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Mode-Dependent Finite-Time H_∞ Filtering for Stochastic Nonlinear Systems with Markovian Switching

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Abstract

This paper addresses the problem of finite-time H_∞ filter design for a class of non-linear stochastic systems with Markovian switching. Based on stochastic differential equations theory, a mode-dependent finite-time H_∞ filter is designed to ensure finite-time stochastic stablility (FTSS) of filtering error system and satisfies a prescribed H_∞ performance level in some given finite-time intervals. Moreover, sufficient conditions are presented for the existence of a finite-time H_∞ filter for the stochastic system under consideration by employing the linear matrix inequality technique. Finally, the explicit expression of the desired filter parameters is given.

Keywords

Stochastic Systems, H_∞ Filter, Finite-Time Stability, Linear Matrix Inequalities (LMIS)

1. Introduction

Since filtering plays an important role in control systems, signal processing and communication, there has been a rapidly growing interest in filter designing due to its advantages over the traditional Kalman filtering. In the past few years, many contributions on filtering for stochastic systems can be found in the literature [1]-[14], because it is an important research topic and has found many practical applications. In [1], a H_∞ filter was designed for nonlinear stochastic systems. H_∞ filtering problems for discrete-time nonlinear stochastic systems were addressed in [2]. Delay-dependent H_∞ filtering for discrete-time singular systems and fuzzy discrete-time systems were reported respectively in [3] [4] [5] [6]. In [7], a H_∞ filter was designed for discrete-time systems with stochastic incomplete measurement and mixed delays. Optimal filter was studied for Itô-stochastic continuous-time systems in [8]. Dissipativity-based filtering and H_∞ filtering were presented for fuzzy switched systems respectively in [9] [10] [11]. Fault detection filtering and distributed filter were proposed for fault detection filtering for nonlinear stochastic systems in [12] [13]. In [14], event-based variance-constrained H_∞ filter wfas reported for stochastic parameter systems.

As is well known, the previously mentioned literature was based on Lyapunov asymptotic stability which focuses on the steady-state behavior of plants over an infinite-time interval. But in many practical systems, it is only required that the system states remain within the given bounds. In these cases, the introduction of finite-time stability or short-time stability was needed, which has caused extensive attention [15]-[23]. The problem of finite-time stability and stabilization for a class of linear systems with time delay was addressed in [15]. In [16], the sufficient conditions were achieved for the finite-time stability of linear time-varying systems with jumps. The problem of robust finite-time stabilization for impulsive dynamical linear systems was investigated in [17]. In [18], fuzzy control method was adopted to solve finite-time stabilization of a class of stochastic system. A robust finite-time filter was established for singular discrete-time stochastic system in [19]. Finite-time H_∞ filtering was proposed respectively for T-S fuzzy systems, switched systems, nonlinear singular systems, Itô stochastic Markovian jump systems in [20] [21] [22] [23]. Motivated by the contributions mentioned above, we investigated the mode-dependent finite-time filtering problems for stochastic nonlinear systems, which could be used to detect generation of residuals for fault diagnosis problems.

This paper will study the H_∞ filtering problem for a class of Markov Jump stochastic systems with Lipschitz nonlinearlity. The main purpose of this study is to construct a H_∞ filter such that the resulting filter error augmented system is FTSS. The sufficient condition for FTSS of the filter error system is obtained by constructing the Lyapunov-Krasovskii functional candidate combined with LMIs. We present an approach to design the desired FTSS filter.

This paper is organized as follows. Some corresponding definitions and lemmas and the problem formulation are introduced in Section 2. In Section 3, we give a sufficient condition for FTSS of the mentioned filtering error system in terms of LMIs. Moreover, an approach of a finite-time H_∞ filter is presented. Some conclusions are drawn in section 4.

We use R^n to denote the n-dimensional Euclidean space. The notation $X > Y$ (respectively, $X \geq Y$), where X and Y are real symmetric matrices, means that the matrix $X - Y$ is positive definite (respectively, positive semi-definite). I and 0 denote the identity and zero matrices with appropriate dimensions. $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ denotes the maximum and the minimum of the eigenvalues of a real symmetric matrix Q. The superscript T denotes the transpose for vectors or matrices. The symbol * in a matrix denotes a term that is defined by symmetry of the matrix.

