Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2018, 6, 949-959
http://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Solitary Wave Solutions and Rational Solutions for Modified Zakharov-Kuznetsov Equation with Initial Value Problem

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How to cite this paper: Dong, Z.Z. and Wang, L. (2018) Solitary Wave Solutions and Rational Solutions for Modified Zakharov-Kuznetsov Equation with Initial Value Problem. *Journal of Applied Mathematics and Physics*, **6**, 949-959.
https://doi.org/10.4236/jamp.2018.65081

Received: March 8, 2018
Accepted: May 12, 2018
Published: May 15, 2018

Abstract

The modified Zakharov-Kuznetsov equation with the initial value problem is studied numerically by means of homotopy perturbation method. The analytical approximate solutions of the modified Zakharov-Kuznetsov equation are obtained. Choosing the form of the initial value, the single solitary wave, two solitary waves and rational solutions are presented, some of which are shown by the plots.

Keywords

Modified Zakharov-Kuznetsov Equation, Homotopy Perturbation Method, Soliton Solution

1. Introduction

Partial differential equations widely describe many phenomena in the world. Although many mathematicians and physicists presented various methods to find the explicit solutions of the partial differential equations, it is a difficult and important task to build the solutions of initial and boundary value problem. Recently, homotopy perturbation method (HPM) has been applied into many problems [1]-[10] and tested to be an effective tool. Here, the initial value problem of the modified Zakharov-Kuznetsov (mZK) equation is studied by using HPM.

The initial value problem of mZK equation is as following:

$$\begin{cases} u_t + u^2 u_x + u_{xxx} + u_{xyy} = 0, \\ u(x,y,0) = f(x,y). \end{cases} \tag{1}$$

Many authors have studied the mZK equation [11]-[16]. The authors in Ref. [11] applied the asymptotic approach into mZK equation and found for the mZK equation that critical collapse in two dimensions is accompanied by damping both the momentum and energy of the perturbed solitary waves and that slows down the rate of the singularity formation. Ref. [12] considered the two-dimensional solitary wave (lump) interactions and the formation of singularities in mZK equation. Ref. [13] obtained a class of approximate periodic solutions for mZK equation by using the homotopy analysis method. The authors in Ref. [14] obtained many solitary waves and periodic waves and kink waves of mZK equation by using the theory of bifurcations of dynamical systems. Uisng the extended tanh method, Ref. [15] got new travelling wave solutions with solitons and periodic structures. In Ref. [16], the first integral method was used to construct travelling wave solutions of mZK equation. Peng [17] developed the extended mapping method to study the traveling wave solution for the mZK equation. Ref. [18] applied the exp-function method to construct generalized solitary and periodic solutions of mZK equation. Authors in Ref. [19] applied the improved (G'/G)-expansion method to construct abundant new exact traveling wave solutions of mZK equation. Ref. [20] employed the complex method to obtain the exact solutions of mZK equation.

This paper is arranged as follows: In Section 2, by using HPM, we obtain the analytical approximate solution of Equation (1). By taking the form of the initial value, some exact solutions of mZK equation are obtained in Section 3. And some pictures are given to show the structure of the obtained solutions. Finally, some conclusions and discussions are given in Section 4.

2. The Homotopy Perturbation Method to mZK Equation

In order to obtain the analytical approximate solution of Equation (1), we consider the one-parameter family of Equation (1) as follows

$$\left(u-u_0\right)_t + p\left(u^2u_x + u_{xxx} + u_{xyy}\right) = 0, \tag{2}$$

where the parameter $p \in [0,1]$ and $u_0 = f(x,y)$.

If $p=0$, we meet $u=u_0$.

If $p=1$, we come back to the original problem (1). Let the solution $u(x,y,t)$ of the system (2) be written in the form of an infinite series,

$$u(x,y,t) = \sum_{i=0}^{\infty} u_i(x,y,t)\, p^i. \tag{3}$$

Then $u(x,y,t) = \sum_{i=0}^{\infty} u_i(x,y,t)$ is a series solution of Equation (1).

