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Voltage Control in Smart Grids: An Approach Based on Sensitivity Theory

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ABSTRACT

Due to the development of Distributed Generation (DG), which is installed in Medium-Voltage Distribution Networks (MVDNs) such as generators based on renewable energy (e.g., wind energy or solar energy), voltage control is currently a very important issue. The voltage is now regulated at the MV busbars acting on the On-Load Tap Changer of the HV/MV transformer. This method does not guarantee the correct voltage value in the network nodes when the distributed generators deliver their power. In this paper an approach based on Sensitivity Theory is shown, in order to control the node voltages regulating the reactive power exchanged between the network and the dispersed generators. The automatic distributed voltage regulation is a particular topic of the Smart Grids.

Keywords: *Voltage Regulation, Reactive Power Injection, Distributed Generation, Smart Grids, Sensitivity Theory, Renewable Energy*

1. Introduction

Due to the development of Distributed Generation (DG), which is installed in Medium-Voltage Distribution Networks (MVDNs) such as generators based on renewable energy (*e.g.*, wind energy or solar energy), voltage control is currently a very important issue.

The voltage of MVDNs is now regulated acting only on the On-Load Tap Changer (OLTC) of the HV/MV transformer [1]. The OLTC control is typically based on the compound technique, and this method does not guarantee the correct voltage value in the network nodes when the generators deliver their power [2,3].

When a generator injects power in the network, the voltage tends to rise. In HV networks this phenomenon happens mainly when reactive power is injected, because the resistance is negligible if compared with the inductive reactance [4]. Instead in MVDNs the resistance is not negligible and the result is that an injection of active power also increases the voltage.

In other words the so-called Pq - QV decoupling [5], which is a typical of HV networks, is inexistent in MVDNs. The P variations are "coupled" with the voltage variations.

If no precautions are taken, in particular network conditions the overcome of the maximum admissible voltage can happen in any nodes.

When a generator injects power, the voltage rises in all network nodes, but some nodes are mainly influenced than others by the power injection. This influence can be obtained using a Sensitivity method.

In this paper an approach based on Sensitivity Theory is shown, in order to control the network voltage using the reactive power exchanged between network and the distributed generators. This approach allows to control the voltage in the long term period. Besides, fast-dynamic voltage disturbances are not taken into account [6].

After the theoretical analysis, a numerical example is shown, in order to validate the proposed theory.

The proposed method differs from the others used in HV networks analysis, based on the Jacobian Matrix [1,2-4] and its application is easy.

The topological proprieties that results from the theoretical analysis imply that the proposed sensitivity method can be easily implemented in automatic voltage control devices, in order to obtain the distributed voltage regulation.

The automatic voltage regulation in a distributed manner is a typical topic of the Smart Grids context.

The paper is structured in the following way. In Section 1, the proposed voltage control method is shown, and an overview on the voltage profiles with DG, are given. In Section 3, the proposed Sensitivity approach is studied, referring to a MV test network, composed by four nodes. Finally, in Section 4, a numerical application is presented, in order to validate the proposed theory.

2. The Proposed Criteria to Control the Network Voltage with Distributed Generation

Many methods can be used to control the voltage in network nodes (network voltages). The proposed method varies the reactive power exchanged between the generators and the network while maintaining the OL-TC in a fixed position for a particular load condition.

Let us suppose that the Automatic Voltage Regulator (AVR) that controls the OLTC maintains the MV bus-bar voltage at the rated value (1 p.u.), assuming that the transformer taps are adequate.

For passive grids, when no generators are connected to the MVDN, the voltage profile (VP; *i.e.*, the voltage values along a line) decreases monotonically (see profile a in **Figure 1**) due to the load absorptions. When the generators are connected and inject power into the MVDN, the nodal voltages increase and the VP is no longer monotonic, as shown in profile b in **Figure 1** (profile b). This phenomenon also occurs if generators work at unitary power factor (*i.e.*, only active power is injected due to the non-negligible network resistance) [7].

It is important to note that, in steady-state, the condition maintained at the MV busbar by the AVR decouples the MV feeders, and the result is that each feeder works without the influence of the other lines. In other words, the loads and generators connected to other feeders do not influence the VP of the considered line.

