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Diversity of Interaction Solutions to the (2 + 1)-Dimensional Sawada-Kotera Equation

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Abstract

In this paper, based on Hirota bilinear form, we aim to show the diversity of interaction solutions to the (2 + 1)-dimensional Sawada-Kotera (SK) equation. By introducing an arbitrary differentiable function in assumption form, we can obtain abundant interaction solutions which can provide the possibility for exploring the interactions between lump waves and other kinds of waves. By choosing some particular functions and values of the involved parameters, we give four illustrative examples of the resulting solutions, and explore some novel interaction behaviors in (2 + 1)-dimensional SK equation.

Keywords

Hirota Bilinear Form, Lump Solution, Interaction Solution, (2 + 1)-Dimensional Sawada-Kotera Equation

1. Introduction

As we all known, integrable nonlinear evolution equations have soliton solutions, which reflect a common nonlinear phenomenon in nature. In the past few decades, many researchers have paid attention to the study of exact solutions. For instance, the rational rogue waves and lump waves exponentially localized solutions in certain directions. Compared with soliton solution, lump wave is a kind of special wave, rationally localized in all directions in the space. The lump solution for its significant physical meanings was first discovered by Manakov *et al.* [1]. Many integrable equations have been found to possess lump solutions, such as the KP equation [2] [3] [4], the two-dimensional nonlinear Schrödinger type equation [3], the three-dimensional three wave resonant interaction equation [5] and the Ishimori equation [6].

With Hirota bilinear method, the lump solution can be transformed into a new equation which is called the Hirota equation [7] [8] [9] [10] [11]. It is natural and interesting to use this method to search for lump solutions of nonlinear partial differential equations. Based on this method, the lump solutions of some more integrable equations have been found, such as dimensionally reduced p-gKP and p-gBKP equations [1], Boussinesq equation [12] and dimensionally reduced Hirota bilinear equation [13]. Recently, lump solutions are being raised more questions about interaction solutions [14], especially interaction solutions between lumps and either solution or kinks [15] [16] [17].

In this paper, we will study the (2 + 1)-dimensional Sawada-Kotera (2DSK) equation

$$u_t-\left(u_{4x}+5uu_{xx}+\frac{5}{3}u^3+5u_{xy}\right)_x-5uu_y+5\int u_{yy}\mathrm{d}x-5u_x\int u_y\mathrm{d}x=0 \tag{1}$$

which was first proposed by Konopelchenko and Dubrovsky [18], where u is a function of the variables x, y and t. It was widely used in many physical branches, such as two-dimensional quantum gravity, conformal field theory and conserved current of Liouville equation [19] [20]. The 2DSK equation's Lax pair was found in Ref. [21] [22] [23], bilinear Bell polynomials were obtained in Ref. [24], symmetry analysis was studied in Ref. [25] [26] [27] [28] and four sets of bilinear Bäcklund transformations were constructed to derive multisoliton solutions in Ref. [29].

Through the transformation

$$u=6(\ln f)_{xx}=\frac{6\left(f_{xx}f-f_x^2\right)}{f^2}, \tag{2}$$

Equation (1) can be turned into the Hirota bilinear form

$$\begin{aligned}&\left(D_x^6-D_xD_t+5D_x^3D_y-5D_y^2\right)f\cdot f\\&=2\Big[\left(ff_{6x}-6f_xf_{5x}+15f_{2x}f_{4x}-10f_{3x}^2\right)-\left(ff_{xt}-f_xf_t\right)\\&\quad+5\left(3f_{2x}f_{xy}-3f_xf_{xxy}+ff_{xxxy}-f_{xxx}f_y\right)-5\left(ff_{yy}-f_y^2\right)\Big]=0\end{aligned} \tag{3}$$

Therefore, if f solves bilinear Equation (3), then $u=6(\ln f)_{xx}$ will solve the 2DSK Equation (1).

