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Flow and Heat Transfer of Basalt Melt in the Feeder of the Smelter Furnace

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Abstract

The flow and heat transfer of the basalt melt in the boundary layer on a flat plate is considered. The conditions of formation of the layer and the intensity of heat transfer are determined. A self-similar analysis using the symmetry method was used. A system of ordinary differential equations in self-similar form is obtained. The fluid flow and heat transfer of molten basalt at a laminar steady-state flow in the feeder furnaces are numerically researched. The term "protective layer" on the interface "basalt melt-lining" is introduced. The dependences for the calculation of dimensionless shear stresses and the Nusselt number on the lining surface are obtained. The conditions of rational organization of the technological process of basalt melt feeding in the furnace feeder are formulated.

Keywords

Basalt Fiber, Smelter Furnace, Energy Efficiency, Heat Technological Processes

1. Introduction

It is known that the technology of production of high-performance thermal insulation based on staple basalt fiber has the potential for modernization [1]. In particular, due to the rational organization of hydrodynamic and heat transfer processes in the elements of technological equipment. According to current concepts, basalt melt is a multiphase heterogeneous system containing liquid, crystalline and gaseous phases. The structure and action of this system are largely determined by the temperature and pressure at which it is located [2] [3]. The peculiarity of the basalt melt is its ability to restore in time its complex crystal structure from the liquid state when found in a certain temperature range—between the lower and upper limits of crystallization. In relation to the basalts of Ukrainian deposits [4], the critical temperature range—between the lower and upper crystallization limits—is between 900˚C and 1270˚C. The occurrence in the smelter furnace of conditions under which the basalt melt acquires a critical temperature leads to the process of secondary crystallization of the melt. At the point of contact between the melt and the furnace lining, secondary crystallization contributes to the formation of a protective layer that impedes the destruction of the latter—the advantage—guaranteeing the long-term operation of the lining. On the other hand, the occurrence of conditions under which it is possible to recrystallize the basalt melt at the place of forming coarse fibers is a problem that leads to a decrease in the quality of fibers, a violation of the technological process, a decrease in plant productivity and, as a consequence, the increased production price of the final product. Studying the flow and heat transfer of basalt melt on the flat surface gives qualitative and quantitative characteristics of the process, and opens up the possibility of developing technical proposals for process control.

The theme of fluid flow and heat transfer on a flat plate—both in the approximation of the boundary layer, analytically, and in its full formulation, numerically—is widely discussed in numerous publications and monographs [5] [6]. The peculiarity of the formulation under consideration is that the classical phenomenological approach is applied to a deliberately heterogeneous system, with a significant dependence of viscosity on temperature—basalt melt. As will be shown below, the viscosity of the basalt melt in the temperature range under consideration varies by 14 orders of magnitude. The purpose of the study was to quantify the processes of flow and heat transfer of the basalt melt and obtain the characteristics of resistance, heat transfer intensity, heat flux values, and sizes of the zone of secondary crystallization of the melt. The task was solved analytically and numerically.

2. Study of the Flow and Heat Transfer of Basalt Melt on a Flat Plate

2.1. Physical Model

The influence of basalt melt viscosity on the flow and heat transfer in the laminar boundary layer on a flat surface under the condition of the formation of a sedentary liquid layer near the wall is considered. The viscosity of the basalt melt is a nonlinear function that depends significantly on the temperature [7] (**Figure 1**).

2.2. Mathematical Model

The stationary problem of flow and heat transfer in the boundary layer of a liquid on a flat surface is considered. For this problem a system of equations of hydrodynamics and heat transfer in a boundary layer on a flat plate in a two-dimensional approximation [5] is considered

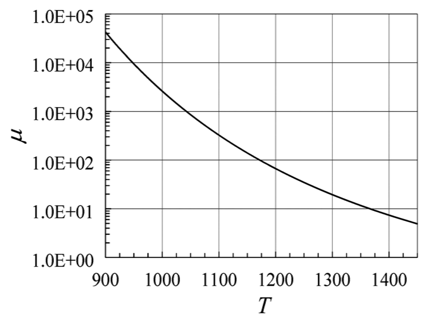

Figure 1. Dependence of the coefficient of dynamic viscosity on temperature for basalt from the Ivanova Dolina deposit.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right). \tag{3}$$

