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Exact Traveling Wave Solutions of Equalwidth Equation

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Abstract

Combining the principle of homogeneous balance method, the exp function expansion method and traveling wave transformation method are applied to the Equalwidth equation to obtain the exact solution of the Equalwidth equation. The obtained solutions include trigonometric functions, hyperbolic functions and rational functions. The method can solve the exact traveling wave solutions of other nonlinear evolution equations.

Keywords

Expansion Method, Homogeneous Balance Principle, Travelling Wave Solution

1. Introduction

Nonlinear differential equations [1] are widely used in various fields of science and engineering, especially in mathematical biology, which is also widely involved in biological mathematics, nonlinear optics, fluid mechanics, optical fiber and chemical dynamics. Development equations have been widely used in biology and mechanics, but in order to further explain many physical phenomena in life, many scholars have spent a lot of energy in the construction of exact solutions. In recent years, many effective methods have been putting forward. For example, Tanh functions method is also widely involved in physics. Similarly, in the field of mathematics, it is very important to find the exact traveling wave solution of a nonlinear development equation, and many scholars have made great achievements in the process of solving nonlinear equations [2].

There are many methods to solve nonlinear equations, such as separation of variables method, homogeneous equilibrium method, etc. In this paper, a new method called $\exp(-\varphi(\xi))$ expansion is adopted [3], which has been applied to solve precise solutions of multiple equations [4] [5]. This paper will continue to use this method to solve the exact solution of the generalized Equalwidth equation [6]. Thus, the equation is obtained in the form of $u(\xi)=\alpha_m\left(\exp(-\varphi(\xi))\right)^m+\alpha_{m-1}\left(\exp(-\varphi(\xi))\right)^{m-1}+\cdots$ new traveling wave solutions [7] [8] [9] [10].

2. Solving Steps Of Expansion Method (exp(-$\varphi(\xi)$))

For the $\exp(-\varphi(\xi))$ expansion method, we first consider a nonlinear development equation, which depends on independent variables x and t, dependent variable $u=u(x,t)$, and has the following general form

$$P(u,u_t,u_x,u_{tt},u_{xt},u_{xx},\cdots)=0, \tag{1}$$

where $u=u(x,t)$ is the unknown function and P is a polynomial in terms of $u=u(x,t)$ and its partial derivatives.

The specific solving process can be divided into the following steps.

Step 1: make the traveling wave transformation, let

$$u(x,t)=u(\xi),\xi=x-vt. \tag{2}$$

where v is the wavespeed. By substituting Equation (2) into Equation (1), we can change Equation (1) into an equation of $u=u(\xi)$

$$Q(u,u',u'',\cdots)=0, \tag{3}$$

Step 2: According to the $\left(\exp(-\varphi(\xi))\right)$ expansion, suppose that we can express the solution of Equation (3) in the following polynomial form of

$$u(\xi)=\alpha_m\left(\exp(-\varphi(\xi))\right)^m+\alpha_{m-1}\left(\exp(-\varphi(\xi))\right)^{m-1}+\cdots, \tag{4}$$

where $\varphi(\xi)$ satisfies the following equation

$$\varphi'(\xi)=\exp(-\varphi(\xi))+\mu\exp(\varphi(\xi))+\lambda, \tag{5}$$

From the plus or minus of $\lambda^2-4\mu$, we can divide the solution of Equation (5) into the following five cases:

Case 1: If $\lambda^2-4\mu>0$ and $\mu\neq 0$, then the solution to Equation (5) is

$$\varphi_1(\xi)=\ln\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\sqrt{\lambda^2-4\mu}/2(\xi+C)\right)-\lambda 2\mu\right), \tag{6}$$

Case 2: If $\lambda^2-4\mu<0$ and $\mu\neq 0$, then the solution to Equation (5) is

$$\varphi_2(\xi)=\ln\left(\frac{\sqrt{4\mu-\lambda^2}\tan\left(\sqrt{4\mu-\lambda^2}/2(\xi+C)\right)-\lambda}{2\mu}\right), \tag{7}$$

