@scirp.org

Reprints (minimum quantity 100 copies)
Reprints Co-ordinator, Scientific Research Publishing, Inc., USA.
E-mail: sub@scirp.org

COPYRIGHT

PRODUCTION INFORMATION

For manuscripts that have been accepted for publication, please contact:
E-mail: jemaa@scirp.org

Scientific Research Publishing

Journal of Electromagnetic Analysis and Applications, 2017, 9, 113-121
http://www.scirp.org/journal/jemaa
ISSN Online: 1942-0749
ISSN Print: 1942-0730

Electromagnetic Wave Propagation in Waveguide Loaded by Split Ring Resonator of Negative Permeability

Abd El Moneim M. Alaa, Mostafa El Said, Samir F. Mahmoud

Electronics and Communications Department, Faculty of Engineering, Cairo University, Cairo, Egypt
Email: abiza2@msn.com

How to cite this paper: Alaa, A.E.M.M., El Said, M. and Mahmoud, S.F. (2017) Electromagnetic Wave Propagation in Waveguide Loaded by Split Ring Resonator of Negative Permeability. *Journal of Electromagnetic Analysis and Applications*, **9**, 113-121.
https://doi.org/10.4236/jemaa.2017.98010

Received: August 12, 2017
Accepted: August 28, 2017
Published: August 31, 2017

Abstract

This paper aims to study and analyze the electromagnetic propagation in media with negative transverse permeability and how this leads into some physical phenomena such as the appearance of backward waves and the propagation below cutoff. This study is done through the use of metamaterials of split ring resonators. It is shown that the waveguide dimensions needed to transmit a certain frequency band, can be miniaturized to half its dimension. The analytical determination of the propagation inside a waveguide in the presence of two slabs with dielectric permittivity and negative transverse permeability is derived. Finally it is shown by simulation, how to obtain a backward wave with lower loss than reported earlier in the literature.

Keywords

Negative Transverse Permeability, Metamaterials, Backward Waves, Miniaturization of Waveguide, Propagation Below Cutoff

1. Introduction

Rectangular waveguides are required for most of applications in microwaves. It can be used as a basic guided structure in radar application. The important application of the waveguide is to radiate element in multi-frequency interlaced antenna arrays. There are several methods to reduce the size of waveguide. One of the most important methods is metamaterial that it is used to reduce the size of the waveguide and to obtain the desired resonant frequency bands. At a particular frequency, metamaterials exhibit both negative permittivity and permeability [1] [2] [3] [4] [5].

In 1968 Veselago [1] analyzed electromagnetic wave propagation through media with negative electric permittivity ε and magnetic permeability μ. The fields and the wave propagation form a left-handed system in these materials, but the nonexistence of transparent left-handed media in nature made Veselago's results just a theory. Recently, Smith *et al.* [2] [3] have demonstrated microwave propagation through an artificial left-handed medium (metamaterial).

Several names and terminologies have been suggested for metamaterials with negative permittivity and permeability, such as "left-handed", "backward-wave media" and "double-negative". Nowadays, many researchers are studying various aspects of this class of metamaterials, and several ideas and suggestions for future applications have been proposed [4] [5] [6].

The edge coupled split ring resonators (EC-SRR) are proposed by Pendry *et al.* [7] and experimentally tested by Smith *et al.* [2] and Marque's, R., *et al.* [8] (**Figure 1(b)**). They are composed of electrically small resonant rings, which show a very high diamagnetic susceptibility above and around its resonance frequency.

The magnetic and electric dipole of the EC-SRR can be expressed by [9]:

$$m_x = \alpha_{xx}^{mm} B_x^{ext} - \alpha_{yx}^{em} E_y^{ext} \quad (1)$$

$$P_y = \alpha_{yy}^{ee} E_y^{ext} + \alpha_{xy}^{em} B_x^{ext} \quad (2)$$

The Bianistropy terms α_{yx}^{em} & α_{xy}^{me} occurred due to the fact that SRR does not act only as a magnetic dipole [9] [10], but also as an electric dipole.