Equations (1)-(3) are analyzed under the following boundary conditions

$$y=0: u=v=0; T=T_w \tag{4}$$

$$y\to\infty: u=U_\infty; v=0; T=T_\infty \tag{5}$$

To solve the system of differential equations with partial derivatives (1)-(3), we convert them to ordinary differential equations. For this purpose, we use symmetry analysis (analysis of Lie groups) [8]. The algorithm for symmetry analysis for heat and mass transfer and hydrodynamics problems is given in [9]. The transformation of the system of Equations (1)-(3) into a system of ordinary differential equations is considered in the work [10]. The following system of ordinary differential equations in self-similar form was obtained

$$\frac{1}{2}\cdot f(\eta)\cdot f''(\eta)+M'\left[\Theta(\eta)\right]\cdot\Theta'(\eta)\cdot f''(\eta)+M\left[\Theta(\eta)\right]\cdot f^{(3)}(\eta)=0, \tag{6}$$

$$\frac{1}{2}\cdot Pr_\infty\cdot f(\eta)\cdot\Theta'(\eta)+\Theta''(\eta)=0. \tag{7}$$

The self-similar variable was defined by the expression

$$\eta=y\sqrt{\frac{U_\infty\rho}{\mu_\infty x}}. \tag{8}$$

The velocity u in self-similar variables are represented in the form

$$u=U_\infty f'(\eta). \tag{9}$$

The velocity v in self-similar variables are represented in the form

$$v=\frac{1}{2}\sqrt{\frac{U_\infty\cdot\mu_\infty}{\rho\cdot x}}\left(f'(\eta)\cdot\eta-f(\eta)\right). \tag{10}$$

Self-similar function for temperature is introduced as

$$T = T_w + (T_\infty - T_w) \cdot \Theta(\eta). \quad (11)$$

The dependence of the dynamic viscosity coefficient of the liquid on the temperature is described by the equation

$$\mu(T) = 3.83 \times 10^{-3} \cdot \mathrm{e}^{\frac{2.41\times10^6}{(\sqrt[4]{T})^7}}. \quad (12)$$

In the self-similar form equation (12) is presented as

$$\mu(\Theta(\eta)) = \mu_\infty \cdot M[\Theta(\eta)], \quad (13)$$

where μ_∞ is liquid viscosity outside the boundary layer.

The system of Equations (6), (7) was solved under the following boundary conditions

$$\eta = 0: f(0) = 0;\ f'(0) = 0;\ \Theta(0) = 0; \quad (14)$$

$$\eta \to \infty: f'(\infty) = 1, \Theta(\infty) = 1. \quad (15)$$

2.3. Results

Based on the system of Equations (6), (7) with boundary conditions (14), (15), numerical simulation of fluid motion and heat transfer in the boundary layer over a flat surface in a wide range of parameters μ and Θ is performed. Numerical solutions were obtained using the 4th order Runge-Kutta method [11].

For the numerical solution of system (6), (7), we replace the variables and perform its transformation

$$f(\eta) = y_1,\ f'(\eta) = \frac{\mathrm{d}y_1}{\mathrm{d}\eta} = y_2,\ f''(\eta) = \frac{\mathrm{d}y_2}{\mathrm{d}\eta} = y_3, \Theta(\eta) = t_1, \Theta'(\eta) = \frac{\mathrm{d}t_1}{\mathrm{d}\eta} = t_2. \quad (16)$$

The system takes the form

$$\frac{\mathrm{d}y_1}{\mathrm{d}\eta} = y_2, \quad (17)$$

$$\frac{\mathrm{d}y_2}{\mathrm{d}\eta} = y_3, \quad (18)$$

$$\frac{\mathrm{d}y_3}{\mathrm{d}\eta} = -\frac{1}{2M[t_1]} \cdot y_1 \cdot y_3 - \frac{M'[t_1]}{M[t_1]} \cdot t_2 \cdot y_3, \quad (19)$$

$$\frac{\mathrm{d}t_1}{\mathrm{d}\eta} = t_2, \quad (20)$$

$$\frac{\mathrm{d}t_2}{\mathrm{d}\eta} = -\frac{1}{2} \cdot Pr_\infty \cdot y_1 \cdot t_2. \quad (21)$$

The system was closed by the following boundary conditions

$$\eta = 0,\ f(0) = y_1(0) = 0,\ f'(0) = y_2(0) = 0, \Theta(0) = t_1(0) = 0; \quad (22)$$

the quantities $f''(0) = y_3(0)$ and $\Theta'(0) = t_2(0)$ were determined by the shooting method [11], with the aim of satisfying conditions (15).

The grid spacing along η was taken equal to $h = 1.0e-06$.