Typically, the generators installed in Smart Grids are based on renewable energy; therefore, their power-time profiles are unknown. Due to the high generated power and a possibly low load condition, the voltage in some nodes can thus exceed the maximum admissible value (V_{max}; *i.e.*, the voltage threshold [8]) defined by the standards.

Of course the voltage threshold is strictly related with the settings of the voltage relays installed in the network, *e.g.* at the generator nodes [9].

If the generators are able to control the injected or absorbed reactive power, the network voltage profiles can be modified by acting on the reactive powers. It is clear that each controllable generator needs a Generator Remote Terminal Unit (GRTU) that is connected to a central control system to set the generator reactive power, (*i.e.*, to control the exciter of the synchronous generators [1] or act on the inverter control if the generator is inverter-based) [10,11]. In this work, the central control is called the Generator Control Centre (GCC). In addition, we use a hierarchical control structure [12,13].

Let us suppose that the voltage is measured only in the generator nodes by the GRTUs. This assumption does not affect the generality of the proposed method because a Measuring Remote Terminal Unit connected to the GCC can be installed in each node that must be controlled.

When the voltage in the i^{th} node exceeds V_{max}, the GRTU installed in the same node sends the signal "Voltage Threshold Overall" (VTO) to the GCC using a communication channel. The GCC then selects the generator in the j^{th} node that has the maximum influence on the voltage of the i^{th} node, the "Best Generator" (BG), and switches it to the reactive power absorption (RPA) mode. Therefore, the voltage in the i^{th} node tends to decrease.

The problem is thus to determine the best generator and ensure that the GCC chooses it. In this work, a sensitivity-based method is proposed to select the BG.

Moreover, we suppose that the generators can only be switched in the RPA mode by the GCC by a constant power factor. Therefore, if P_j is the active power injected by the generator connected to the j^{th} node, then it absorbs the reactive power $Q_j = P_j \tan\varphi_j$ (where $\cos\varphi_j$ is the minimum power factor of the generator) when it is switched during RPA. In other words, we assume that no continuous reactive power modulation is possible.

An example of the procedure described above is shown in **Figure 2**. Let us suppose that load Ld suddenly decreases its power (for example, due to a trip) and V_2 exceed V_{max}.

The GRTUs of G2 send the signal VTO to the GCC that must choose the BG using the sensitivity method. Assuming that the BG is G1, it will be switched by the

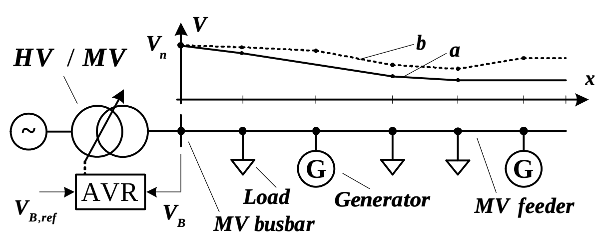

Figure 1. Voltage profiles in a MV feeder with and without Distributed Generators

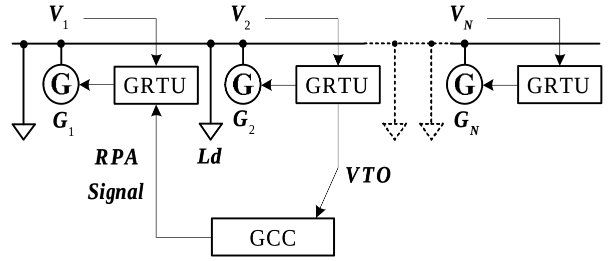

Figure 2. Voltage control using GRTU and GCC

GCC in the RPA mode; therefore, the reactive power absorbed by G1 becomes $Q_1 = P_1 \tan\varphi_1$.

As explained in the following, the GCC must know the reactive power that each controllable generator can absorb in order to choose the BG. We suppose that this information is acquired by the GCC using a polling technique on each GRTU.