Substituting Equation (3) into Equation (2), and equating the coefficients of $p, p^2, \cdots$, we have

$$u_{1,t} + u_0^2 u_{0,x} + u_{0,xxx} + u_{0,xyy} = 0, \tag{4}$$

$$u_{2,t} + u_0^2 u_{1,x} + 2u_0u_1u_{0,x} + u_{1,xxx} + u_{1,xyy} = 0, \tag{5}$$

$$u_{3,t} + u_0^2 u_{2,x} + u_1^2 u_{0,x} + 2u_0u_1u_{1,x} + 2u_0u_2u_{0,x} + u_{2,xxx} + u_{2,xyy} = 0, \tag{6}$$

and so on. Solving Equation (4), (5) and (6), one can obtain

$$u_1(x,y,t) = -\left(u_0^2 u_{0,x} + u_{0,xxx} + u_{0,xyy}\right)t, \tag{7}$$

$$\begin{aligned} u_2(x,y,t) = \frac{1}{2}\Big(&4u_0^3u_{0,x}^2 + u_0^4u_{0,xx} + 2u_0^2u_{0,xxxx} + 2u_0^2u_{0,xxyy} + 12u_{0,x}^2u_{0,xx} \\ &+ 6u_0u_{0,xx}^2 + 10u_0u_{0,x}u_{0,xxx} + 2u_{0,x}^2u_{0,yy} + 8u_{0,x}u_{0,y}u_{0,xy} \\ &+ 4u_0u_{0,xy}^2 + 6u_0u_{0,x}u_{0,xyy} + 2u_{0,y}^2u_{0,xx} + 4u_0u_{0,y}u_{0,xxy} \\ &+ 2u_0u_{0,xx}u_{0,yy} + u_{0,xxxxxx} + 2u_{0,xxxxyy} + u_{0,xxyyyy}\Big)t^2, \end{aligned} \tag{8}$$