2. Preliminaries

Consider a class of Itô stochastic nonlinear system with Markovian switching, which can be described as follows:

$$\begin{aligned} \mathrm{d}x(t) = & \left[A(\eta_t)x(t)+F(\eta_t)f(x(t))+A_1(\eta_t)v(t)\right]\mathrm{d}t \\ & +\left[B(\eta_t)x(t)+G(\eta_t)g(x(t))+B_1(\eta_t)v(t)\right]\mathrm{d}w(t) \end{aligned} \tag{1}$$

$$\mathrm{d}y(t)=\left[C(\eta_t)x(t)+C_1(\eta_t)v(t)\right]\mathrm{d}t+\left[D(\eta_t)x(t)+D_1(\eta_t)v(t)\right] \tag{2}$$

$$z(t)=L(\eta_t)x(t), x(0)=x_0 \tag{3}$$

where, $x(t)\in R^n$, $y(t)\in R^m$, $v(t)\in R^p$, $z(t)\in R^q$ are state vector, measurement, external disturbance, and controlled output respectively, where $v(t)$ satisfies the constraint condition with respect to the finite-time interval $[0,T]$

$$\int_0^T v^{\mathrm{T}}(t)v(t)\le d, d\ge 0, \tag{4}$$

and $w(t)\in R$ is a standard Wiener process satisfying $\Xi\{\mathrm{d}w(t)\}=0$, $\Xi\{\mathrm{d}w^2(t)\}=\mathrm{d}t$, which is assumed to be independent of the system mode $\{\eta_t, t\ge 0\}$. The random form process $\{\eta_t\}$ is a continuous-time discrete-state Markov process taking values in a finite set $N\triangleq\{1,2,\cdots,s\}$. The set N comprises the operation modes of the system. The transition probabilities for the process $\{\eta_t\}$ are defined as

$$p_{ij}=prob(\eta_{t+\Delta t}=j\,|\,\eta_t=i)=\begin{cases}\sigma_{ij}\Delta t+o(\Delta t), i\ne j,\\ 1+\sigma_{ii}\Delta t+o(\Delta t), i=j,\end{cases} \tag{5}$$

where $\Delta t>0$, $\lim_{\Delta t\to 0}\left(\frac{o(\Delta t)}{\Delta t}\right)=0$ and $\sigma_{ij}\ge 0$ for $i\ne j$ is the transition probability rate from mode i at time t to mode j at time $t+\Delta t$ and $\sigma_{ii}=-\sum_{j=1,j\ne i}^{s}\sigma_{ij}\le 0$.

For each possible value of $\eta_t=i, i\in N$ in the succeeding discussion, we denote the matrices with the ith mode by

$$A_i\triangleq A(\eta_t),\ F_i\triangleq F(\eta_t),\ A_{1i}\triangleq A_1(\eta_t),$$
$$B_i\triangleq B(\eta_t),\ G_i\triangleq G(\eta_t),\ B_{1i}\triangleq B_1(\eta_t),$$
$$C_i\triangleq C(\eta_t),\ C_{1i}\triangleq C_1(\eta_t),\ D_i\triangleq D(\eta_t),\ D_{1i}\triangleq D_1(\eta_t),\ L_i\triangleq L(\eta_t),$$

where $A_i, F_i, A_{1i}, B_i, G_i, B_{1i}, C_i, C_{1i}, D_i, D_{1i}, L_i$ for any $i\in N$ are known constant matrices of appropriate dimensions.

Assumption 1. The nonlinear functions $f(x(t))$ and $g(x(t))$ satisfy the following quadratic inequalities:

$$\left\|f(x(t))\right\|^2\le\mu_1^2\left\|x(t)\right\|^2,\ \left\|g(x(t))\right\|^2\le\mu_2^2\left\|x(t)\right\|^2. \tag{6}$$

We now consider the following filter for system (1) - (3):

$$\mathrm{d}\hat{x}(t)=A_{fi}\hat{x}(t)+B_{fi}\mathrm{d}y(t) \tag{7}$$

$$\hat{z}(t)=C_{fi}\hat{x}(t), \hat{x}(0)=x_0 \tag{8}$$

where $\hat{x}(t)\in R^n$ is the filter state, A_{fi},B_{fi},C_{fi} are the filter parameters with compatible dimensions to be determined.