Avoiding Bianistropy of the EC-SRR by a modification to the Broadside-coupled split ring resonator (**Figure 1(a)**), the BC-SRR has inversion symmetry with regard to the center of both rings. Therefore the cross-polarizability terms must vanish. So the Bianistropy terms α_{yx}^{em} & α_{xy}^{me} are equal to zero, and the magnetic and electric dipole can be written as [10] [11]:

$$m_x = \alpha_{xx}^{mm} B_x^{ext} \quad (3)$$

$$P_y = \alpha_{yy}^{ee} E_y^{ext} \quad (4)$$

The SRR loaded waveguide supports the propagation of backward waves below the cut-off frequency of the air-filled waveguide [12] [13] [14] [15] [16]. It provides the miniaturization of waveguide. Hrabar *et al.* [13], showed that

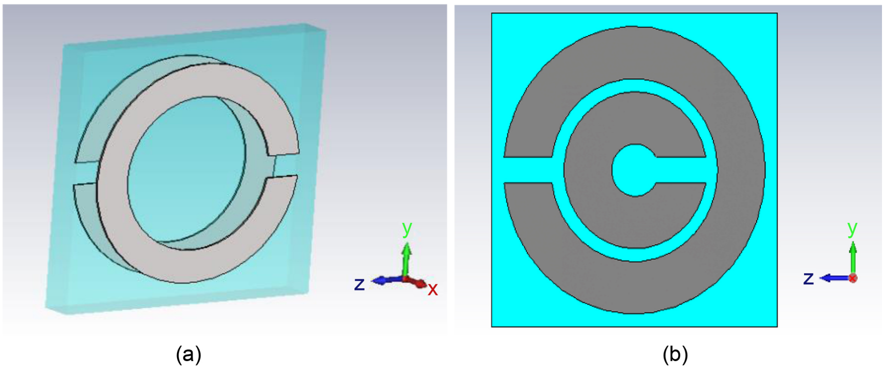

Figure 1. (a) (BC-SRR); (b) (EC-SRR).

backward propagation occurs when the longitudinal permeability is positive and the transversal permeability is negative, but it is noticed that a large insertion loss of almost 25 dB occurs in the S_{12} measurement results in the backward wave, and also it has a very narrow bandwidth.

The aim of this paper is to increase the bandwidth and decrease the losses of the backward wave, through maximizing the negative magnetic permeability.

2. The Proposed Design

From the proposed configuration of Schelkunoff's [17], the magnetic polarizability of a closed metallic loop of radius r loaded by a capacitor is expressed as:

$$\alpha_{xx}^{mm} = \frac{\pi^2 r^4}{L}\left(\frac{\omega_o^2}{\omega^2} - 1\right)^{-1} \quad (5)$$

where ω_o is the resonant frequency of the LC circuit formed by the loop and the capacitor. It is shown from Equation (5) that, just above the frequency of resonance, the polarizability becomes negative and very large. Therefore, it is expected that a regular array of capacitive loaded metallic loops will show a negative magnetic permeability just above the frequency of resonance of the loops [11]. According to schelkunoff's [17] and Marque's [11], if two or more split rings resonator are formed in a regular array, it will show a large negative magnetic permeability just above the resonance frequency of the rings.

By separating the two rings each on a single substrate and with opposite slots as shown in **Figure 2**, a regular array of capacitive loading ring "the gap capacitance of the slot and the surface capacitance" will show a large negative magnetic permeability in the direction of the magnetic dipole. In addition to the advantage of avoiding the bianistropy, where the electric polarization of the upper half side ($y > 0$) must equal to the opposite electric polarization of the lower half side ($y < 0$) of the rings, so the design is not bianisotropic. The magnetic dipole of the proposed design resulting from the regular array of the two rings, can be expressed by:

$$\sum_{i=1}^{n} M_{x_i} = \alpha_{xx}^{mm} B_x^{ext} \quad (6)$$

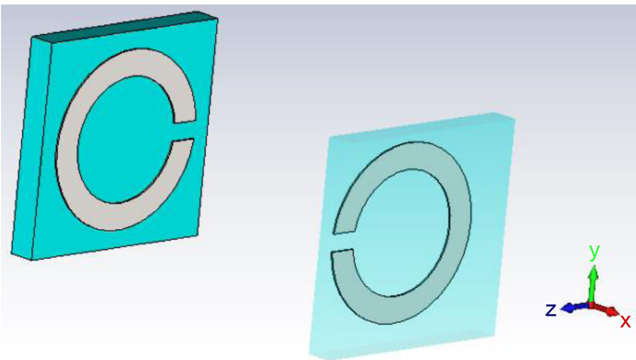

Figure 2. Two separated substrates with opposite single split ring resonator.

where, n is the number of rings in x-direction.

3. Theoretical Analysis

A rectangular waveguide is loaded by two slabs each of ε_r and negative transverse permeability μ_{tr} due to the presence of split ring resonator, the two slabs are located in the waveguide as shown in Figure 3.

The electric field E_y in the different regions is given as:

In air

$$E_y = \begin{cases} A\sin\left(k_{xo}(x)\right)e^{-i\beta z}, & 0 \le x \le m \\ B\cos\left(k_{xo}\left(\frac{a}{2}-x\right)\right)e^{-i\beta z}, & (d+m) \le x \le (a-m-d) \\ C\sin\left(k_{xo}(a-x)\right)e^{-i\beta z}, & (a-m) \le x \le a \end{cases}$$

In slab

$$E_y = \begin{cases} \left[\frac{A\sin\left(k_{xo}(m)\right)}{\sin\left(k_x(m)\right)}\right]\sin\left(k_x(x)\right)e^{-i\beta z}, & m \le x \le (d+m) \\ B\cos\left(k_{xo}\left(m+d-\frac{a}{2}\right)\right)\left[\frac{\sin\left(k_x(a-x)\right)}{\sin\left(k_x(m+d)\right)}\right]e^{-i\beta z}, & (a-m-d) \le x \le (a-m) \end{cases} \quad (7)$$

While, from the basic of Electromagnetic propagation inside a waveguide

$$E_x = 0, \quad E_z = 0, \quad H_y = 0$$

The magnetic field H is obtained from the Maxwell's equation:

$$\nabla XE = -j\omega[\mu]H$$

And, the magnetic permeability tensor is:

$$\mu = \mu_o \begin{bmatrix} \mu_{tr} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (8)$$

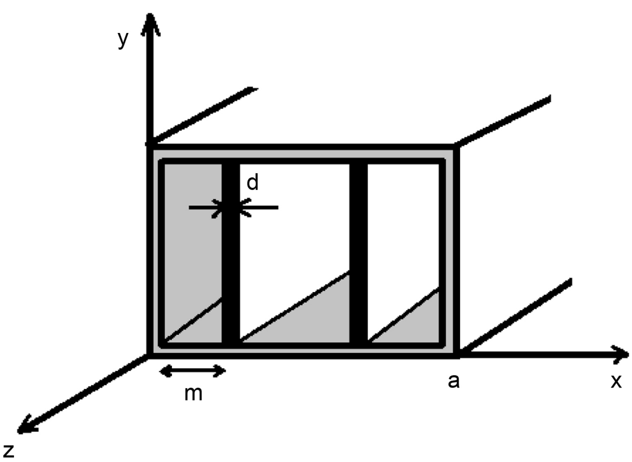

Figure 3. Two slabs with negative transverse permeability located in a waveguide.

We can assume a magnetic wall in the middle of the waveguide as shown in Figure 4, while the walls of the waveguide are electric walls.