$$\begin{aligned} u_3(x,y,t) = -\frac{1}{6}\Big(&30u_0u_{0,xyy}u_{0,xxxx} + 66u_{0,xx}^2u_{0,xyy} + 76u_0u_{0,x}u_{0,y}^2u_{0,xx} \\ &+ 92u_0^2u_{0,x}u_{0,xxy}u_{0,xxx} + 84u_0^2u_{0,y}u_{0,xx}u_{0,xy} + 46u_0^2u_{0,x}u_{0,xx}u_{0,yy} \\ &+ 160u_0u_{0,x}^2u_{0,y}u_{0,xy} + 3u_0^2u_{0,xxxxxx} + 3u_0^4u_{0,xxyy} + 216u_{0,xx}^2u_{0,xxx} \\ &+ 150u_{0,x}u_{0,xxx}^2 + 12u_0u_{0,y}u_{0,xxyyy} + 46u_0u_{0,xyy}u_{0,xxyy} \\ &+ 66u_0^3u_{0,xx}u_{0,xxx} + 3u_0^4u_{0,xxxxx} + 174u_0^2u_{0,x}^2u_{0,xxx} + 18u_0^5u_{0,x}u_{0,xx} \\ &+ 84u_0u_{0,xxx}u_{0,xxxx} + 168u_{0,x}u_{0,xy}u_{0,xxxy} + 12u_0u_{0,y}u_{0,xxxxy} \\ &+ 84u_{0,y}u_{0,xxx}u_{0,xxy} + 84u_{0,y}u_{0,xx}u_{0,xxxy} + 48u_{0,x}u_{0,y}u_{0,xxxy} \\ &+ 42u_0^3u_{0,x}u_{0,xxxx} + 264u_0u_{0,x}^3u_{0,xx} + 24u_{0,x}u_{0,yy}u_{0,xxxx} + u_0^6u_{0,xxx} \\ &+ 8u_0u_{0,yyy}u_{0,xxxy} + 30u_0^4u_{0,x}^3 + 56u_{0,x}u_{0,xy}u_{0,xyyy} + 84u_0u_{0,xy}u_{0,xxxy} \\ &+ 6u_0u_{0,yy}u_{0,xxxxx} + 14u_0^2u_{0,y}^2u_{0,xxx} + 24u_{0,y}u_{0,yy}u_{0,xxxy} + 8u_{0,x}^2u_{0,xyyyy} \\ &+ 14u_0u_{0,yy}u_{0,xxxyy} + 40u_{0,x}u_{0,yy}u_{0,xxyy} + 72u_{0,xy}u_{0,yy}u_{0,xxy} \\ &+ 6u_{0,x}u_{0,xx}u_{0,yyyy} + 24u_{0,xx}u_{0,xy}u_{0,yyy} + 36u_0u_{0,x}u_{0,xxxyy} \\ &+ 54u_0u_{0,xxx}u_{0,xxyy} + 24u_0^3u_{0,xx}u_{0,xyy} + 28u_{0,y}u_{0,xx}u_{0,xyyy} \\ &+ 8u_0u_{0,xx}u_{0,xyyyy} + 48u_{0,y}u_{0,xy}u_{0,xxxx} + 28u_0u_{0,xxy}u_{0,xyyy} \\ &+ 54u_0u_{0,xx}u_{0,xxxxx} + 24u_0u_{0,x}u_{0,xxxxx} + 54u_0u_{0,xx}u_{0,xxxyy} \\ &+ 234u_{0,x}u_{0,xx}u_{0,xxxx} + 48u_0u_{0,xy}u_{0,xxxxy} + 144u_{0,x}u_{0,xx}u_{0,xxyy} \\ &+ 76u_{0,y}u_{0,xxy}u_{0,xyy} + 24u_{0,x}^5 + 2u_0u_{0,xxx}u_{0,yyyy} + 24u_{0,x}^3u_{0,y}^2 \\ &+ 80u_{0,y}u_{0,xy}u_{0,xxyy} + 32u_0u_{0,xy}u_{0,xxyyy} + 32u_{0,x}u_{0,y}u_{0,xxyyy} \\ &+ 12u_0u_{0,x}u_{0,xxyyyy} + 28u_0u_{0,x}^3u_{0,yy} + 38u_{0,xx}u_{0,yy}u_{0,xyy} \\ &+ 42u_{0,xx}u_{0,yy}u_{0,xxx} + 24u_{0,x}u_{0,xxy}u_{0,yyy} + 240u_{0,xx}u_{0,xy}u_{0,xxy} \\ &+ 30u_0^3u_{0,x}u_{0,xxyy} + 222u_0^2u_{0,x}u_{0,xx}^2 + 92u_0^2u_{0,x}u_{0,xy}^2 + 6u_0^3u_{0,yy}u_{0,xxx} \\ &+ 36u_0^3u_{0,xy}u_{0,xxy} + 6u_0^2u_{0,xxxxyy} + 12u_0^3u_{0,y}u_{0,xxxy} + 68u_0^2u_{0,x}^2u_{0,xyy} \\ &+ 96u_{0,x}u_{0,xxx}u_{0,xyy} + 8u_{0,y}u_{0,xxx}u_{0,yyy} + 6u_{0,yy}^2u_{0,xxx} + 42u_{0,x}^2u_{0,xyy} \\ &+ 14u_{0,y}^2u_{0,xxyy} + 76u_{0,xy}^2u_{0,xyy} + 54u_{0,x}^2u_{0,xxxx} + 6u_{0,y}^2u_{0,xxxx} \\ &+ 84u_{0,xy}^2u_{0,xxx} + 54u_{0,x}^2u_{0,xxyy} + 120u_{0,x}u_{0,xy}^2u_{0,xxy} + 3u_0^2u_{0,xxyyyy} \\ &+ u_{0,xxxxxxxx} + 3u_{0,xxxxxxyy} + 3u_{0,xxxxyyyy} + u_{0,xxyyyyyy}\Big)t^3. \end{aligned} \tag{9}$$

Hence, we obtain the solution of Equation (1)

$$u(x,y,t) = f(x,y) + u_1(x,y,t) + u_2(x,y,t) + u_3(x,y,t) + \cdots, \tag{10}$$

where u_1, u_2 and u_3 are given by Equation (7), (8) and (9) respectively.