Define $\xi^{\mathrm{T}}(t)=\left[x^{\mathrm{T}}(t)\quad \hat{x}^{\mathrm{T}}(t)\right]^{\mathrm{T}}$ and $e(t)=z(t)-\hat{z}(t)$ then we can obtain the following filtering error system:

$$\mathrm{d}\xi(t)=\left[\bar{A}_i\xi(t)+\bar{F}_iF(x(t))+\bar{A}_{1i}v(t)\right]\mathrm{d}t+\left[\bar{B}_i\xi(t)+\bar{G}_iG(x(t))+\bar{D}_iv(t)\right]\mathrm{d}w(t) \quad (9)$$

$$e(t)=\bar{L}\xi(t),\xi(0)=\xi_0 \quad (10)$$

where

$$\bar{A}_i=\begin{bmatrix}A_i & 0\\ B_{fi}C_i & A_{fi}\end{bmatrix},\ \bar{F}_i=\begin{bmatrix}F_i & 0\end{bmatrix},\ \bar{A}_{1i}=\begin{bmatrix}A_{1i}\\ B_{fi}C_{1i}\end{bmatrix},\ \bar{B}_i=\begin{bmatrix}B_i\\ B_{fi}D_i\end{bmatrix},\ \bar{G}_i=\begin{bmatrix}G_i & 0\end{bmatrix},$$

$$\bar{D}_i=\begin{bmatrix}D_{1i}\\ B_{fi}D_{1i}\end{bmatrix},\ \bar{L}=\begin{bmatrix}L_i & -C_{fi}\end{bmatrix},\ F(x(t))=\begin{bmatrix}f(x(t))\\ 0\end{bmatrix},\ G(x(t))=\begin{bmatrix}g(x(t))\\ 0\end{bmatrix}.$$

We introduce the following definitions and lemmas, which will be useful in the succeeding discussion.

Definition 1 ([24]): The filtering error system (9) (10) with $v(t)=0$ is said to be finite-time stochastic stable (FTSS) with respect to (c_1,c_2,T,R), where $R>0$, $0<c_1<c_2$ if for a given time-constant $T>0$, the following relation holds:

$$\Xi\left[x^{\mathrm{T}}(0)Rx(0)\right]<c_1\Rightarrow\Xi\left[x^{\mathrm{T}}(t)Rx(t)\right]<c_2,\ \ \forall t\in[0,T].$$

Definition 2: The filtering error system (9) (10) with $v(t)$ is said to be finite-time stochastic stable (FTSS) with respect to (c_1,c_2,T,R,d) if it is stochastic finite-time stable in the sense of definition 1 for all nonzero $v(t)$ satisfying the constraint condition (4) for all $T>0$ under the zero-initial condition.

Definition 3: Given a disturbance attenuation level $\gamma>0$, the filtering error system (9) (10) with $v(t)$ satisfying (4) is said to be H_∞ finite-time stochastic stable (FTSS) with respect to (c_1,c_2,T,R,d) with a prescribed disturbance attenuation level γ, if it is stochastic finite-time stable in the sense of Definition 1 and

$$\Xi\left\{\int_0^T e^{\mathrm{T}}(t)e(t)\mathrm{d}t\right\}\le\gamma^2\int_0^T v^{\mathrm{T}}(t)v(t)\mathrm{d}t. \quad (11)$$

Lemma 1 (Gronwall inequality [25]): Let $v(t)$ be a nonnegative function such that

$$v(t)\le a+b\int_0^t v(s)\mathrm{d}s, 0<t<T, \quad (12)$$

for some constants $a,b\ge 0$, then we have $v(t)\le a\exp(bt), 0<t<T$.

Lemma 2 (Schur complement [26] [27]) Given a symmetric matrix $\phi=\begin{bmatrix}\phi_{11} & \phi_{12}\\ \phi_{21} & \phi_{22}\end{bmatrix}$, the following three conditions are equivalent to each other:

1) $\phi<0$;
2) $\phi_{11}<0$, and $\phi_{22}-\phi_{12}^{\mathrm{T}}\phi_{11}^{-1}\phi_{12}<0$;
3) $\phi_{22}<0$, and $\phi_{11}-\phi_{12}\phi_{22}^{-1}\phi_{12}^{\mathrm{T}}<0$.