The results of the calculations are presented in the graphical form. The increased viscosity of the medium and its dependence on the temperature on the surface of the plate significantly affects the velocity distribution in the boundary layer. In **Figure 2**, the comparison of the classical solution of Blasius for a self-similar velocity function $f'(\eta)$ with the solution of the system (6), (7) for different temperatures on the wall surface is displayed. The deformation of the velocity profiles within the temperature boundary layer at decrease of T_w is clearly visible. Curve 1 is the classic Blasius solution obtained for isothermal flow on the plate surface. Curves 2-4 are the solutions obtained for the nonisothermal flow of the melt on the plate at different surface temperatures—1300˚C, 1100˚C, 900˚C, respectively. The melt temperature and the Prandtl number in the free flow are T_∞ = 1450˚C; Pr_∞ = 5180. Accepted conventions are also relevant to **Figure 3**.

To calculate the shear stress on the wall, the dependence of the second derivative of the self-similar velocity function on the wall $f''(\eta)_{\eta=0}$ on the wall temperature T_w is obtained (**Figure 3**).

The dimensionless shear stress on the wall is determined by the expression

$$\frac{1}{2}c_f = \frac{f''(\eta)_{\eta=0}}{\sqrt{Re_x}}. \tag{23}$$

Tabulated values $f''(\eta)_{\eta=0}$ is presented in **Table 1**.

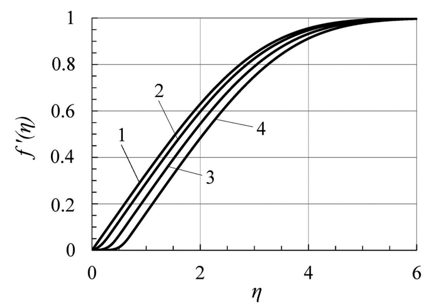

Figure 2. Distribution of velocity profiles in the boundary layer at: 1—Blasius solution; 2—T_w = 1300˚C; 3—T_w = 1100˚C; 4—T_w = 900˚C; T_∞ = 1450˚C; Pr_∞ = 5180.

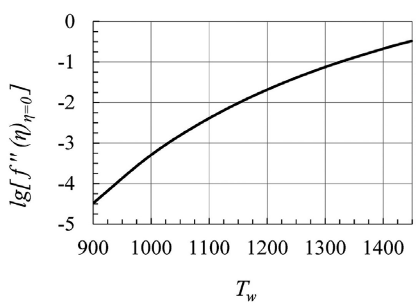

Figure 3. Dependence of the second derivative of the self-similar function on the wall temperature T_w (˚C): T_∞ = 1450˚C; Pr_∞ = 5180.

Table 1. Dependence of the second derivative of the self-similar function on the wall temperature.

$f''(\eta)_{\eta=0}$	3.285E-05	5.108E-04	4.095E-03	2.071E-02	0.0750	0.2123	0.3291
T_w, ˚C	900	1000	1100	1200	1300	1400	1449

Figure 4 shows the dependence of the self-similar temperature function on the self-similar coordinate. The classical solution obtained by Levek is compared with the solution of the system (6), (7) for different temperatures on the wall surface. As the temperature on the wall decreases, the temperature distribution in the temperature boundary layer approaches the linear one, which is typical for non-movable layers.

As already noted, near a flat surface with a temperature $T_w < T_\infty$ basalt melt forms a sedentary layer that protects the surface from destruction. We formalize the concept of "sedentary layer". We will term the melt layer near the surface, in which the velocity of the medium does not exceed 1% of the velocity in the undisturbed melt flow a protective layer. Using the dependence of the self-similar velocity function on the self-similar coordinate **Figure 2**, we calculate the value of the protective layer formed on a flat surface for different temperatures T_w. The calculated area is selected taking into account the actual geometric dimensions of the melting plants. The results of calculations are presented in graphical form in **Figure 5**. It should be noted that in the temperature range T_w = 900˚C - 1100˚C the thickness of the protective layer exceeds the size of the feeder in the melting plant. Based on this, it was concluded that it was not advisable to maintain such temperatures. On the other hand, in the temperature range T_w = 1300˚C - 1400˚C the size of the protective layer of the same order with the geometric dimensions of the feeder in the melting plant and is sufficient to fulfill its technological purpose. The specified temperature range is advisable to choose as the determining, when calculating the value of the heat flux density on the surface of the lining.