In order to get lump solutions, the following quadratic function can be assumed [30]:

$$f=g^2+h^2+a_9,\ g=a_1x+a_2y+a_3t+a_4,\ h=a_5x+a_6y+a_7t+a_8, \tag{4}$$

where $a_i\ (1\le i\le 9)$ are real parameters to be determined. By a direct calculation, three families of lump solutions for 2DSK Equation (1) have been previously presented in Ref. [30].

$$\begin{aligned}u_1=\frac{12}{f^2}\Bigg[&\left(a_1^2+a_5^2\right)f-2\Bigg(a_1\left(a_1x+a_2y+\frac{5\left(-a_1a_2^2-2a_2a_5a_6+a_1a_6^2\right)t}{a_1^2+a_5^2}+a_4\right)\\&+a_5\left(a_5x+a_6y-\frac{5\left(-a_2^2a_5+2a_1a_2a_6+a_5a_6^2\right)t}{a_1^2+a_5^2}+a_8\right)\Bigg)^2\Bigg],\end{aligned} \tag{5}$$

where f is defined in Equation (4)

$$f=\frac{3\left(a_1^2+a_5^2\right)^2\left(a_1a_2+a_5a_6\right)}{\left(a_2a_5-a_1a_6\right)^2}+\left(a_1x+a_2y+\frac{5\left(-a_1a_2^2-2a_2a_5a_6+a_1a_6^2\right)t}{a_1^2+a_5^2}+a_4\right)^2+\left(a_5x+a_6y-\frac{5\left(-a_2^2a_5+2a_1a_2a_6+a_5a_6^2\right)t}{a_1^2+a_5^2}+a_8\right)^2,$$

$$u_2=\frac{12a_1^2\left[\frac{3a_1^3a_2}{a_6^2}-\left(a_1x+a_2y+a_4+\frac{5\left(-a_2^2+a_6^2\right)t}{a_1}\right)^2+\left(a_6y-\frac{10a_2a_6t}{a_1}+a_8\right)^2\right]}{\left[\frac{3a_1^3a_2}{a_6^2}+\left(a_1x+a_2y+a_4+\frac{5\left(-a_2^2+a_6^2\right)t}{a_1}\right)^2+\left(a_6y-\frac{10a_2a_6t}{a_1}+a_8\right)^2\right]^2}, \quad (6)$$

and

$$u_3=\frac{3a_1^2\left[\frac{\left(a_2^2+a_6^2\right)^2}{a_2^2a_6^2}f-2\left(\frac{a_1\left(a_2^2+a_6^2\right)^2x}{2a_2^2a_6^2}+\frac{\left(a_2^2+a_6^2\right)y}{a_2}+\frac{10\left(a_2^2-a_6^2\right)t}{a_1}+2a_4-\frac{a_8\left(a_2^2-a_6^2\right)}{a_2a_6}\right)^2\right]}{\left[\left(a_1x+a_2y+a_4\right)^2+\left[\frac{a_1\left(a_6^2-a_2^2\right)x}{2a_2a_6}+a_6y-\frac{10a_2a_6t}{a_1}+a_8\right]^2+\frac{3a_1^3\left(a_2^2+a_6^2\right)^3}{8a_2^5a_6^2}\right]^2}. \quad (7)$$

In Ref. [30], it has been shown that the solutions (5) and (7) can exhibit the bright lump wave structure (one peak and two valleys), while the solution (6) displays a bright-dark lump wave (one peak and one valley).

In this paper, our intention is to further extend the assumption in Equation (4) by introducing an arbitrary function which is more generalized than some other assumption forms. It can provide the possibility for exploring the interactions between lump waves and other kinds of waves in Equation (1). We will give some examples to show the diversity of interaction solutions to the (2 + 1)-dimensional Sawada-Kotera equation.

2. Interaction Solutions to the (2 + 1)-Dimensional Sawada-Kotera Equation

We assume that f has the combined solutions of the form

$$f=g^2+h^2+\omega(k)+a_{13}, \quad (8)$$

where ω is a function and three linear wave variables are

$$\begin{aligned}g&=a_1x+a_2y+a_3t+a_4,\\h&=a_5x+a_6y+a_7t+a_8,\\k&=a_9x+a_{10}y+a_{11}t+a_{12},\end{aligned} \quad (9)$$

where the parameters $a_i\left(1\le i\le 13\right)$ are all real constants to be determined. It is noted that this ansatz (8) can generate a class of lump and interaction solutions. In particular, combined solutions (8) can reduce to the lump solutions when the function $\omega(k)$ disappears.