Lemma 3 (Itô formula [28]) Let $x(t)$ be an n-dimensional Itô process on $t \geq 0$ with the stochastic differential

$$\mathrm{d}x(t) = f(t)\mathrm{d}t + g(t)\mathrm{d}w(t), \tag{13}$$

where $f(t) \in R^n$ and $g(t) \in R^{n\times m}$, $V(x(t),t) \in (R^n \times R^+, R^+)$. Then $V(x(t),t)$ is a real-valued Itô process with its stochastic differential

$$\mathrm{d}V(x(t),t) = \mathcal{L}V(x(t),t) + V_x(x(t),t)g(t)\mathrm{d}w(t) \tag{14}$$

where the weak infinitesimal operator

$$\mathcal{L}V(x(t),t) = V_t(x(t),t) + V_x(x(t),t)f(t) + \frac{1}{2}tr\left[g^{\mathrm{T}}(t)V_{xx}(x(t),t)g(t)\right]. \tag{15}$$

3. Main Results

Theorem 1: Suppose that the filter parameters A_{fi}, B_{fi}, C_{fi} in (7) (8) are given. The filtering error system (9) (10) is FTSS with respect to (c_1, c_2, T, R, d), if there exist scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \alpha, \gamma > 0$ and symmetric positive definite matrices $P_i, i \in N$ satisfying

$$P_i = \begin{bmatrix} P_{1i} & 0 \\ 0 & P_{2i} \end{bmatrix} = R^{\frac{1}{2}} Q_i R^{\frac{1}{2}}, \tag{16}$$

such that the following LMIs hold

$$\Pi = \begin{bmatrix} \Pi_{11} & P_i\bar{A}_{1i} & P_i^{\mathrm{T}} & \bar{B}_i^{\mathrm{T}}P_i^{\mathrm{T}} & 0 & \bar{B}_i^{\mathrm{T}} \\ * & -\gamma^2 I & 0 & 0 & \bar{D}_i^{\mathrm{T}}P_i^{\mathrm{T}} & \bar{D}_i^{\mathrm{T}} \\ * & * & -\varepsilon_1^{-1}I & 0 & 0 & 0 \\ * & * & * & -\varepsilon_2^{-1}I & 0 & 0 \\ * & * & * & * & -\varepsilon_3^{-1}I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{17}$$

and

$$\frac{\mathrm{e}^{\alpha T}\left\{\sup_{i\in N}\left[\lambda_{\max}(Q_i)\right]c_1 + \gamma^2 d\,\dfrac{1-\mathrm{e}^{-\alpha T}}{\alpha}\right\}}{\inf_{i\in N}\left[\lambda_{\min}(Q_i)\right]} \leq c_2, \tag{18}$$

where $\Pi_{11} = P_i\bar{A}_i + \bar{A}_i^{\mathrm{T}}P_i + \Lambda_1 + \Lambda_2 + \Lambda_3 + \Lambda_4 + \sum_{j=1}^{s}\sigma_{ij}P_j - \alpha P_i$, and "*" denotes the transposed elements in the symmetric positions.

Proof: Define the following stochastic Laypunov-Krasovskii functional candidate:

$$V(\xi(t),i) = \xi^{\mathrm{T}}(t)P_i\xi(t), \tag{19}$$

By Itô formula, we have the weak infinitesimal operator of $V(\xi(t),i)$ as follows:

$$\begin{aligned}
&\mathcal{L}V(\xi(t),i) \\
&= \xi^{\mathrm{T}}(t)P_i\left[\bar{A}_i\xi(t) + \bar{F}_iF(x(t)) + \bar{A}_{1i}v(t)\right] \\
&\quad + \left[\bar{A}_i\xi(t) + \bar{F}_iF(x(t)) + \bar{A}_{1i}v(t)\right]^{\mathrm{T}}P_i\xi(t) \\
&\quad + \left[\bar{B}_i\xi(t) + \bar{G}_iG(x(t)) + \bar{D}_iv(t)\right]^{\mathrm{T}}P_i\left[\bar{B}_i\xi(t) + \bar{G}_iG(x(t)) + \bar{D}_iv(t)\right] \\
&\quad + \sum_{j=1}^{s}\sigma_{ij}\xi^{\mathrm{T}}(t)P_j\xi(t)
\end{aligned}$$

