Theoretical Investigation of Convergence of Max-SINR Algorithm in the MIMO Interference Network

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Abstract

Design of transceiver for interference channel (IC) is an important research area. Max-SINR algorithm is an iterative algorithm for multi-input multi-output (MIMO) IC. Nodes in the MIMO IC, work in a time division duplex mode, where half of them are equipped with M antennas while the others have N antennas. It is shown how the Max-SINR algorithm proposed by Gomadam *et al.* converges by considering an equivalent problem, *i.e.* a constrained maximization problem.

Keywords

Converge, Interference Channel, MIMO, Max-SINR

1. Introduction

To date, different approaches have been developed to address interference management. Beside the conventional methods for interference management [1], a new method termed "interference alignment" (IA) has been proposed by the researchers. The basic idea behind the IA is to fit undesirable signals into a small portion of the signal space, observed by each receiver (interference subspace), and then leave the remaining signal space free of any interference for the desired signal (signal subspace). In [1]-[12], and [13], the authors implement the IA for different scenarios.

Transceiver for multi-input multi-output (MIMO) interference channel (IC) has been designed by progressive minimization of the leakage interference, Algorithm 1 in [4]. In this scheme, the IA is achieved only at very high SNRs. The Max-SINR algorithm, Algorithm 2 in [4], is another approach to obtain IA. This technique shows significant improvements in terms of sum rate in the range of

low-to-intermediate SNRs and achieves the IA at high SNR.

The algorithm 1 seeks perfect interference alignment. In particular, it seeks to create an interference-free subspace of the required number of dimensions that is designated as the *desired* signal subspace. However, Algorithm 1 makes no attempt to maximize the desired signal power within the desired signal subspace. In fact, Algorithm 1 does not depend at all on the direct channels through which the desired signal arrives at the intended receiver. Therefore, while the interference is eliminated within the desired space, no coherent combining gain (array gain) for the desired signal is obtained with Algorithm 1. While this is optimal as all signal powers approach infinity, it is not optimal in general at intermediate SNR values. Therefore, Algorithm 2 may be designed which will perform better than Algorithm 1 at intermediate SNR values.

Authors show that Algorithm 1 converges and convergence of Max-SINR is not presented. Since all the iterative algorithms have meaning when converges. No convergence proof has been presented in literature for Max SINR algorithm. In this paper, convergence issue of Max-SINR is investigated. It is shown how the Max-SINR algorithm proposed by Gomadam *et al.* converges by considering an equivalent problem, *i.e.* a constrained maximization problem.

Convergence of robust MMSE is shown in [14] theoretically. Total fraction of interference leakage to received signal parameter is used to show convergence numerically. To validate the correctness of the proposed proof, numerical convergence behavior of the Max SINR is compared with robust MMSE.

2. System Model

In a K-user MIMO IC, transmitter j and receiver k have M and N antennas, respectively. Independent symbols D^j with power P are sent by the jth transmitter. Channel matrices between transmitter j and receiver k is denoted by H^{kj}. The received signal at receiver k is expressed by

$$Y^k = \sum_{j=1}^{K} H^{kj} X^j + Z^k \tag{1}$$

where X^j is the $M \times 1$ signal vector transmitted by the transmitter j and $Z^k \sim CN(0, N_0 I)$ is additive white Gaussian noise (AWGN) vector. Beam-forming strategy is used based on the interference alignment. In particular, transmitter j precodes symbol vector by using the precoder matrix. V^j is the $M \times D^j$ precoder matrix. Columns of V^j, v_d^j, are unit norm vectors. Receiver k estimates the transmitted symbol vector s^k by using the interference suppression matrix U^k. The received signal is filtered by U^k as $\overline{Y^k} = U^{k\dagger} Y^k$.

Each node works in a time division duplex (TDD) mode. At two consecutive time slots, first, nodes on the left-hand side send the data to the nodes on the right-hand side. Then the role of nodes is switched and the nodes on the left-hand side receive the data, as illustrated in Figure 1.

The relation between the original and reciprocal channel matrices is $\overleftarrow{H^{jk}} = H^{kj\dagger}$ [4]. Since the receivers of the reciprocal channel play the roles of
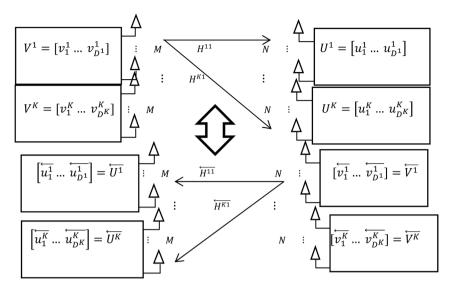
Figure 1. System model.

original network's transmitters and vice versa, then $\overleftarrow{V^k} = U^k$ and $\overleftarrow{U^j} = V^j$.