With the aid of symbolic computation, substituting Equation (8) into Equation (3) and eliminating the coefficients of the polynomial yield the following constraining equations on the function and parameters:

$$\begin{cases} a_2 = -\frac{3}{2}a_5 a_9^2 |c_1|,\ a_3 = \frac{45}{4}a_1 a_9^4 c_1^2,\ a_6 = \frac{3}{2}a_1 a_9^2 |c_1|, \\ a_7 = \frac{45}{4}a_5 a_9^4 c_1^2,\ a_{10} = \frac{1}{2}a_9^3 c_1,\ a_{11} = \frac{9}{4}a_9^5 c_1^2, \\ \omega^{(5)} = c_1 \omega''' = c_1^2 \omega',\ \omega^{(6)} = c_1 \omega^{(4)} = c_1^2 \omega'', \\ (\omega'')^2 = c_1 (\omega')^2 - c_3,\ \omega'' = c_1 \omega - c_2, \end{cases} \tag{10}$$

where $c_i\ (i = 1,2,3)$ are all arbitrary real constants. Therefore, we can say that if $\omega(k)$ and parameters obey constraining conditions (10), the resulting combined solutions (8) will generate many classes of interaction solutions. Furthermore, if we require $\omega(k) + a_{13} > 0$, the function f in Equation (8) is positive and interaction solutions have no singularity.

In the following, to illustrate the resulting interaction solutions, we give four examples to show the diversity of interaction solutions to the (2 + 1)-dimensional Sawada-Kotera equation.

Case I: When $c_1 = -1, c_2 = 0, c_3 = -1$ and $\omega(k) = \sin(k)$, we have

$$\begin{aligned} u_1 = \frac{6}{f^2}\Bigg[& \left(2a_1^2 + 2a_5^2 - a_9^2 \sin\left(a_9 x - \frac{1}{2}a_9^3 y + \frac{9}{4}a_9^5 t + a_{12}\right)\right) \\ & - \frac{1}{4}\Bigg(\left(a_1^2 + a_5^2\right)\left(4x + 45 a_9^4 t\right) + 4a_1 a_4 + 4a_5 a_8 \\ & + 2a_9 \cos\left(a_9 x - \frac{1}{2}a_9^3 y + \frac{9}{4}a_9^5 t + a_{12}\right)\Bigg)^2\Bigg], \end{aligned} \tag{11}$$

with

$$\begin{aligned} f = & \left(a_1 x - \frac{3}{2}a_5 a_9^2 y + \frac{45}{4}a_1 a_9^4 t\right)^2 \\ & + \left(a_5 x + \frac{3}{2}a_1 a_9^2 y + \frac{45}{4}a_5 a_9^4 t\right)^2 \\ & - \frac{a_9^2}{2\left(a_1^2 + a_5^2\right)} + \sin\left(a_9 x - \frac{1}{2}a_9^3 y + \frac{9}{4}a_9^5 t + a_{12}\right). \end{aligned} \tag{12}$$

Case II: When $c_2 = c_3 = 0$ and $\omega(k) = \mathrm{e}^{-2(a_9 x + a_{10} y + a_{11} t + a_{12})}$, we have

$$\begin{cases} c_1 = 4,\ a_2 = -6a_5 a_9^2,\ a_3 = 180 a_1 a_9^4,\ a_6 = 6a_1 a_9^2, \\ a_7 = 180 a_5 a_9^4,\ a_{10} = 2a_9^3,\ a_{11} = 36 a_9^5,\ a_{13} = 0. \end{cases} \tag{13}$$

$$\begin{aligned} u_2 = \frac{6}{f^2}\Bigg[& 2\left(a_1^2 + a_5^2 + 2a_9^2 \mathrm{e}^{-2\left(a_9 x + 2a_9^3 y + 36 a_9^5 t + a_{12}\right)}\right) f \\ & - 4\left(a_1 a_4 + a_5 a_8 - a_9 \mathrm{e}^{-2\left(a_9 x + 2a_9^3 y + 36 a_9^5 t + a_{12}\right)} + \left(a_1^2 + a_5^2\right)\left(x + 180 a_9^4 t\right)\right)^2\Bigg], \end{aligned} \tag{14}$$

with

$$f=\left(a_1x-6a_5a_9^2y+180a_1a_9^4t+a_4\right)^2 \\ +\left(a_5x+6a_1a_9^2y+180a_5a_9^4t+a_8\right)^2+\mathrm{e}^{-2\left(a_9x+2a_9^3y+36a_9^5t+a_{12}\right)}. \tag{15}$$