Applying (6) and the following well-known fact:

$$X^{\mathrm{T}}Y+Y^{\mathrm{T}}X\le\varepsilon X^{\mathrm{T}}X+\varepsilon^{-1}Y^{\mathrm{T}}Y,\varepsilon>0\,, \tag{20}$$

it follows that

$$\xi^{\mathrm{T}}(t)P_i\bar{F}_iF(x(t))+\left[\xi^{\mathrm{T}}(t)P_i\bar{F}_iF(x(t))\right]^{\mathrm{T}}\le\varepsilon_1\xi^{\mathrm{T}}(t)P_i^{\mathrm{T}}P_i\xi(t)+\Lambda_1\xi^{\mathrm{T}}(t)\xi(t), \tag{21}$$

$$\begin{aligned}&\xi^{\mathrm{T}}(t)\bar{B}_i^{\mathrm{T}}P_i\bar{G}_iG(x(t))+\left[\xi^{\mathrm{T}}(t)\bar{B}_i^{\mathrm{T}}P_i\bar{G}_iG(x(t))\right]^{\mathrm{T}}\\&\le\varepsilon_2\xi^{\mathrm{T}}(t)\bar{B}_i^{\mathrm{T}}P_i^{\mathrm{T}}P_i\bar{B}_i\xi(t)+\Lambda_2\xi^{\mathrm{T}}(t)\xi(t),\end{aligned} \tag{22}$$

$$G^{\mathrm{T}}(x(t))\bar{G}_i^{\mathrm{T}}P_i\bar{G}_iG(x(t))\le\Lambda_3\xi^{\mathrm{T}}(t)\xi(t), \tag{23}$$

$$\begin{aligned}&G^{\mathrm{T}}(x(t))\bar{G}_i^{\mathrm{T}}P_i\bar{D}_iv(t)+\left[G^{\mathrm{T}}(x(t))\bar{G}_i^{\mathrm{T}}P_i\bar{D}_iv(t)\right]^{\mathrm{T}}\\&\le\varepsilon_3v^{\mathrm{T}}(t)\bar{D}_i^{\mathrm{T}}P_i^{\mathrm{T}}P_i\bar{D}_iv(t)+\Lambda_4\xi^{\mathrm{T}}(t)\xi(t),\end{aligned} \tag{24}$$

$$\Lambda_1=\begin{bmatrix}\varepsilon_1^{-1}\mu_1^2\lambda_{\max}\left(\bar{F}_i^{\mathrm{T}}\bar{F}_i\right)I & 0\\ 0 & 0\end{bmatrix},\ \ \Lambda_2=\begin{bmatrix}\varepsilon_2^{-1}\mu_2^2\lambda_{\max}\left(\bar{G}_i^{\mathrm{T}}\bar{G}_i\right)I & 0\\ 0 & 0\end{bmatrix},$$

$$\Lambda_3=\begin{bmatrix}\mu_2^2\lambda_{\max}\left(\bar{G}_i^{\mathrm{T}}P_i\bar{G}_i\right)I & 0\\ 0 & 0\end{bmatrix},\ \ \Lambda_4=\begin{bmatrix}\varepsilon_3^{-1}\mu_2^2\lambda_{\max}\left(\bar{G}_i^{\mathrm{T}}\bar{G}_i\right)I & 0\\ 0 & 0\end{bmatrix}.$$

Let $\zeta(t)=\left[\xi^{\mathrm{T}}(t)\quad v^{\mathrm{T}}(t)\right]^{\mathrm{T}}$, from (21) - (24), it follows

$$\mathcal{L}V(\xi(t),i)\le\zeta^{\mathrm{T}}(t)\begin{bmatrix}\Lambda & \bar{B}_i^{\mathrm{T}}P_i\bar{D}_i+P_i\bar{A}_{1i}\\ * & \bar{D}_i^{\mathrm{T}}P_i\bar{D}_i+\varepsilon_3\bar{D}_i^{\mathrm{T}}P_i^{\mathrm{T}}P_i\bar{D}_i\end{bmatrix}\zeta(t), \tag{25}$$

where

$$\begin{aligned}\Lambda=&P_i\bar{A}_i+\bar{A}_i^{\mathrm{T}}P_i+\bar{B}_i^{\mathrm{T}}P_i\bar{B}_i+\varepsilon_1P_i^{\mathrm{T}}P_i+\varepsilon_2\bar{B}_i^{\mathrm{T}}P_i^{\mathrm{T}}P_i\bar{B}_i\\&+\Lambda_1+\Lambda_2+\Lambda_3+\Lambda_4+\textstyle\sum_{j=1}^{s}\sigma_{ij}P_j\end{aligned}. \tag{26}$$