Then, the wave equation in air region is applied to get:

$$k_{Xo}^2 + \beta^2 = k_o^2 \tag{9}$$

And the boundary condition is applied at $x = m + d$, we get:

$$\frac{\tan\left[k_X\left(m+d\right)\right]}{k_X} - \frac{\cot\left[k_{xo}\left(\frac{a}{2} - m - d\right)\right]}{k_{Xo}} = 0 \tag{10}$$

And from Maxwell's equation:

$$\nabla XH = j\omega\varepsilon E$$

Applying wave equation in slab region to get:

$$k_x^2 + \frac{\beta^2}{\mu_r} = \omega^2 \mu_o \varepsilon_o \varepsilon_r \tag{11}$$

From Equation (11), the wave propagation factor can be expressed as:

$$\beta = \sqrt{\mu_r \varepsilon_r \left(k_o^2 - \frac{k_x^2}{\varepsilon_r}\right)} \tag{12}$$

We define the cutoff frequency of the partially filled waveguide with metamaterial as f_{cp}.

It can be shown that:

$$f_{cp} > f_c = \frac{f_{co}}{\sqrt{\varepsilon_r}}$$

where f_{co} is the cutoff frequency of air-filled waveguide, therefore:

$$\beta = k_o \sqrt{\mu_r \varepsilon_r \left(1 - \left(\frac{f_{cp}}{f}\right)^2\right)} \tag{13}$$

In Equation (13), if ε_r and μ_{tr} are positive, propagation above cutoff occurs.

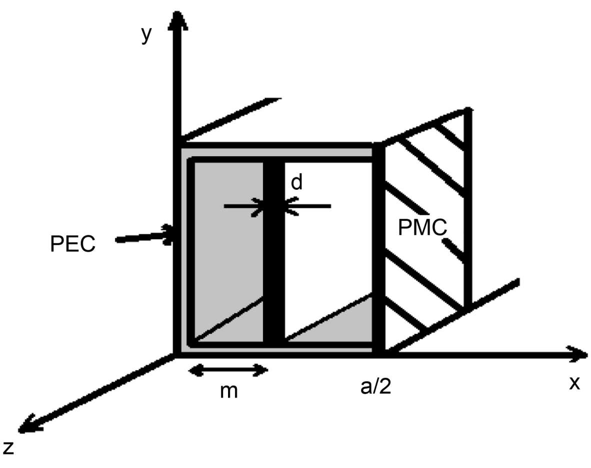

Figure 4. Equivalent waveguide.

Also if ε_r is negative, μ_{tr} must be negative for propagation to occur. The interesting case is when the μ_{tr} is negative and $f_{cp} > f$, where propagation below cut off occurs.

A simulation for the propagation constant β versus frequency is shown in **Figure 5** at different values of negative transverse permeability μ_{tr} and (m = 2.6 mm, a = 12 mm).

It seems that at a certain frequency, the propagation constant increased as the negative permeability increased. Meanwhile, at same negative permeability, the propagation constant is decreasing with the increasing of the frequency.

4. Results

We have designed two rings of opposite slots direction at resonance frequency $f_o = 8.25\ \text{GHz}$ with the following parameters, $R_i = 1.75\ \text{mm}$, $R_o = 2.5\ \text{mm}$ and slot width 0.5 mm, and etched on copper cladding substrate with thickness 0.35 mm, copper thickness 0.02 mm, and dielectric permittivity $\varepsilon_r = 2.6$. By using CST MW Studio, the simulated result of S12 is shown in **Figure 6**. The 10 db bandwidth of S12 extends from 8.1 to 8.5 GHz.

Two slabs each of ten SRRs with opposite slots direction are inserted symmetrically along the center of waveguide of dimensions (12 mm × 12 mm), the