3. The Proposed Sensitivity Approach

3.1 Classical Sensitivity Theory Overview

The classical sensitivity theory used in HV network analysis to perform primary and secondary voltage regulation [14] is based on the Jacobian Matrix and reveals the relationships between the nodal voltages (magnitude and phase) and the nodal power injections (active and reactive). The relationships mentioned above are represented by the following matrix expression [2]:

$$\begin{bmatrix}[\Delta E]\\ [\Delta J]\end{bmatrix} = \begin{bmatrix}\left[\frac{\partial P}{\partial V}\right] & \left[\frac{\partial P}{\partial J}\right]\\ \left[\frac{\partial Q}{\partial V}\right] & \left[\frac{\partial Q}{\partial J}\right]\end{bmatrix}^{-1} \begin{bmatrix}[1] & [0]\\ [0] & [1]\end{bmatrix} \begin{bmatrix}[\Delta P^*]\\ [\Delta Q^*]\end{bmatrix} \quad (1)$$

where $[\Delta E]$ and $[\Delta J]$ are, respectively, the nodal voltage magnitudes (rms) and phase variations corresponding to the nodal active or reactive power injections $[\Delta P^*]$ and $[\Delta Q^*]$ ($[1]$ is the identity matrix).

Equation (1) can be rewritten in the following compact form:

$$\begin{bmatrix}[\Delta E]\\ [\Delta J]\end{bmatrix} = [s] \begin{bmatrix}[\Delta P^*]\\ [\Delta Q^*]\end{bmatrix} \quad (2)$$

where:

$$[s] @ \begin{bmatrix}\left[\frac{\partial P}{\partial V}\right] & \left[\frac{\partial P}{\partial J}\right]\\ \left[\frac{\partial Q}{\partial V}\right] & \left[\frac{\partial Q}{\partial J}\right]\end{bmatrix}^{-1} \begin{bmatrix}[1] & [0]\\ [0] & [1]\end{bmatrix} \quad (3)$$

is the (injection) sensitivity matrix. The method descries above is generally valid, but its computational complexity is too high for practical voltage analysis in MVDNs. For radial networks, only the voltage magnitude is needed to control the nodal voltages. The proposed theory is easier than classical theory, and it is suitable for radial MVDNs.

3.2 The Proposed Theory

In this section, the proposed theory for choosing the BG is outlined. The method is first described in general and considers the possibility of reactive power regulation for all nodes.

After the general treatment, the analysis focuses on a realistic network in which the reactive power can only be controlled in some nodes (generator nodes).

Let us consider the network depicted in **Figure 3**, which is a four-node test MVDN.

The general loads $Ld_1 \ldots Ld_4$ are represented using constant PQ models. Positive P (or Q) corresponds to the absorbed power by the load. Negative P (or Q) corresponds to the injected power in the network *(i.e.*, the general load is really a generator). The per-phase equivalent circuit is shown in **Figure 4**.

The lines $L_{01} \ldots L_{24}$ are modeled using the RL-direct sequence equivalent circuit [15], but the shunt admittances are neglected. The node 0 represents the MV busbar, which is regulated at a constant voltage value E_0 by the AVR of the OLTC. This reference voltage coincides with the rated value $E_0 = V_n/\sqrt{3}$.

Because the busbar is regulated at E_0, we can characterize the generic node i using the difference V_{0i} between the magnitude of the busbar voltage and the node voltage E_i. In other words, we can write:

$$V_{0i} = E_0 - E_i \quad (4)$$

In radial networks, (4) can be calculated as the sum of the voltage differences between adjacent nodes from the ith node toward the MV busbar. For example, if $i = 3$ (see **Figure 4**), (4) becomes:

$$V_{03} = E_0 - E_3 \quad (5)$$

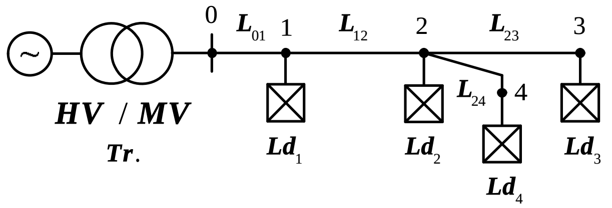

Figure 3. The considered four nodes test MVDN

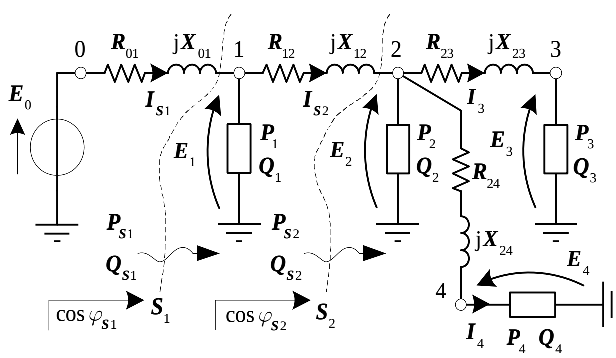

Figure 4. The per-phase equivalent circuit

By adding and subtracting E_1 and E_2 in (5), we obtain:

$$\begin{aligned} V_{03} &= (E_0 - E_1) + (E_1 - E_2) + (E_2 - E_3) \\ &= V_{01} + V_{12} + V_{23} \end{aligned} \quad (6)$$

where V_{03} is the sum of the voltage differences V_{01}, V_{12} and V_{23}.

V_{23} can be calculated considering the network parameters and the line power flows as follows:

$$\begin{aligned} V_{23} &= E_2 - E_3 \\ &= R_{23} I_3 \cos j_3 + X_{23} I_3 \sin j_3 \\ &= \frac{R_{23} E_3 I_3 \cos j_3 + X_{23} E_3 I_3 \sin j_3}{E_3} \\ &= \frac{R_{23} P_3 + X_{23} Q_3}{E_3} \end{aligned} \quad (7)$$

where $\cos j_3$, P_3 and Q_3 are the power factor and the active and reactive (per-phase) powers of the load Ld_3, respectively. I_3, R_{23} and X_{23} are the current, resistance and reactance of the line L_3.

Normally, the nodal voltages are close to the rated voltage E_n. Applying this assumption to (7) leads to:

$$V_{23} \cong \frac{R_{23} P_3 + X_{23} Q_3}{E_n} \quad (8)$$

Similarly, considering nodes 1 and 2, we can write:

$$\begin{aligned} V_{12} &= E_1 - E_2 \\ &= R_{12} I_2 \cos j_{S2} + X_{12} I_2 \sin j_{S2} \\ &= \frac{R_{12} E_2 I_2 \cos j_{S2} + X_{12} E_2 I_2 \sin j_{S2}}{E_2} \\ &\cong \frac{R_{12} P_{S2} + X_{12} Q_{S2}}{E_n} \end{aligned} \quad (9)$$

where P_{S2} and Q_{S2} are the active and reactive powers through the section S_2 and $\cos j_{S2}$ is the power factor for the same section. For P_{S2} and Q_{S2}, we can write:

$$P_{S2} = P_2 + P_3 + P_4 + P_{R23} + P_{R24} \quad (10)$$

$$Q_{S2} = Q_2 + Q_3 + Q_4 + Q_{X23} + Q_{X24} \quad (11)$$

where P_{R23} and P_{R24} are the power losses in R_{23} and R_{24}, while Q_{X23} and Q_{X24} are the reactive powers absorbed by X_{23} and X_{24}. These active and reactive losses are negligible compared to the load powers. Applying this assumption to (9), (10) and (11) leads to:

$$P_{S2} \cong P_2 + P_3 + P_4 \quad (12)$$

$$Q_{S2} \cong Q_2 + Q_3 + Q_4 \quad (13)$$

and:

$$V_{12} \cong \frac{R_{12}(P_2 + P_3 + P_4) + X_{12}(Q_2 + Q_3 + Q_4)}{E_n} \quad (14)$$

Finally, the voltage difference V_{01} is:

$$V_{01} = E_0 - E_1 \cong \frac{R_{01} P_{S1} + X_{01} Q_{S1}}{E_n} \quad (15)$$

where:

$$P_{S1} \cong P_1 + P_2 + P_3 + P_4 \quad (16)$$

$$Q_{S1} \cong Q_1 + Q_2 + Q_3 + Q_4 \quad (17)$$

are the powers through section S_1.