The dependence of the first derivative of the self-similar temperature function at $\eta = 0$ on the temperature on the wall surface, in order to calculate the intensity of heat exchange on the plate is shown in **Figure 6**. Traditionally, the heat transfer intensity on a flat surface is described by the dimensionless Nusselt number. In relation to the problem under consideration, the Nusselt number is determined by the expression

$$Nu_x = \left(\frac{\mathrm{d}\Theta}{\mathrm{d}\eta}\right)_{\eta=0} \cdot \left(Re_x\right)^{1/2}, \tag{24}$$

where $\left(\frac{\mathrm{d}\Theta}{\mathrm{d}\eta}\right)_{\eta=0}$ is obtained from the solution of the system (6), (7)

$$\left(\frac{\mathrm{d}\Theta}{\mathrm{d}\eta}\right)_{\eta=0} = 0.218 \cdot \mathrm{e}^{2.269\times10^{-3} T_w}. \tag{25}$$

The average heat flux density on the wall q_{ave} is determined by integrating an expression for the local heat flux density

$$q_x = Nu_x \cdot \left(\frac{\lambda_\infty}{x}\right) \cdot \left(T_\infty - T_w\right), \tag{26}$$

$$q_{ave} = \frac{1}{L}\int_0^L q_x \mathrm{d}x. \tag{27}$$

Figure 7 shows the dependence of the average heat flux density q_{ave} on the temperature on the wall T_w.

As it can be noted, the q_{ave} dependence on the T_w temperature is nonlinear.

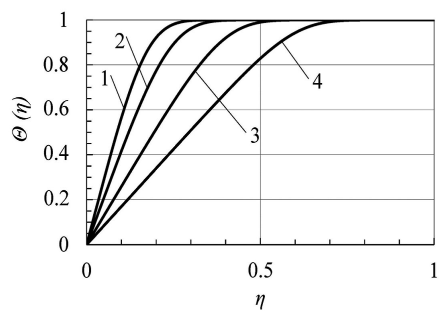

Figure 4. Distribution of temperature in the boundary layer at: 1—Leveque solution; 2—T_w = 1300˚C; 3—T_w = 1100˚C; 4—T_w = 900˚C; T_∞ = 1450˚C; Pr_∞ = 5180.

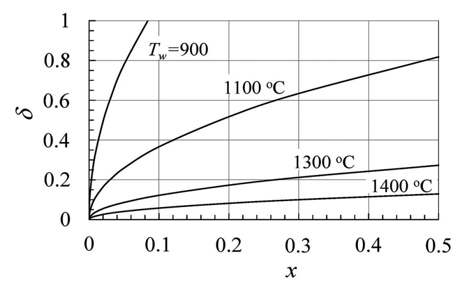

Figure 5. Dependence of the thickness of the protective layer δ (m) on the x (m) coordinate (for different temperatures on the wall T_w (˚C)): T_∞ = 1450˚C; Pr_∞ = 5180.

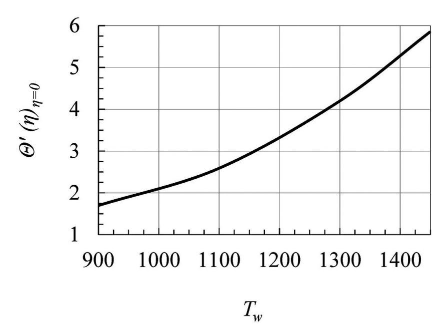

Figure 6. Dependence of the first derivative of the self-similar function on the temperature on the wall T_w (˚C): T_∞ = 1450˚C; Pr_∞ = 5180.

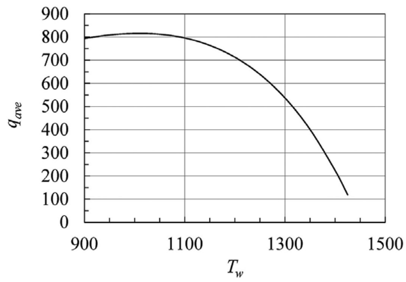

Figure 7. Dependence of the average heat flux density q_{ave} (W/m^2) on the temperature on the wall T_w (˚C): T_∞ = 1450˚C; Pr_∞ = 5180.

The resulting nonlinearity is explained by the change in the heat transfer mechanism in the layer. Heat transfer changes from predominantly convective (molar), at a temperature on the surface of the wall close to the temperature of the undisturbed flow, to predominantly conductive (molecular) transfer, at surface temperatures below 1100˚C.

Its maximum value of 813 W/m^2 q_{ave} reaches at a temperature on the surface T_w = 1007˚C.

Of practical interest are the values of q_{ave} corresponding to the temperature range on the wall surface T_w = 1300˚C - 1400˚C. They constitute, respectively, q_{ave} = 532 - 218 W/m^2.

3. Conclusions

The concept of "protective layer" in relation to the sedentary area of the melt on the interface "basalt melt-lining" is formulated. The thermophysical conditions determining the formation and size of the protective layer on the interface "basalt melt-lining" are identified. The temperature range on the wall surface is 1300˚C - 1400˚C. The corresponding range of heat flux densities is 532 - 218 W/m^2.

The dependences for the calculation of dimensionless shear stresses and the Nusselt number on the interface "basalt melt-lining" are obtained.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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