Case III: When $c_2\neq 0, c_3\neq 0$ and $\omega(k)=\left[\sinh\left(a_9x+a_{10}y+a_{11}t+a_{12}\right)\right]^2$, we have

$$\begin{cases} c_1=4,\ c_2=-2,\ c_3=-4,\ a_2=-6a_5a_9^2,\ a_3=180a_1a_9^4, \\ a_6=6a_1a_9^2,\ a_7=180a_5a_9^4,\ a_{10}=-2a_9^3,\ a_{11}=36a_9^5,\ a_{13}=\dfrac{-4a_1^2-4a_5^2-4a_9^2}{8\left(a_1^2+a_5^2\right)}, \end{cases} \tag{16}$$

$$u_3=\frac{6}{f^2}\Big[\Big(2a_1^2+2a_5^2+2a_9^2\cosh\left(a_9x+2a_9^3y+36a_9^5t+a_{12}\right)^2 \\ +2a_9^2\sinh\left(a_9x+2a_9^3y+36a_9^5t+a_{12}\right)^2\Big)f \\ -\Big(2\left(a_1^2+a_5^2\right)\left(x+180a_9^4t\right)+2a_1a_4+2a_5a_8 \\ +2\cosh\left(a_9x+2a_9^3y+36a_9^5t+a_{12}\right)\sinh\left(a_9x+2a_9^3y+36a_9^5t+a_{12}\right)\Big)^2\Big], \tag{17}$$

with

$$f=\left(a_1x-6a_5a_9^2y+180a_1a_9^4t+a_4\right)^2 \\ +\left(a_5x-6a_1a_9^2y+180a_5a_9^4t+a_8\right)^2 \\ +\frac{a_1^2+a_5^2+a_9^2}{2a_1^2+2a_5^2}+\sinh\left(a_9x+2a_9^3y+36a_9^5t+a_{12}\right)^2. \tag{18}$$

Case IV: When $c_2=0, c_3\neq 0$ and $\omega(k)=\sinh\left(a_9x+a_{10}y+a_{11}t+a_{12}\right)+2\cosh\left(a_9x+a_{10}y+a_{11}t+a_{12}\right)$, we have

$$\begin{cases} c_1=1,\ c_3=-3,\ a_2=-\dfrac{3}{2}a_5a_9^2,\ a_3=\dfrac{45}{4}a_1a_9^4,\ a_6=\dfrac{3}{2}a_1a_9^2, \\ a_7=\dfrac{45}{4}a_5a_9^4,\ a_{10}=\dfrac{a_9^3}{2},\ a_{11}=\dfrac{9a_9^5}{4},\ a_{13}=\dfrac{3a_9^2}{2\left(a_1^2+a_5^2\right)}. \end{cases} \tag{19}$$

$$u_4=\frac{6}{f^2}\Bigg[\Bigg(2a_1^2+2a_5^2+\Bigg(2\cosh\left(a_9x+\frac{1}{2}a_9^3y+\frac{9}{4}a_9^5t+a_{12}\right) \\ +\sinh\left(a_9x+\frac{1}{2}a_9^3y+\frac{9}{4}a_9^5t+a_{12}\right)\Bigg)a_9^2\Bigg)f \\ -\frac{1}{4}\Bigg(\left(a_1^2+a_5^2\right)\left(4x+45a_9^4t\right)+4a_1a_4+4a_5a_8 \\ +2\Bigg(\cosh\left(a_9x+\frac{1}{2}a_9^3y+\frac{9}{4}a_9^5t+a_{12}\right) \\ +2\sinh\left(a_9x+\frac{1}{2}a_9^3y+\frac{9}{4}a_9^5t+a_{12}\right)\Bigg)a_9\Bigg)^2\Bigg], \tag{20}$$

with

$$
\begin{aligned}
f = & \left(a_1 x - \frac{3}{2} a_5 a_9^2 y + \frac{45}{4} a_1 a_9^4 t + a_4\right)^2 \\
& + \left(a_5 x - \frac{3}{2} a_1 a_9^2 y + \frac{45}{4} a_5 a_9^4 t + a_8\right)^2 \\
& + \frac{3a_9^2}{2a_1^2 + 2a_5^2} + 2\cosh\left(a_9 x + \frac{1}{2} a_9^3 y + \frac{9}{4} a_9^5 t + a_{12}\right) \\
& + \sinh\left(a_9 x + \frac{1}{2} a_9^3 y + \frac{9}{4} a_9^5 t + a_{12}\right).
\end{aligned} \tag{21}
$$

Then, we will discuss the interaction between lump solutions and soliton solutions for Equations (14), (17) and (20), respectively. In order to get the collision phenomena, $a_3^2 + a_7^2 + a_{11}^2 \neq 0$ is essential. So the asymptotic behaviors of the obtained solutions (14), (17) and (20) can be got: $u_i \to 0\left(i = 1,2,3,4\right)$ as $t \to \pm\infty$.