Using (6) with (15), (9) and (8), we can say that V_{03} is a function of all loads and active and reactive powers, *i.e.*, $P_1 \ldots P_4$ and $Q_1 \ldots Q_4$. The same observation is valid for E_3:

$$E_3 = E_0 - V_{03} = E_0 - (V_{01} + V_{12} + V_{23}) \quad (18)$$

because E_0 is constant. In other words, we can write:

$$E_3 = f(P_1, \ldots, P_4, Q_1, \ldots, Q_4) \quad (19)$$

Equation (19) shows that an active/reactive power variation (in the general j node) that is defined as:

$$\Delta P_j = P_j^f - P_j^0 \quad (20)$$

$$\Delta Q_j = Q_j^f - Q_j^0 \quad (21)$$

where P_j^f (Q_j^f) and P_j^0 (Q_j^0) are the final and initial power values, respectively, produces a voltage variation in node 3 that is defined as:

$$\Delta E_3 = E_3^f - E_3^0 \quad (22)$$

In this treatment, we only consider the reactive power variations (*i.e.*, $\Delta P_j = 0$) because we assume that only the reactive power can be used to control the node voltages.

The variation ΔE_3 can be calculated by linearizing (19) and considering only the reactive power variations. In particular, we can write:

$$\begin{aligned} \Delta E_3 = &\frac{\partial E_3}{\partial Q_1} \Delta Q_1 + \frac{\partial E_3}{\partial Q_2} \Delta Q_2 + \\ &+ \frac{\partial E_3}{\partial Q_3} \Delta Q_3 + \frac{\partial E_3}{\partial Q_4} \Delta Q_4 \end{aligned} \quad (23)$$

The terms $\partial E_i / \partial Q_j$ in (23) indicate the "gain" from the voltage variation ΔE_i in node i when a reactive power variation ΔQ_j occurs in node j. In other words, they are sensitivity terms.

According to (18), we can obtain:

$$\frac{\partial E_3}{\partial Q_2} = -\frac{X_{01} + X_{12}}{E_n}$$
$$\frac{\partial E_3}{\partial Q_3} = -\frac{X_{01} + X_{12} + X_{23}}{E_n} \qquad (24)$$
$$\frac{\partial E_3}{\partial Q_4} = -\frac{X_{01} + X_{12}}{E_n}$$

Substituting equation group (24) into (23) has important implications. If we have a reactive injection in any node, *i.e.*, $\Delta Q_j < 0$ (in this case $j = 1...4$), then $\Delta E_3 > 0$ in node 3 (*i.e*., the voltage increases). Then, if we were to reduce the voltage in any node, we must abs- orb reactive power from the network (*i.e*., $\Delta Q_j > 0$) by using, for example, the distributed generators.

If the above analysis that focuses on node 3 is extended to all network nodes, (23) has a general matrix relationship:

$$\begin{bmatrix} \Delta E_1 \\ \Delta E_2 \\ \Delta E_3 \\ \Delta E_4 \end{bmatrix} = \begin{bmatrix} \frac{\partial E_1}{\partial Q_1} & \frac{\partial E_1}{\partial Q_2} & \frac{\partial E_1}{\partial Q_3} & \frac{\partial E_1}{\partial Q_4} \\ \frac{\partial E_2}{\partial Q_1} & \frac{\partial E_2}{\partial Q_2} & \frac{\partial E_2}{\partial Q_3} & \frac{\partial E_2}{\partial Q_4} \\ \frac{\partial E_3}{\partial Q_1} & \frac{\partial E_3}{\partial Q_2} & \frac{\partial E_3}{\partial Q_3} & \frac{\partial E_3}{\partial Q_4} \\ \frac{\partial E_4}{\partial Q_1} & \frac{\partial E_4}{\partial Q_2} & \frac{\partial E_4}{\partial Q_3} & \frac{\partial E_4}{\partial Q_4} \end{bmatrix} \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_4 \end{bmatrix} \qquad (25)$$

which in a compact form yields:

$$[\Delta E] = [s_Q][\Delta Q] \qquad (26)$$

where $[s_Q]$ is the reactive sensitivity matrix, $[\Delta Q]$ is the reactive power-variations vector and $[\Delta E]$ is the nodal voltages vector.

Calculating the partial derivatives contained in $[s_Q]$, we have Equation (27).

After analyzing this form of (27), we can say that this matrix can be built using the following inspection rule:

"*The element* i, j *is the arithmetic sum of the reactance of the branches in which both the powers absorbed by node* i *and node* j *flow multiplied by* $-1/E_n$ ".