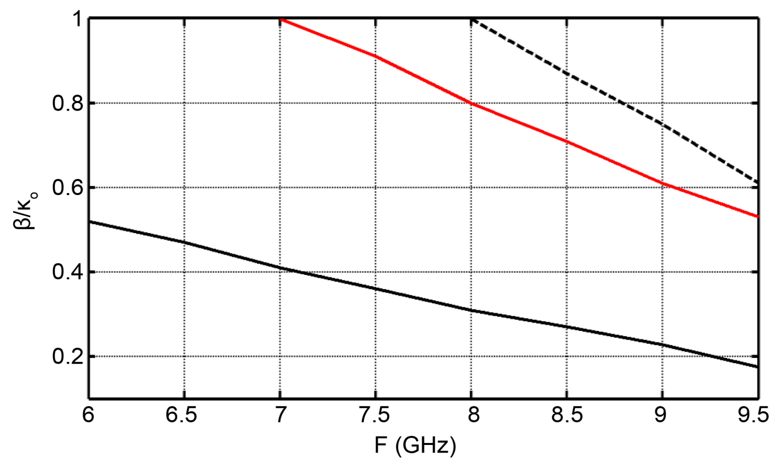

Figure 5. The propagation constant β/k_o versus resonance frequency f_o.

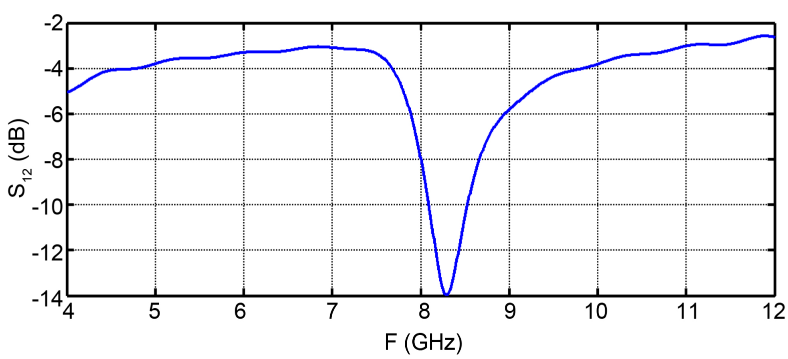

Figure 6. The result of S_{12} for a Single split ring resonator.

lattice constant is 6 mm and distance between two slabs = 6.5 mm. By using CST MW Studio, the simulated results S_{12} are shown in **Figure 7**.

In **Figure 7**, when a regular array of capacitive loaded rings are inserted in a waveguide, a large negative magnetic permeability in the direction of the magnetic dipole at ($f_o = 8.7\text{ GHz}$) occurs just above the frequency of resonance of the rings (8.25 GHz). The S12 reached 0db at no losses, while by adding losses of the substrate and the copper clad $\left(\sigma = 2\times10^7\text{ s/m}\right)$, the S_{12} reaches −4 db, while in [14] the S_{12} reached −10 db in lossless case and −28 db in lossy case.

The result of the 3 db bandwidth for the backward wave of **Figure 7** is shown in **Figure 8**.

In **Figure 8**, it is shown that a bandwidth of 95 MHz has been achieved, which is wider than the bandwidth reported in the literature [12], where it was about 70 MHz. This means that, the bandwidth of the proposed design has increased by 30% relative to that reported in literature [14] [15].

By changing the following parameters (R_i , R_o , m/a) and applying simulation program CST MW studio, we have got the influences of these parameters on the resonant frequency f_o as shown in **Table 1** and plotted in **Figure 9**.

5. Conclusion

A waveguide filled with negative permeability metamaterial SRR of resonant

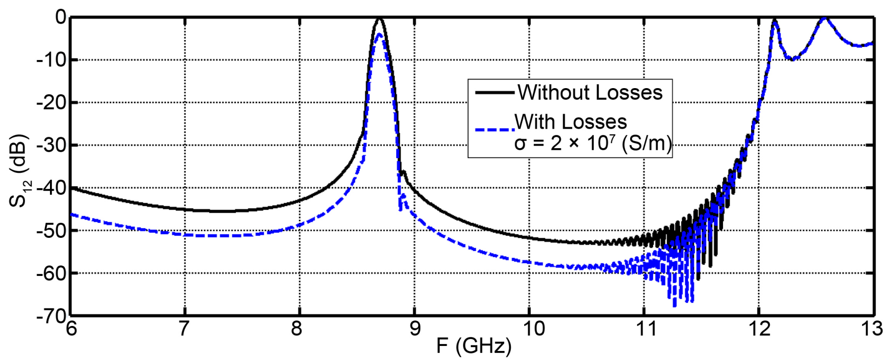

Figure 7. The result of S_{12}, the solid line of a waveguide filled with the propose design and the dotted line with adding losses to Cu Clad and substrate with $\sigma = 2\times10^7\text{ s/m}$.