Case 3: If $\lambda^2-4\mu=0,\mu=0$ and $\lambda\neq 0$, then the solution to Equation (5) is

$$\varphi_3(\xi)=-\ln\left(\frac{\lambda}{\cosh(\lambda(\xi+C))+\sinh(\lambda(\xi+C))-1}\right), \tag{8}$$

Case 4: If $\lambda^2-4\mu=0,\mu\neq 0$ and $\lambda\neq 0$, then the solution to Equation (5) is

$$\varphi_4(\xi) = \ln\left(-\frac{2(\lambda(\xi+C)+2)}{\lambda^2(\xi+C)}\right), \tag{9}$$

Case 5: If $\lambda^2 - 4\mu = 0, \mu = 0$ and $\lambda = 0$, then the solution to Equation (5) is

$$\varphi_5(\xi) = \ln(\xi + C). \tag{10}$$

where $a_m \neq 0, \cdots, C, \lambda, \mu$ is a constant to be determined, and then the desired m is obtained by homogeneous equilibrium method through the highest derivative term and nonlinear term in Equation (3).

Step 3: substitute Equation (4) into Equation (3), and then use Equation (5) to transform the left side of Equation (3) into a polynomial of $(\exp(-\varphi(\xi)))$. Then combine similar terms, and equal the coefficients of each polynomial to zero, so as to obtain a system of equations.

Step 4: after solving the system of equations and determining the constant $\alpha_m, \cdots, C, \lambda, \mu$ substitute the general solution (6)-(10) of Equation (5) into (4) to obtain the new traveling wave solution of nonlinear evolution Equation (1).

In this section, we study the traveling wave solutions of the generalized Equal width equation on the premise of finding the exact solution of the Equal width equation. The obtained solution is richer and more efficient. The obtained solutions include trigonometric functions, hyperbolic functions and Rational function. Since the algorithm is fast and efficient, the $(\exp(-\varphi(\xi)))$ method can be extended to physics, engineering, and other nonlinear sciences.

3. The Traveling Wave Solution of Equalwidth Equation Is Generalized

The traveling wave solution of the generalized Equalwidth equation will be solved by using the above method,

$$u_t + u^n u_x + u u_{xxt} = 0, \tag{11}$$

In order to get the traveling wave solution of Equation (11), set
$u(x,t) = u(\xi), \xi = x - vt$.
where v is the wave velocity, you get

$$-vu' + u^n u' - vuu''' = 0, \tag{12}$$

Then, the highest derivative term and nonlinear term of $u^n u'$ and vuu''' in Equation (12) are carried out by means of homogeneous equilibrium method. We get

$$mn + m + 1 = m + m + 3 \Rightarrow m = \frac{2}{n-1},$$

1) When $n = 2$, we have $m = 2$.

2) When $n = 3$, we have $m = 1$.

3.1. The Traveling Wave Solutions of the Generalized Equalwidth Equation in the Case *n* = 2

When $n = 2$, we have $m = 2$. Equation (12) becomes

$$-vu'+u^2u'-vuu'''=0. \tag{13}$$

Let's assume that Equation (13) has a solution of the following form

$$u(\xi)=\alpha_2\left(\exp(-\varphi(\xi))\right)^2+\alpha_1\left(\exp(-\varphi(\xi))\right)+\alpha_0,\alpha_1\neq 0, \tag{14}$$

where $\varphi(\xi)$ satisfies Equation (5), $\alpha_2,\alpha_1,\alpha_0,\lambda$ and μ are non-zero constants, and then using Equations (14) and(5), we can get

$$u^2(\xi)=\alpha_0^2+\alpha_1^2e^{-2\varphi}+\alpha_2^2e^{-4\varphi}+2\alpha_0\alpha_1e^{-\varphi}+2\alpha_0\alpha_1e^{-2\varphi}+2\alpha_1\alpha_2e^{-3\varphi}, \tag{15}$$

$$u'(\xi)=-2\alpha_2e^{-3\varphi}-(2\alpha_2+\alpha_1)e^{-2\varphi}-(\alpha_1\lambda+2\alpha_2\mu)e^{-\varphi}-\mu\alpha_1, \tag{16}$$

$$\begin{aligned}u'''(\xi)=&6\alpha_2e^{-4\varphi}+6\mu\alpha_2e^{-2\varphi}+6\lambda\alpha_2+4\alpha_2e^{-3\varphi}+4\lambda^2\alpha_1e^{-3\varphi}\\&+3\alpha_1\lambda e^{-2\varphi}+\left(2\alpha_1\mu+\alpha_1\lambda^2\right)e^{-\varphi}+\alpha_1\mu\lambda.\end{aligned} \tag{17}$$