For Equation (14), the collision behavior shows the single stripe soliton wave feature. From the expression of u_1, it is algebraically decaying and also exponentially decaying. Hence it is a mixed exponential-algebraic solitary wave solution. It reflects the completely non-elastic interaction between lump solution and single stripe soliton. Without loss of generality, we take $a_4 = a_8 = 0$.

In order to investigate the interaction phenomena in $a_{11} > 0$ and $a_{11} < 0$, we can change a_9 in Equation (14). When $a_{11} > 0$, as shown in **Figure 1**, the collision behavior of lump solution and single stripe soliton will occur. It is clear that when $t \to -\infty$, the solution u_1 represents two waves: the lump solution and the single stripe soliton. When $t \to +\infty$, the lump solution disappears, and only the single stripe soliton exists. It reflects the completely non-elastic interaction between two different waves. When $a_{11} < 0$, oppositely, we can see from **Figure 2** that when $t \to -\infty$, only the single stripe soliton exists. When $t \to +\infty$, the lump solution appears and the solution u_1 represents the lump solution and the single stripe soliton. It also reflects the completely non-elastic interaction between two different waves. From above two behaviors, we can see that the interaction phenomena both happen near $t = 0$, when $a_{11} > 0$, the lump solution is drowned or swallowed by the single stripe soliton after $t = 0.3$, and when $a_{11} < 0$, the lump solution rises up and appears before $t = -0.3$.

For Equation (17) and Equation (20), the collision behaviors show the soliton wave feature. From the expression of u_2 and u_3, they are mixed exponential-algebraic solitary wave solutions, too. The difference is that they reflect the elastic interaction between lump solution and soliton solution. After a series of the same steps as u_1, the collision behaviors indicate that the interaction phenomena in $a_{11} > 0$ and $a_{11} < 0$ are consistent. It is clear that when $t \to -\infty$, only the soliton solution exists. When $t \to 0$, the lump solution appears and the solution u_2 and u_3 severally represent two waves: the lump solution and the soliton solution. When $t \to +\infty$, the lump solution disappears, and only the soliton solution exists. The process of interaction can be seen from **Figure 3** and **Figure 4**.