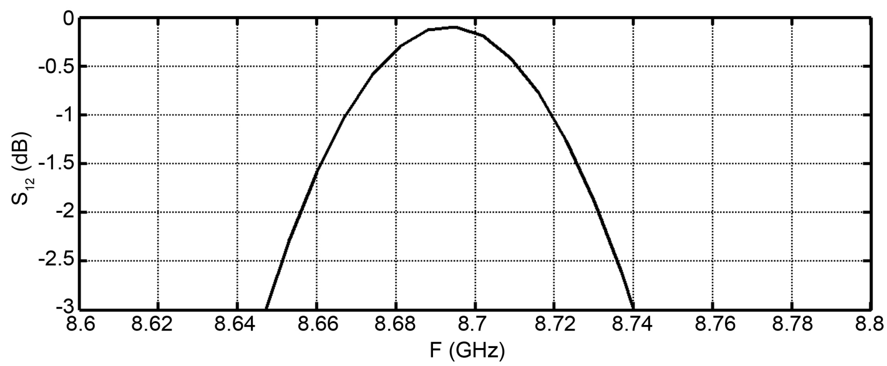

Figure 8. The result of 3 db of the backward wave for the proposed design.

Table 1. Parametric study of different resonant frequency.

	R_i	R_o	m/a	f_o
a	2 mm	2.5 mm	0.225	8.3 GHz
b	1.75 mm	2.5 mm	0.2	8.75 GHz
c	1.7 mm	2 mm	0.175	9.2 GHz
d	1.7 mm	2.4 mm	0.14	10 GHz
e	1.5 mm	2.4 mm	0.1	10.5 GHz

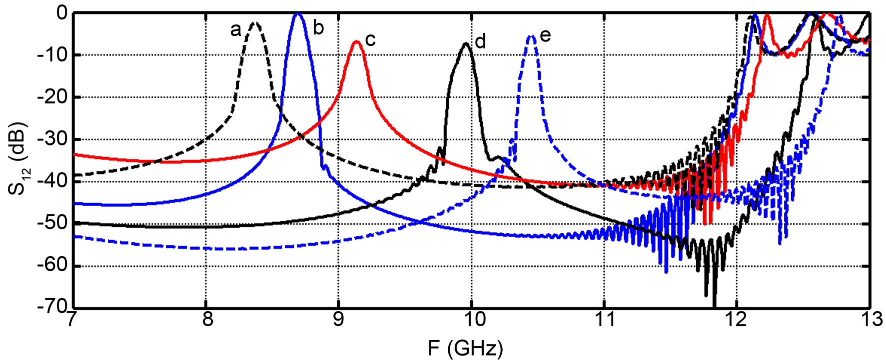

Figure 9. The result of S_{12} for each case in **Table 1**.

frequency $f_o = 7.8\text{ GHz}$ has been analyzed theoretically and simulated by CST MW Studio, and is shown to support a backward propagation below cutoff when the transverse permeability is negative. We have added two slabs symmetrically in waveguide to increase magnetic dipole of the rings. The transverse width of the waveguide can just be miniaturized to smaller than the half of an empty waveguide needed to propagate this frequency. As a result, the losses of the backward wave are decreased to 0 db and the bandwidth becomes wider than that reported in the literature by about 30%.