Substitute (14), (15), (16) and (17) into (13), we can get

$$\begin{aligned}&\left(24v\alpha_2^2-2\alpha_2^3\right)e^{-7\varphi}+\left(54v\lambda\alpha_2^2-2\lambda\alpha_2^3+30v\alpha_1\alpha_2-5\alpha_1\alpha_2^2\right)e^{-6\varphi}\\&+\left(38v\lambda^2+40v\mu\alpha_2^2+66v\lambda\alpha_1\alpha_2-2\mu\alpha_2^2-5\lambda\alpha_1\alpha_2^2+24v\alpha_0\alpha_2+6v\alpha_1^2\right.\\&\left.-4\alpha_0\alpha_2^2-4\alpha_1^2\alpha_2\right)e^{-5\varphi}+\left(12v\lambda^3\alpha_2^2+52v\mu\lambda\alpha_2^2+48v\mu\alpha_1\alpha_2+54v\lambda\alpha_0\alpha_2\alpha_1^2\right.\\&\left.-5\mu\alpha_1\alpha_2^2-4\lambda\alpha_0\alpha_2^2-4\lambda\alpha_1^2\alpha_2-\alpha_1^3\right)e^{-4\varphi}+\left(22v\mu\lambda^2\alpha_2^2+9v\lambda^3\alpha_1\alpha_2\right.\\&\left.+68v\mu\lambda\alpha_1\alpha_2+7v\lambda^2\alpha_1^2+40v\mu\alpha_0+8v\mu\alpha_1^2+\mu\alpha_1^2\alpha_2-\alpha_1^3-4\alpha_0^2\alpha_2+6v\alpha_2\right)e^{-3\varphi}\\&+\left(6v\lambda\mu^2\alpha_2^2+22v\mu\lambda^2\alpha_1\alpha_2+\lambda^3\alpha_1^2+52v\mu\lambda\alpha_0\alpha_2+12v\mu\lambda\alpha_1^2+6v\lambda^2\alpha_0\alpha_1\right.\\&\left.-2\lambda\alpha_0^2\alpha_2-\alpha_0^2\alpha_1+2v\alpha_1\right)e^{-2\varphi}+\left(12v\lambda\mu^2\alpha_1\alpha_2+8v\mu\lambda^2\alpha_0\alpha_2+v\mu\lambda^2\alpha_1^2\right.\\&\left.+16v\mu^2\alpha_0\alpha_2+v\mu^2\alpha_1^2-2\mu\alpha_0\alpha_1^2-\lambda\alpha_0^2\alpha_1+2v\mu\alpha_2+4v\lambda\alpha_1\right)e^{-\varphi}\\&+6v\mu^2\lambda\alpha_0\alpha_2+v\mu\lambda^2\alpha_0\alpha_1+2v\mu^2\alpha_0\alpha_1-\mu\alpha_0^2\alpha_1+v\mu\alpha_1.\end{aligned}$$

Now we have the following algebraic system of equations for $\alpha_2,\alpha_1,\alpha_0,\lambda$ and μ.