Applying Schur complement, we have the following inequality by taking (17) into consideration:

$$\Xi\mathcal{L}V(\xi(t),i)<\alpha\Xi V(\xi(t),i)+\gamma^2v^{\mathrm{T}}(t)v(t). \tag{27}$$

Multiplying the above inequality by $\mathrm{e}^{-\alpha t}$ and by Gronwall inequality (12), we obtain the following inequality

$$\Xi\left[\mathrm{e}^{-\alpha t}V(\xi(t),i)\right]-\Xi\left[V(\xi(0),i)\right]<\gamma^2\int_0^t\mathrm{e}^{-\alpha s}v^{\mathrm{T}}(s)v(s)\mathrm{d}s. \tag{28}$$

Then, we have

$$\begin{aligned}\Xi\left[V(\xi(t),i)\right]&<\mathrm{e}^{\alpha t}V(\xi(0),i)+\gamma^2d\mathrm{e}^{\alpha t}\int_0^t\mathrm{e}^{-\alpha s}\mathrm{d}s\\&<\mathrm{e}^{\alpha t}\left[V(\xi(0),i)+\gamma^2d\frac{1-\mathrm{e}^{-\alpha t}}{\alpha}\right]\\&<\mathrm{e}^{\alpha T}\left[V(\xi(0),i)+\gamma^2d\frac{1-\mathrm{e}^{-\alpha T}}{\alpha}\right]\end{aligned} \tag{29}$$

$$V(\xi(0),i)\le\sup_{i\in N}\left[\lambda_{\max}(Q_i)\right]\xi^{\mathrm{T}}(0)R\xi(0). \tag{30}$$

$$V(\xi(t),i)\ge\inf_{i\in N}\left[\lambda_{\min}(Q_i)\right]\xi^{\mathrm{T}}(t)R\xi(t). \tag{31}$$

Taking (29)-(31) into account, we obtain

$$\Xi\left[V\left(\xi(t),i\right)\right]\le\frac{e^{\alpha T}\left\{\sup_{i\in N}\left[\lambda_{\max}\left(Q_i\right)\right]c_1+\gamma^2 d\frac{1-e^{-\alpha T}}{\alpha}\right\}}{\inf_{i\in N}\left[\lambda_{\min}\left(Q_i\right)\right]}. \tag{32}$$

Therefore, it follows that condition (18) implies $\Xi\left[x^{\mathrm{T}}(t)Rx(t)\right]<c_2$. The filtering error system is finite-time bounded with respect to $\left(c_1,c_2,T,R,d\right)$. This completes the proof.

Theorem 2: The filtering error system (9) (10) is FTSS with respect to $\left(c_1,c_2,T,R,d\right)$ and satisfies the condition (11), if there exist positive constant $\varepsilon_1,\varepsilon_2,\varepsilon_3,\alpha,\gamma$ and symmetric positive definite matrices $P_i, i\in N$ such that (16) (18) hold and

$$\Omega=\begin{bmatrix}\Omega_{11} & P_i\bar{A}_{1i} & P_i^{\mathrm{T}} & \bar{B}_i^{\mathrm{T}}P_i^{\mathrm{T}} & 0 & \bar{B}_i^{\mathrm{T}}\\ * & -\gamma^2 I & 0 & 0 & \bar{D}_i^{\mathrm{T}}P_i^{\mathrm{T}} & \bar{D}_i^{\mathrm{T}}\\ * & * & -\varepsilon_1^{-1}I & 0 & 0 & 0\\ * & * & * & -\varepsilon_2^{-1}I & 0 & 0\\ * & * & * & * & -\varepsilon_3^{-1}I & 0\\ * & * & * & * & * & -I\end{bmatrix}<0 \tag{33}$$

where $\Omega_{11}=P_i\bar{A}_i+\bar{A}_i^{\mathrm{T}}P_i+\Lambda_1+\Lambda_2+\Lambda_3+\Lambda_4+\sum_{j=1}^{s}\sigma_{ij}P_j+\bar{L}^{\mathrm{T}}\bar{L}-\alpha P_i$.