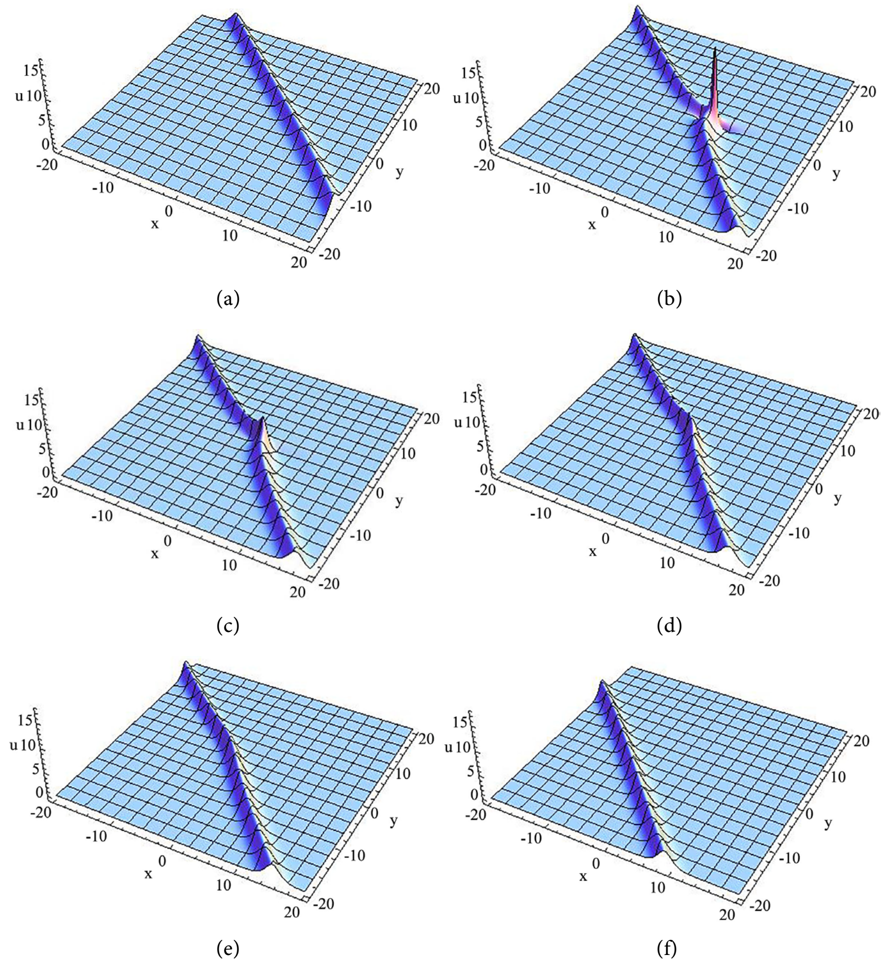
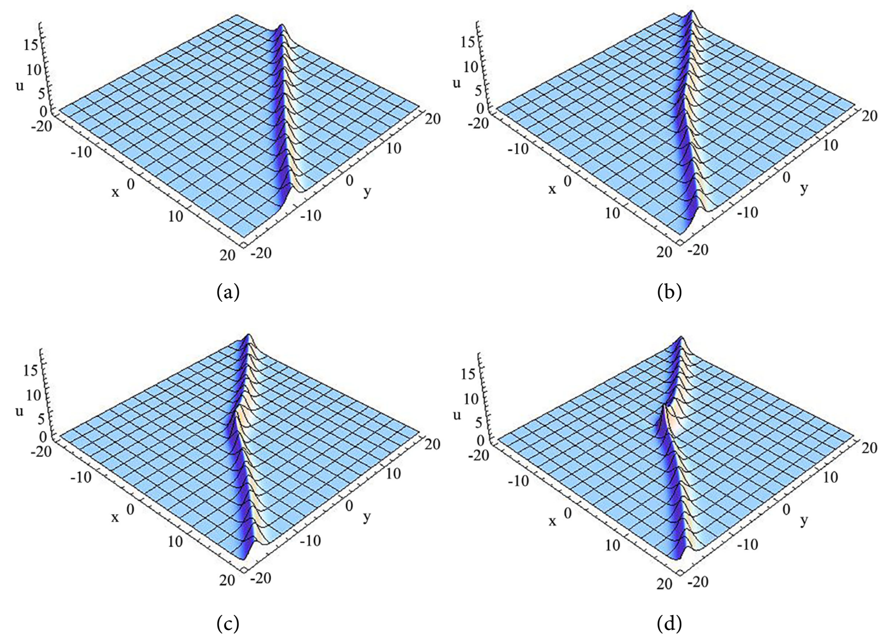
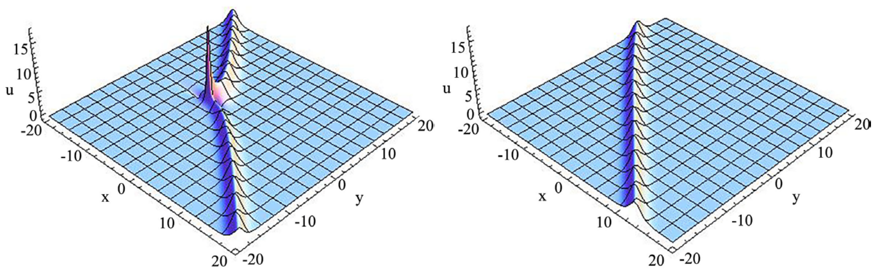


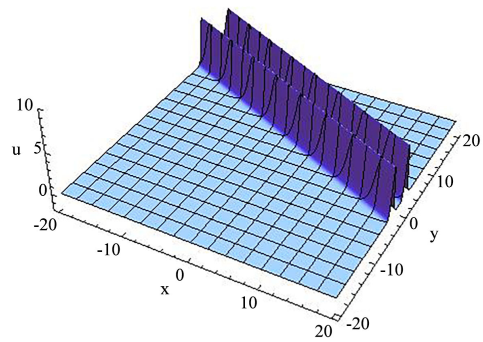
Figure 1. Evolution plot of lump solution and single stripe soliton with $a_1 = 0.1$, $a_4 = 0$, $a_5 = -0.1$, $a_8 = 0$, $a_9 = 0.7$, $a_{12} = -1$ in Equation (14). (a) $t = -1$; (b) $t = -0.1$; (c) $t = -0$; (d) $t = 0.1$; (e) $t = -0.3$; (f) $t = 1$.