$$24v\alpha_2^2-2\alpha_2^3=0,$$

$$54v\lambda\alpha_2^2-2\lambda\alpha_2^3+30v\alpha_1\alpha_2-5\alpha_1\alpha_2^2=0,$$

$$\begin{aligned}&38v\lambda^2+40v\mu\alpha_2^2+66v\lambda\alpha_1\alpha_2-2\mu\alpha_2^2-5\lambda\alpha_1\alpha_2^2\\&+24v\alpha_0\alpha_2+6v\alpha_1^2-4\alpha_0\alpha_2^2-4\alpha_1^2\alpha_2=0,\end{aligned}$$

$$\begin{aligned}&8v\lambda^3\alpha_2^2+52v\mu\lambda\alpha_2^2+45v\lambda^2\alpha_1\alpha_2+48v\mu\alpha_1\alpha_2+54v\lambda\alpha_0\alpha_2+12v\lambda\alpha_1^2\\&-5\mu\alpha_1\alpha_2^2-4\lambda\alpha_0\alpha_2^2-4\lambda\alpha_1^2\alpha_2+6v\alpha_0\alpha_1-6\alpha_0\alpha_1\alpha_2-\alpha_1^3=0,\end{aligned}$$

$$\begin{aligned}&14v\mu\lambda^2\alpha_2^2+9v\lambda^3\alpha_1\alpha_2+16v\mu^2\alpha_2^2+60v\mu\lambda\alpha_1\alpha_2+38v\lambda^2\alpha_0\alpha_2+7v\lambda^2\alpha_1^2\\&+40v\mu\alpha_0\alpha_2+8v\mu\alpha_1^2+12v\lambda\alpha_0\alpha_1-4\mu\alpha_0\alpha_2^2-4\mu\alpha_1^2\alpha_2+6\lambda\alpha_0\alpha_1\alpha_2\\&-\lambda\alpha_1^3-2\alpha_0^2\alpha_2-2\alpha_0\alpha_1^2+2v\alpha_2=0,\end{aligned}$$

$$\begin{aligned}&6v\lambda\mu^2\alpha_2^2+15v\mu\lambda^2\alpha_1\alpha_2+8v\lambda^3\alpha_0\alpha_2+v\lambda^3\alpha_1^2+18v\mu^2\alpha_1\alpha_2+52v\mu\lambda\alpha_0\alpha_2\\&+8v\mu\lambda\alpha_1^2+7v\lambda^2\alpha_0\alpha_1+8v\mu\alpha_0\alpha_1-6\mu\alpha_0\alpha_1\alpha_2-\mu\alpha_1^3-2\lambda\alpha_0^2\alpha_2\\&-2\lambda\alpha_0\alpha_1^2+2v\lambda\alpha_2-\alpha_0^2\alpha_1+v\alpha_1=0,\end{aligned}$$

$$\begin{aligned}&6v\lambda\mu^2\alpha_1\alpha_2+14v\mu\lambda^2\alpha_0\alpha_2+v\mu\lambda^2\alpha_1^2+v\lambda^3\alpha_0\alpha_1+16v\mu^2\alpha_0\alpha_2+2v\mu^2\alpha_1^2\\&+8v\mu\lambda\alpha_0\alpha_1-2\mu\alpha_0^2\alpha_2-2\mu\alpha_0\alpha_1^2-\lambda\alpha_0^2\alpha_1+2v\mu\alpha_2+v\lambda\alpha_1=0,\end{aligned}$$

$$6\nu\mu^2\lambda\alpha_0\alpha_2 + \nu\mu\lambda^2\alpha_0\alpha_1 + 2\nu\mu^2\alpha_0\alpha_1 - \mu\alpha_0^2\alpha_1 + \nu\mu\alpha_1 = 0.$$

We can solve the above equations

$$\text{v} = \nu,$$
$$\alpha_0 = \lambda^2\nu^3 - 8\mu\nu^3, \alpha_1 = 12\nu\lambda, \alpha_2 = 12\text{v}. \quad (18)$$

where v is wave velocity, λ and μ are arbitrary constants, if we substitute $\alpha_0, \alpha_1, \alpha_2$ from (18) into (14), we get

$$u(\xi) = 12\nu\left(\exp(-\varphi(\xi))\right)^2 + 12\nu\lambda\left(\exp(-\varphi(\xi))\right) + \lambda^2\nu^3 - 8\mu\nu^3, \alpha_0, \alpha_1, \alpha_2 \neq 0,$$
(19)

The above is the solution formula of Equation (13). Accordingly, we substitute Equation (6) to (10) into Equation (19), and then we can get the new five types of traveling wave solutions of Equation (13).