For example, in (27), the element 2, 4 is

$$-(X_{01} + X_{12})/E_n$$

because the powers delivered by node 2 and node 4 flow in branches 01 and 12.

3.3 The Choice of the Best Generator

The BG is the generator that has the greatest influence on node i, which is the node where the voltage exceeds the threshold.

Thus, after analyzing (25), we can say that the BG is the generator that maximizes the following product, which we call the "*sensitivity product*":

$$\frac{\partial E_i}{\partial Q_j}\Delta Q_j \qquad (28)$$

For example, if the node with a voltage that exceeds V_{max} is $i = 2$ and the BG is connected to node $j = 4$, the sensitivity product $(\partial E_2/\partial Q_4)\Delta Q_4$ is the highest compared to the other products contained in row 2 of the sensitivity matrix. In addition, in order to choose the BG, it is necessary to evaluate the single products (28) of the row that represents node i. Thus, the value ΔQ_j is needed and is acquired as the GCC polls the GRTUs, as stated previously.

The procedure described above suggests a way of defining the "*sensitivity table*" $[T_S]$ that contains the single sensitivity products. For the MVDN represented in **Figure 4**, $[T_S]$ takes the following form

$$\begin{bmatrix} \frac{\partial E_1}{\partial Q_1}\Delta Q_1 & \frac{\partial E_1}{\partial Q_2}\Delta Q_2 & \frac{\partial E_1}{\partial Q_3}\Delta Q_3 & \frac{\partial E_1}{\partial Q_4}\Delta Q_4 \\ \frac{\partial E_2}{\partial Q_1}\Delta Q_1 & \frac{\partial E_2}{\partial Q_2}\Delta Q_2 & \frac{\partial E_2}{\partial Q_3}\Delta Q_3 & \frac{\partial E_2}{\partial Q_4}\Delta Q_4 \\ \frac{\partial E_3}{\partial Q_1}\Delta Q_1 & \frac{\partial E_3}{\partial Q_2}\Delta Q_2 & \frac{\partial E_3}{\partial Q_3}\Delta Q_3 & \frac{\partial E_3}{\partial Q_4}\Delta Q_4 \\ \frac{\partial E_4}{\partial Q_1}\Delta Q_1 & \frac{\partial E_4}{\partial Q_2}\Delta Q_2 & \frac{\partial E_4}{\partial Q_3}\Delta Q_3 & \frac{\partial E_4}{\partial Q_4}\Delta Q_4 \end{bmatrix} \qquad (29)$$

Row i represents the node in which we want to control the voltage, and column j represents the nodes in which we can control the reactive power. The BG is the generator connected to node j that has the maximum absolute value of the sensitivity product in position i, j. By finding the maximum sensitivity product in row i,

$$[s_Q] = -\frac{1}{E_n}\begin{bmatrix} X_{01} & X_{01} & X_{01} & X_{01} \\ X_{01} & X_{01}+X_{12} & X_{01}+X_{12} & X_{01}+X_{12} \\ X_{01} & X_{01}+X_{12} & X_{01}+X_{12}+X_{23} & X_{01}+X_{12} \\ X_{01} & X_{01}+X_{12} & X_{01}+X_{12} & X_{01}+X_{12}+X_{24} \end{bmatrix} \qquad (27)$$

we automatically choose the BG because the location corresponds to column j of the maximum sensitivity product.

It is clear that, for a general network with N nodes, the sensitivity table takes the following form:

$$\begin{bmatrix} \frac{\partial E_1}{\partial Q_1}\Delta Q_1 & \frac{\partial E_1}{\partial Q_2}\Delta Q_2 & \cdots & \frac{\partial E_1}{\partial Q_N}\Delta Q_N \\ \frac{\partial E_2}{\partial Q_1}\Delta Q_1 & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial E_N}{\partial Q_1}\Delta Q_1 & \cdots & \cdots & \frac{\partial E_N}{\partial Q_N}\Delta Q_N \end{bmatrix} \qquad (30)$$

It is important to note that, if it is not possible to regulate the reactive power (*e.g.*, if in that node there is a load or a non-controllable generator) in a node j, then $\Delta Q_j = 0$ and, consequently, the sensitivity product in the position i, j of the sensitivity table is 0.