References

[1] Veselago, V.G. (1968) The Electrodynamics of Substances with Simultaneously Negative Values of ε and μ. *Soviet Physics Uspekhi*, **10**, 509. https://doi.org/10.1070/PU1968v010n04ABEH003699

[2] Smith, D.R., Padilla, W.J., Vier, D.C., Nemat-Nasser, S.C. and Schultz, S. (2000) Composite Medium with Simultaneously Negative Permeability and Permittivity. *Physical Review Letters*, **84**, 4184. https://doi.org/10.1103/PhysRevLett.84.4184

[3] Shelby, R.A., Smith, D.R., Nemat-Nasser, S.C. and Schultz, S. (2001) Microwave Transmission through a Two-Dimensional, Isotropic, Left-Handed Metamaterial. *Applied Physics Letters*, **78**, 489-491.

[4] Engheta, N. and Ziolkowski, R.W. (2006) Metamaterials: Physics and Engineering Explorations. IEEE Press, The Institute of Electrical and Electronics Engineers, Inc.

[5] Indhumathi, G., Karthika, G.S. and Muthulakshmi, S. (2017) Metamaterial Triangular Split Ring Resonator Antenna for Wireless Applications. *SSRG International Journal of Electronics and Communication Engineering*, (*ICRTECITA*-2017), Special Issue, March 2017.

[6] Bage, A. and Das, S. (2016) A Compact, Wideband Waveguide Bandpass Filter Using Complementary Loaded Split Ring Resonators. *Progress in Electromagnetics Research C*, **64**, 51-59. https://doi.org/10.2528/PIERC16040102

[7] Pendry, J.B., Holden, A.J., Robbins, D.J. and Stewart, W.J. (1999) IEEE Transactions on Microwave Theory and Techniques. *Magnetism from Conductors and Enhanced Nonlinear Phenomena*, **47**.

[8] Marqués, R., Martel, J., Mesa, F. and Medina, F. (2002) Left-Handed-Media Simulation and Transmission of EM Waves in Subwavelength Split-Ring-Resonator-Loaded Metallic Waveguides. *Physical Review Letters*, **89**, 183901-1-183901-4.

[9] Marqués, R., Mesa, F., Martel, J. and Medina, F. (2003) Comparative Analysis of Edge- and Broadside-Coupled Split Ring Resonators for Metamaterial Design—Theory and Experiments. *Transactions on Antennas and Propagation*, **51**, 2572-2581.

[10] Marqués, R., Medina, F. and Rachid Rafii-El-Idrissi, R. (2002) Role of Bianisotropy in Negative Permeability and Left-Handed Metamaterials. *Physical Review B*, **65**, 144440-1-14440-6.

[11] Marqués, R., Martin, F. and Sorolla, M. (2008) Metamaterials with Negative Parameters Theory, Design, and Microwave Applications. John Willey and Sons.

[12] Hrabar, S. (2006) Basic Radiation Properties of Waveguides Filled with Uniaxial Single Negative Metamaterials. *Microwave and Optical Technology Letters*, **48**, 2587-2591. https://doi.org/10.1002/mop.21993

[13] Sipus, Z. (2004) Miniaturization of Rectangular Waveguide Using Uniaxial Negative Permeability Meta-Material. *Proceedings of the 12th IEEE Mediterranean Electrotechnical Conference*, 12-15 May 2004, 495-498.

[14] Meng, F., Wu, Q., Erni, D. and And Li, L. (2011) Controllable Metamaterial-Loaded Waveguides Supporting Backward and Forward Waves. *Transactions on Antennas and Propagation*, **59**, 3400-3411. https://doi.org/10.1109/TAP.2011.2161540

[15] Hrabar, S., Bartolic, J. and Sipus, Z. (2005) Waveguide Miniaturization Using Uniaxial Negative Permeability Metamaterial. *IEEE Transactions on Antennas and Propagation*, **53**, 110-119.

[16] Mohamed, K. and Nidal, A. (2013) Rectangular Waveguide Radiator Miniaturization using Electromagnetic Infinity-Shaped Metamaterial Resonator. *IUG Journal of Natural and Engineering Studies*, **21**, 69-82.

[17] Shelkunoff, S.A. and Friis Antennas, H.T. (1966) Theory and Practice. 3rd Edition, Wiley, New York.