Case 1: If $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$, then the traveling wave solution to Equation (11) is

$$u(\xi) = 12\text{v}\left(\frac{-2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + C)\right) + \lambda}\right)^2 + \frac{24\nu\mu\lambda}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + C)\right) + \lambda} + \lambda^2\nu^3 - 8\mu\nu^3,$$

C is an arbitrary constant.

Case 2: If $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$, then the traveling wave solution to Equation (11) is

$$u(\xi) = 12\nu\left(\frac{2\mu}{-\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(\xi + C)\right) - \lambda}\right)^2 + \frac{24\nu\mu\lambda}{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu - \lambda^2 4\mu}}{2}(\xi + C)\right) - \lambda} + \lambda^2\nu^3 - 8\mu\nu^3,$$

C is an arbitrary constant.

Case 3: If $\lambda^2 - 4\mu > 0, \mu = 0$ and $\lambda \neq 0$, then the traveling wave solution to Equation (11) is

$$u(\xi) = 12\nu\left(\frac{\lambda}{\exp(\lambda(\xi + C)) - 1}\right)^2 + \frac{-24\nu\lambda^2}{\exp(\lambda(\xi + C)) - 1} + \lambda^2\nu^3,$$

C is an arbitrary constant.

Case 4: If $\lambda^2-4\mu=0, \mu\neq 0$ and $\lambda\neq 0$, then the traveling wave solution to Equation (11) is

$$u(\xi)=12v\left(\frac{\lambda^2(\xi+C)}{2(\lambda(\xi+C)+2)}\right)^2+\frac{6v\lambda^3(\xi+C)}{\lambda(\xi+C)+2}+\lambda^2v^3-8\mu v^3.$$

C is an arbitrary constant.

Case 5: If $\lambda^2-4\mu=0, \mu=0$ and $\lambda=0$, then the traveling wave solution to Equation (11) is

$$u(\xi)=12v\left(\frac{1}{\xi+C}\right)^2+\frac{12v\lambda}{\xi+C}.$$

C is an arbitrary constant.

3.2. The Traveling Wave Solutions of the Generalized Equalwidth Equation in the Case *n* = 3

When $n=3$, we have $m=2$. Equation (20) becomes

$$-vu'+u^3u'-vuu'''=0. \tag{20}$$

Let's assume that Equation (13) has a solution of the following form

$$u(\xi)=\alpha_1(\exp(-\varphi(\xi)))+\alpha_0, \alpha_1\neq 0, \tag{21}$$

where $\varphi(\xi)$ satisfies Equation (5), $\alpha_1,\alpha_0,\lambda$ and μ are non-zero constants. Then, by applying Equation (21) and (5), $u'(\xi),u'''(\xi),u^2(\xi),u^3(\xi)$ can be obtained respectively. By substituting them into Equation (20), we can obtain the following algebraic equations concerning $\alpha_1,\alpha_0,\lambda$ and μ.

$v\mu\lambda^2\alpha_0+2v\mu^2\alpha_0-\mu\alpha_0^3+v\mu=0,$

$v\mu\lambda^2\alpha_1+v\lambda^3\alpha_0+2v\mu^2\alpha_1+8v\mu\lambda\alpha_0-3\mu\alpha_0^2\alpha_1-\lambda\alpha_0^3+v\lambda=0,$

$v\lambda^3\alpha_1+v\mu\lambda\alpha_1+7v\lambda^2\alpha_0-3\mu\alpha_0^2\alpha_1-\lambda\alpha_0^3+v\lambda=0,$

$-\lambda\alpha_1^3+12v\lambda\alpha_1-3\alpha_0\alpha_1^2+6v\alpha_0=0$,

$7v\lambda^2\alpha_1-\mu\alpha_1^3-3\lambda\alpha_0\alpha_1^2+8v\mu\alpha_1+12v\lambda\alpha_0-3\alpha_0^2\alpha_1=0,$

$-\alpha_1^3+6v\alpha_1=0.$

We can solve the above equations

$v=v, \alpha_1=\pm\sqrt{6v},$

1) When $\alpha_1=\sqrt{6v}$, we have $\alpha_0=\frac{\lambda\sqrt{6v}}{2}$.