Comparing (29) with (25), we can say that each element i, j of $[T_S]$ represents the line-to-ground voltage variation in node i when a reactive power variation occurs in node j.

In the following section, a numerical example of the sensitivity method application is shown.

4. Application of the Proposed Method

The network considered in this numerical application is represented in **Figure 5**.

During normal network operation, we have four generators and eight loads. The generator and load characteristics are summarized in **Table 1** (S is the apparent power) and **Table 2**, respectively (three-phase powers are represented in these tables).

We suppose that the generators normally operate with a unitary power factor (*i.e.*, no reactive power is injected in the nodes).

The per-kilometer reactance of the cable lines is $x = 0.17\,\Omega/km$, which is a typical value for Italian MVDNs. The line lengths and parameters are summarized in **Table 3**.

Let us suppose that each generator is connected to its GRTU that measures the nodal voltage and communicates with the GCC. Moreover, let us suppose that G_5 cannot regulate the reactive power because it is not designed for this purpose. The MV busbar is regulated at the rated voltage (1 p.u.), which is 20 kV (line-to-line). In this example, the voltage threshold V_{max} is 1.05 p.u.

Using load-flow software, we calculated the voltage E in the generator nodes (nodes 4, 5, 6, and 7) for normal network operation. The results are shown in **Figure 6** (Normal Operation).

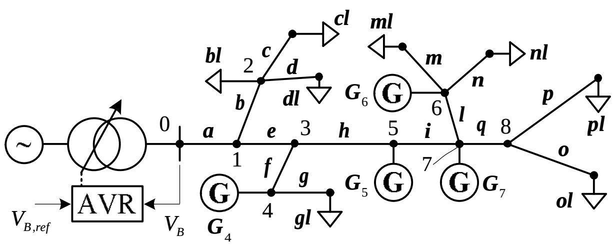

Figure 5. The network considered in the numerical application

Table 1. Loads characteristics

Load	*S* [*MVA*]	*cos*	*P* [*MW*]	*Q* [*MVAR*]
bl, nl	2	0.95	1.9	0.62
cl	7	0.92	6.44	2.74
dl	3	0.92	2.76	1.18
gl	2.08	0.95	1.99	0.62
ml	2.58	0.93	2.4	0.94
ol	1.98	0.96	1.9	0.57
pl	1.5	0.92	1.38	0.59

Table 2. Generators characteristics

Generator	*P* [*MW*]
G_4	6
G_5	1.75
G_6	4.5
G_7	3.75

Table 3. Lines parameters

Line Name	*L* [*km*]	*X* [*Ω*]
a, l, q	2	0.34
b, f, g, i, m, n	1	0.17
c, d, o, p	0.5	0.085
e	15	2.55
h	5	0.85

If line *b* trips (*e.g.*, due to a fault), loads *bl*, *cl* and *dl* are cut off from the supply, which causes the voltage to increase in the network. In particular, if the load-flow is re-computed to take into account the new network configuration, we obtain the results shown in **Figure 6** (Tripped Line).

It is important to note that, if the voltage exceeds the maximum threshold V_{max} in node 5, the GRTU connected to G_5 sends the VTO signal to the GCC that must choose the BG using the sensitivity table.

We suppose that the three-phase reactive powers absorbable by each generator that were collected from the last poll are those summarized in **Table 4**, which also contains the corresponding power factors *cosφ*. To calculate the sensitivity table, we need the single-phase powers. Therefore, the reactive powers shown in **Table 4** have to be divided by three. It is important to note that the reactive powers calculated this way correspond to ΔQ_j because Q_j^0 is zero (see (21)). The ΔQ_j values are shown in **Table 5**.