According to the system model, the SINR value for the dth data stream at k^{th} receiver is expressed by (where, $\|.\|$ denotes the Euclidian norm.)

$$SINR_d^k = \frac{P\left\|u_d^{k\dagger}H^{kk}v_d^k\right\|^2}{P\sum_{j=1}^{K}\sum_{m=1}^{D^j}\left\|u_d^{k\dagger}H^{kj}v_m^j\right\|^2 - P\left\|u_d^{k\dagger}H^{kk}v_d^k\right\|^2 + N_0\left\|u_d^k\right\|^2} \quad (2)$$

Alternatively $SINR_d^k$ can be expressed by

$$SINR_d^k = \frac{u_d^{k\dagger}T_d^k u_d^k}{u_d^{k\dagger}\left[S^k - T_d^k + N_0 I\right]u_d^k} \quad (3)$$

where $S^k = P\sum_{j=1}^{K}\sum_{m=1}^{D^j}H^{kj}v_m^j v_m^{j\dagger}H^{kj\dagger}$ and $T_d^k = PH^{kk}v_d^k v_d^{k\dagger}H^{kk\dagger}$ denote, respectively, the covariance matrix of all data streams observed by the receiver k and covariance matrix of the dth desirable data stream.

3. Proof of Convergence of Max-SINR Algorithm

Max-SINR algorithm is presented in algorithm 2 in [4]. It is repeated in Table 1.

Maximizing (3) over u_d^k can be written as follow

$$\max \frac{u_d^{k\dagger}Gu_d^k}{u_d^{k\dagger}Fu_d^k} \quad (4)$$

where matrices are $G = G^\dagger = T_d^k \geq 0$, and $F = F^\dagger = S^k - T_d^k + N_0 I > 0$. It is shown in [15] that the optimization problem in (4) is equivalent to

$$\max u_d^{k\dagger}Gu_d^k,$$
$$\text{s.t. } u_d^{k\dagger}Fu_d^k = 1. \quad (5)$$

For the equivalent problem, *i.e.* constrained maximization in (5), Lagrangian function is $l\left(u_d^k, \lambda\right) = u_d^{k\dagger}Gu_d^k + \lambda\left(1 - u_d^{k\dagger}Fu_d^k\right)$. Lagrange conditions are

Table 1. Max SINR algorithm.

1: Start with $V^k : M^k \times D^k$.
2: Begin iteration.
3: Compute interference plus noise covariance matrix for B_d^k for stream d at receiver k, $\forall k \in \{1, \cdots, K\}, d \in \{1, \cdots, D^k\}$.
4: Calculate receive combining vectors u_d^k at receiver k, $\forall k \in \{1, \cdots, K\}, d \in \{1, \cdots, D^k\}$.
5: Reverse the communication direction and use the receive combining vectors as precoding vectors: $\overleftarrow{V^k} = U^k, \forall k \in \{1, \cdots, K\}$.
6: In the reciprocal network, compute interference plus noise covariance matrix for $\overleftarrow{B_d^k}$ for stream d at receiver k, $\forall k \in \{1, \cdots, K\}, d \in \{1, \cdots, D^k\}$.
7: Calculate receive combining vectors $\overleftarrow{u_d^k}$, $\forall k \in \{1, \cdots, K\}, d \in \{1, \cdots, D^k\}$.
8: Reverse the communication direction and use the receive combining vectors as precoding vectors: $V^k = \overleftarrow{U^k}, \forall k \in \{1, \cdots, K\}$.
9: Repeat until convergence, or the number of iterations reaches a limit defined earlier.

$\dfrac{\partial l\left(u_d^k, \lambda\right)}{\partial u_d^k} = 0$ and $\dfrac{\partial l\left(u_d^k, \lambda\right)}{\partial \lambda} = 0$. The solution is denoted by u_d^{k*} and Lagrange

multiplier by λ^*. It is also shown in [15] that u_d^{k*} is the eigenvector corresponding to the maximal eigenvalue of $F^{-1}G$ and λ^* is $u_d^{k*\dagger} G u_d^{k*}$.

Therefore, the unit vector that maximizes (3), is given by

$$u_d^k = \vartheta\left[F^{-1}G\right] \tag{6}$$

where operator $\vartheta[.]$ denotes the eigenvector corresponding to the maximal eigenvalue of a matrix.