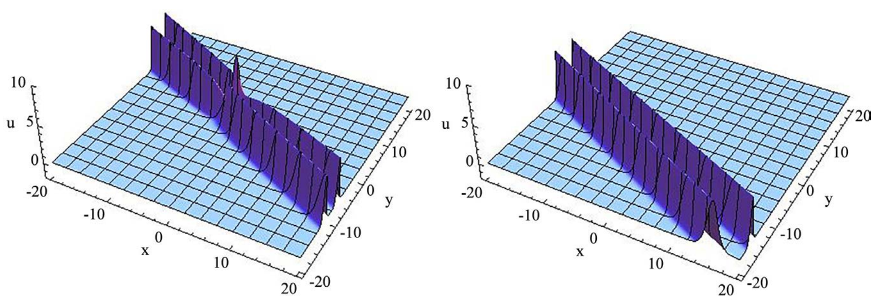


(e) (f)

Figure 2. Evolution plot of lump solution and single stripe soliton with $a_1 = 0.1$, $a_4 = 0$, $a_5 = -0.1$, $a_8 = 0$, $a_9 = -0.7$, $a_{12} = -1$ in Equation (14). (a) t = −1; (b) t = −0.3; (c) t = −0.1; (d) t = 0; (e) t = 0.1; (f) t = −1.

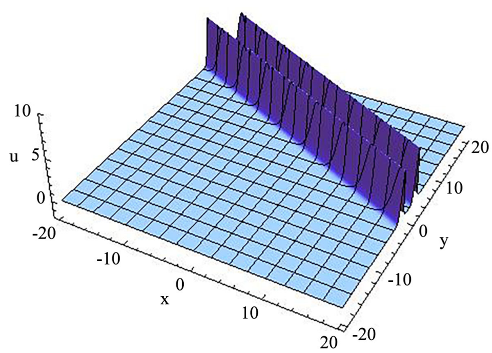


(a)

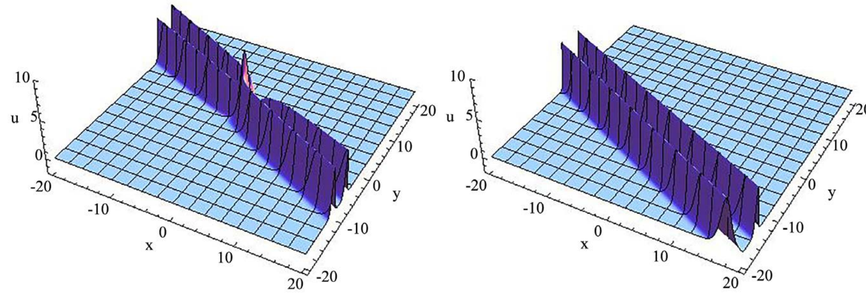


(b) (c)

Figure 3. Evolution plot of lump solution and soliton solution with $a_1 = 0.5$, $a_4 = 0$, $a_5 = -0.5$, $a_8 = 0$, $a_9 = 0.9$, $a_{12} = 2$ in Equation (17). (a) t = −0.5; (b) t = 0; (c) t = 0.5.



(a)



(b) (c)

Figure 4. Evolution plot of lump solution and soliton solution with $a_1 = 0.8$, $a_4 = 0$, $a_5 = -0.8$, $a_8 = 0$, $a_9 = 2$, $a_{12} = -1$ in Equation (20). (a) t = −0.5; (b) t = 0; (c) t = 0.5.

3. Conclusion

In this paper, via the Hirota bilinear form, we have studied the (2 + 1)-dimensional Sawada-Kotera equation. The lump solutions and the mixed exponential-algebraic solitary wave solutions have been obtained. We have presented a class of interaction solutions between lump solutions and other kinds of solitary wave solutions for the (2 + 1)-dimensional Sawada-Kotera equation. This class of the resulting interaction solutions requires a function satisfying four linear ordinary differential equations. All of these have provided abundant interaction solutions and supplemented the existing lump and soliton solutions. Then, we will study other high-dimensional nonlinear problems based on the interaction solutions presented in this paper.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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