2) when $\alpha_1=-\sqrt{6v}$, we have $\alpha_0=\frac{3\lambda\sqrt{6v}}{4}$.

where v is wave velocity and λ is an arbitrary constant, if α_0,α_1 is substituted into (21), then

$$u_1(\xi)=\sqrt{6v}(\exp(-\varphi(\xi)))+\frac{\lambda\sqrt{6v}}{2}, \tag{22}$$

$$u_2(\xi)=-\sqrt{6v}(\exp(-\varphi(\xi)))+\frac{3\lambda\sqrt{6v}}{4}. \tag{23}$$

The above is the solution formula of Equation (20). Accordingly, we substitute Equation (6) to (10) into Equations (22) and (23) respectively, and then we can get the new five types of traveling wave solutions of Equation (20).

Case 1: If $\lambda^2-4\mu>0$ and $\mu\neq 0$, then the traveling wave solution to equation (20) is

$$u_1(\xi)=\frac{-2\mu\sqrt{6\nu}}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C)\right)+\lambda}+\frac{\lambda\sqrt{6\nu}}{2},$$

$$u_2(\xi)=\frac{2\mu\sqrt{6\nu}}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C)\right)+\lambda}+\frac{3\lambda\sqrt{6\nu}}{4}.$$

C is an arbitrary constant.

Case 2: If $\lambda^2-4\mu<0$ and $\mu\neq 0$, then the traveling wave solution to Equation (20) is

$$u_1(\xi)=\frac{2\mu\sqrt{6\nu}}{-\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right)-\lambda}+\frac{\lambda\sqrt{6\nu}}{2},$$

$$u_2(\xi)=\frac{-2\mu\sqrt{6\nu}}{-\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right)-\lambda}+\frac{3\lambda\sqrt{6\nu}}{4}.$$

C is an arbitrary constant.

Case 3: If $\lambda^2-4\mu>0, \mu=0$ and $\lambda\neq 0$, then the traveling wave solution to Equation (20) is

$$u_1(\xi)=\frac{\lambda\sqrt{6\nu}}{\exp(\lambda(\xi+C))-1}+\frac{\lambda\sqrt{6\nu}}{2},$$

$$u_2(\xi)=\frac{-\lambda\sqrt{6\nu}}{\exp(\lambda(\xi+C))-1}+\frac{3\lambda\sqrt{6\nu}}{4}.$$

C is an arbitrary constant.

Case 4: If $\lambda^2-4\mu=0, \mu\neq 0$ and $\lambda\neq 0$, then the traveling wave solution to Equation (20) is

$$u_1(\xi)=\frac{\lambda^2\sqrt{6\nu}(\xi+C)}{2(\lambda(\xi+C)+2)}+\frac{\lambda\sqrt{6\nu}}{2},$$

$$u_2(\xi)=\frac{-\lambda^2\sqrt{6\nu}(\xi+C)}{2(\lambda(\xi+C)+2)}+\frac{3\lambda\sqrt{6\nu}}{4}.$$

C is an arbitrary constant.

Case 5: If $\lambda^2 - 4\mu = 0, \mu = 0$ and $\lambda = 0$, then the traveling wave solution to Equation (20) is

$$u_1(\xi) = \frac{\sqrt{6\nu}}{\xi + C} + \frac{\lambda\sqrt{6\nu}}{2},$$

C is an arbitrary constant.

4. Summary

1) This paper gives a description of the $\exp(-\phi(\xi))$ method and applies it to the generalized Equalwidth equation.

2) Some new exact solutions of the given equation are obtained, including trigonometric functions, hyperbolic functions and rational functions.

Because the algorithm is fast and effective, $\exp(-\phi(\xi))$ methods can be extended to physical mathematics, engineering and other nonlinear sciences.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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