The convergence of the algorithm is proved by considering total Lagrangian function of all data streams in the network $\sum_{k=1}^{K} \sum_{d=1}^{D^j} l\left(u_d^k, \lambda\right)$. The metric is defined in (7). The function is unchanged in the original and reciprocal networks since the transmit and receive filters change their roles. Therefore, each step [4, Algorithm 2] in the algorithm increases the value of the function. This implies that the algorithm converges.

$$\max_{\substack{V^j \text{ and } U^K \\ \forall j \text{ and } k \in \mathcal{K}}} metric = \sum_{k=1}^{K} \sum_{d=1}^{D^j} l\left(u_d^k, \lambda\right) \tag{7}$$

Accordingly:

$$\max_{\substack{U^K \\ \forall k \in \mathcal{K}}} metric = \sum_{k=1}^{K} \sum_{d=1}^{D^j} \max_{u_d^k} l\left(u_d^k, \lambda\right) \tag{8}$$

In other words, given $V^j \; \forall j \in \mathcal{K}$, Step 4 [4, Algorithm 2] increases the value of (7) over all possible choices of $U^k \; \forall k \in \mathcal{K}$. The filter $\overleftarrow{U^j}$ computed in Step 7 [4, Algorithm 2], based on $\overleftarrow{V^k} = U^k$, also maximizes the metric in the reciprocal channel (9).

$$\max_{\substack{\overleftarrow{U^j} \\ \forall j \in \mathcal{K}}} \overleftarrow{metric} = \sum_{j=1}^{K} \sum_{d=1}^{D^j} \overleftarrow{l}\left(\overleftarrow{u_d^j}, \overleftarrow{\lambda}\right) = \sum_{j=1}^{K} \sum_{d=1}^{D^j} \overleftarrow{u_d^j}^{\dagger} \overleftarrow{G} \overleftarrow{u_d^j} + \overleftarrow{\lambda}\left(1 - \overleftarrow{u_d^j}^{\dagger} \overleftarrow{F} \overleftarrow{u_d^j}\right) \tag{9}$$

Since $\overleftarrow{V^k} = U^k$ and $\overleftarrow{U^j} = V^j$, the metric remains unchanged in the original and reciprocal networks, according to following equation:

$$\overline{metric} = \sum_{j=1}^{K} \sum_{d=1}^{D^j} u_d^{j\dagger} T_d^j u_d^j + \sum_{j=1}^{K} \sum_{d=1}^{D^j} \lambda_d^j \left(1 + u_d^{j\dagger} \left[T_d^j - N_0 I\right] u_d^j\right)$$
$$- P \sum_{j=1}^{K} \sum_{d=1}^{D^j} \sum_{k=1}^{K} \sum_{m=1}^{D^j} \lambda_d^j u_m^{k\dagger} H^{kj} v_d^j v_d^{j\dagger} H^{kj\dagger} u_m^k = metric \qquad (10)$$

Therefore, Step 7 [4, Algorithm 2] also can increase the value of (7). Since the value of (7) is monotonically increased after every iteration, convergence of the algorithm is guaranteed.

4. Simulation Results

Simulation results to validate the correctness of the proposed proof are based on fraction of interference leakage to the received signal parameter [14]. **Figure 2** and **Figure 3** show total fraction of interference leakage to received signal. Since convergence of robust MMSE is shown in [14], its total parameter is plotted in **Figure 2** and **Figure 3** as well. Convergence behavior of the algorithm in comparison with robust MMSE implies a convergent scheme.
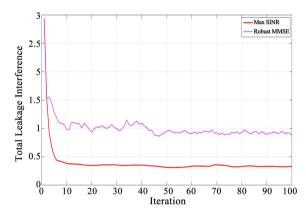

Figure 2. Sum of fraction of interference leakage to received signal at each user. $M = N = 3$, $D = 1$, $K = 4$ interference network with $\sigma^2 = 0.1$, and $\dfrac{P}{N_0} = 10$ dB.
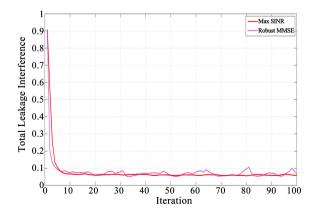

Figure 3. Sum of fraction of interference leakage to received signal at each user. $M = 3$, $N = 4$, $D = 2$, $K = 2$ interference network with $\sigma^2 = 0.1$, and $\dfrac{P}{N_0} = 10$ dB.

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