Proof: For the filtering error system (9) (10), consider the same stochastic Laypunov functional as in (19). Obviously, condition (33) implies that

$$\begin{bmatrix}\Lambda+\bar{L}^{\mathrm{T}}\bar{L}-\alpha P_i & \bar{B}_i^{\mathrm{T}}P_i\bar{D}_i+P_i\bar{A}_{1i}\\ * & \bar{D}_i^{\mathrm{T}}P_i\bar{D}_i+\varepsilon_3\bar{D}_i^{\mathrm{T}}P_i^{\mathrm{T}}P_i\bar{D}_i\end{bmatrix}<0 \tag{34}$$

where Λ is given in (26).

By theorem 1, conditions (17) and (18) guarantee that system (9) (10) is FTSS with respect to $\left(c_1,c_2,T,R,d\right)$.

Therefore, we only need to prove that (11) holds.

Noting that (27) and (34), we obtain

$$\Xi\mathcal{L}V\left(\xi(t),i\right)<\alpha\Xi V\left(\xi(t),i\right)+\gamma^2 v^{\mathrm{T}}(t)v(t)-\Xi\left[v^{\mathrm{T}}(t)v(t)\right].$$

Then using the similar proof as Theorem 1, condition (11) can be easily obtained.

Theorem 3: The filtering error system (9) (10) is FTSS with respect to $\left(c_1,c_2,T,R,d\right)$ and satisfies the condition (11), if there exist positive constant $\varepsilon_1,\varepsilon_2,\varepsilon_3,\alpha,\gamma$ and symmetric positive definite matrices $P_i, i\in N$ and matrices $\Theta_1,\Theta_2,\Theta_3$ such that (16) (18) hold and

$$\Theta=\begin{bmatrix}\Theta_{11} & \bar{A}_{1i}P_i^{-1} & P_i^{-1} & P_i^{-1}\bar{B}_i^{\mathrm{T}} & 0 & P_i^{-1}\bar{B}_i^{\mathrm{T}}P_i^{-1} & P_i^{-1}\bar{L}^{\mathrm{T}}P_i^{-1}\\ * & -\gamma^2 I & 0 & 0 & P_i^{-1}\bar{D}_i^{\mathrm{T}} & P_i^{-1}\bar{D}_i^{\mathrm{T}}P_i^{-1} & 0\\ * & * & -\varepsilon_1^{-1}I & 0 & 0 & 0 & 0\\ * & * & * & -\varepsilon_2^{-1}I & 0 & 0 & 0\\ * & * & * & * & -\varepsilon_3^{-1}I & 0 & 0\\ * & * & * & * & * & -I & 0\\ * & * & * & * & * & * & -I\end{bmatrix}<0 \tag{35}$$

where $\Theta_{11} = \bar{A}_i P_i^{-1} + P_i^{-1} \bar{A}_i^{\mathrm{T}} + P_i^{-1} \left(\Lambda_1 + \Lambda_2 + \Lambda_3 + \Lambda_4 + \sum_{j=1}^{s} \sigma_{ij} P_j \right) P_i^{-1} - \alpha P_i^{-1}$.

In addition, the suitable parameters of the filter (7) (8) are given as follows:

$$A_{fi} = \Theta_1 P_{2i}^{-1}, \quad B_{fi} = P_{2i}^{-1} \Theta_2, \quad C_{fi} = \Theta_3. \tag{36}$$

Proof: By theorem 2, let $\Theta_1 = P_{2i} A_{fi}$, $\Theta_2 = P_{2i} B_{fi}$, $\Theta_3 = C_{fi}$,

Apply Surch complement for (33), then pre- and post-multiply $diag\left\{P_i^{-1}, I, I, I, I, I, I\right\}$ and $diag\left\{P_i^{-1}, I, I, I, I, I, I\right\}$ respectively, we can get inequality (35) from (33).

4. Conclusion

In this paper, we deal with the finite-time H_∞ filter designing problem for a class of stochastic nonlinear systems with Markovian switching. The sufficient conditions for FTSS of the filtering error system have been presented and proved by the Lyapunov-Krasovski approach. The designed filter is provided to ensure the filtering error system FTSS and satisfies a prescribed H_∞ performance level in a given finite-time interval, which can be reduced to feasibility problems involving restricted linear matrix equalities.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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