The voltage exceeds the threshold in node 5. Thus, we only consider the fifth row of the sensitivity table. According to the inspection rule mentioned above, this row is as follows:

$$\begin{bmatrix} 0 & 0 & 0 & t_{S5,4} & t_{S5,5} & t_{S5,6} & t_{S5,7} & 0 \end{bmatrix} \quad (31)$$

where the single sensitivity products $t_{Si,j}$ are:

$$t_{S5,4} = -\frac{1}{E_n}\left(X_a + X_e\right)\Delta Q_4 = -164.53\,V \quad (32)$$

$$t_{S5,5} = -\frac{1}{E_n}\left(X_a + X_e + X_h\right)\Delta Q_5 = 0\,V \quad (33)$$

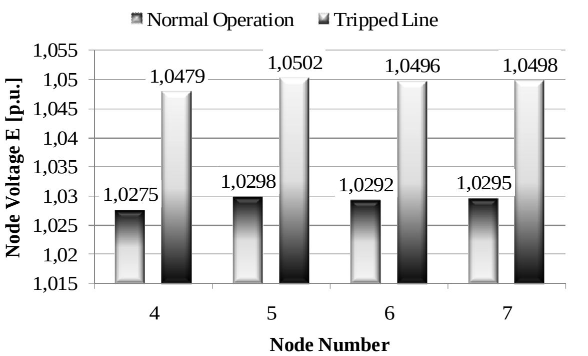

Figure 6. Load-Flow results with the Network Normal Operation

Table 4. Reactive powers absorbable by the generators

Generator	*Q* [*MVAR*]	*cosφ*
G_4	1.97	0.95
G_5	0.00	1
G_6	0.91	0.98
G_7	1.23	0.95

Table 5. Reactive Power Variations in the Generator Nodes

Generator	DQ_j [*MVAR*]
G_4	0.66
G_5	0.00
G_6	0.30
G_7	0.41

$$t_{S5,6} = -\frac{1}{E_n}\left(X_a + X_e + X_h\right)\Delta Q_6 = -98.65\,V \quad (34)$$

$$t_{S5,7} = -\frac{1}{E_n}\left(X_a + X_e + X_h\right)\Delta Q_7 = -133.07\,V \quad (35)$$

The maximum sensitivity product (in absolute value) corresponds to generator 4, (*i.e.*, $j = 4$). Thus, the BG is G_4.

Equation (32) provides important information. If G_4 performs the considered reactive power variation, the line-to-ground voltage variation in node 5 is:

$$\Delta E_5 = -164.53\,V \to -0.0142\ p.u. \quad (36)$$

Then, considering (22) (rewritten for node 5), and $E_5^0 = 1.0502\ p.u.$ from **Figure 6**, we can say that the voltage value after the reactive power variation is:

$$E_5^f = E_5^0 + \Delta E_5 = 1.036\ p.u. \quad (37)$$

which is less than the voltage threshold V_{max}.

Equation (37) shows the theoretical result obtained using the proposed method. We checked this value using load-flow software:

$$E_5^f{}_{load\ flow} = 1.034\ p.u. \quad (38)$$

The percentage error between (38) and (37) is:

$$\varepsilon_{\%} = \frac{E_5^f{}_{load\ flow} - E_5^f}{E_5^f{}_{load\ flow}} \cdot 100 = -0.19\ \% \quad (39)$$

which is negligible and demonstrates the validity of the proposed approach.

5. Conclusions

The proposed sensitivity method allows the voltage within network acting on single generators to be regulated by choosing the most effective generator on the controlled node (*i.e.*, the Best Generator). This is a very important feature in grids that have distributed generation (*e.g.*, in a Smart Grid context).

The proposed method uses a topological approach. Moreover, the sensitivity table can be constructed automatically.

In addition to the BG choice, the proposed method also evaluates the voltage in all network nodes after a reactive power variation.

After choosing the BG, but before its commutation during RPA, it is possible to verify that the voltage variation in the other nodes is tolerable for the connected loads. Moreover, it is necessary to verify that the threshold settings of the voltage relay installed in the same nodes.

When a generator is switched during RPA, it works with a non-unitary power factor; the reactive power flow increases along the lines and increases the power loss [16].

This phenomenon is negligible in HV networks because the line resistance is typically smaller than the line reactance, but is important to consider in MV networks.

Therefore, if network analysis reveals that the RPA-switching produces high losses, voltage control using the reactive power variation must only be used for temporary voltage variation mitigation (*i.e.*, during emergency conditions).

The possible future develops of this work could be focused on the optimization of the forecasted power-time profiles of the loads and generators applying both the se nsitivity approach and distributed voltage measurement.

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