3. Application

In this section, we will study the single soliton, two-soliton and rational solutions of mZK equation.

3.1. Single Solitary Wave Solution

Consider the following case:

$$\begin{cases} u_t + u^2 u_x + u_{xxx} + u_{xyy} = 0, \\ u(x,y,0) = \dfrac{2k\sqrt{6\left(1+m^2\right)}\exp\left(k\left(x+my\right)\right)}{\exp\left(2k\left(x+my\right)\right)+1}. \end{cases} \tag{11}$$

From the above section, we can have

$$u_0(x,y,t) = \frac{2k\sqrt{6\left(1+m^2\right)}\exp\left(k\left(x+my\right)\right)}{\exp\left(2k\left(x+my\right)\right)+1}, \tag{12}$$

$$u_1(x,y,t) = \frac{2k^4\left(1+m^2\right)\sqrt{6\left(1+m^2\right)}\exp\left(k\left(x+my\right)\right)}{\left(\exp\left(2k\left(x+my\right)\right)+1\right)^2}\left(\exp\left(2k\left(x+my\right)\right)-1\right)t, \tag{13}$$

$$\begin{aligned} u_2(x,y,t) = {} & \frac{k^7\left(1+m^2\right)^2\sqrt{6\left(1+m^2\right)}\exp\left(k\left(x+my\right)\right)}{\left(\exp\left(2k\left(x+my\right)\right)+1\right)^3} \\ & \times\left(\exp\left(4k\left(x+my\right)\right)-6\exp\left(2k\left(x+my\right)\right)+1\right)t^2, \end{aligned} \tag{14}$$

$$\begin{aligned} u_3(x,y,t) = {} & \frac{k^{10}\left(1+m^2\right)^3\sqrt{6\left(1+m^2\right)}\exp\left(k\left(x+my\right)\right)}{3\left(\exp\left(2k\left(x+my\right)\right)+1\right)^4}\left(\exp\left(6k\left(x+my\right)\right)\right. \\ & \left.-23\exp\left(4k\left(x+my\right)\right)+23\exp\left(2k\left(x+my\right)\right)-1\right)t^3, \end{aligned} \tag{15}$$

$$\begin{aligned} u(x,y,t) = {} & \frac{2k\sqrt{6\left(1+m^2\right)}\exp\left(k\left(x+my\right)\right)}{\exp\left(2k\left(x+my\right)\right)+1} \\ & + \frac{2k^4\left(1+m^2\right)\sqrt{6\left(1+m^2\right)}\exp\left(k\left(x+my\right)\right)}{\left(\exp\left(2k\left(x+my\right)\right)+1\right)^2}\left(\exp\left(2k\left(x+my\right)\right)-1\right)t \\ & + \frac{k^7\left(1+m^2\right)^2\sqrt{6\left(1+m^2\right)}\exp\left(k\left(x+my\right)\right)}{\left(\exp\left(2k\left(x+my\right)\right)+1\right)^3}\left(\exp\left(4k\left(x+my\right)\right)\right. \\ & \left.-6\exp\left(2k\left(x+my\right)\right)+1\right)t^2 + \frac{k^{10}\left(1+m^2\right)^3\sqrt{6\left(1+m^2\right)}\exp\left(k\left(x+my\right)\right)}{3\left(\exp\left(2k\left(x+my\right)\right)+1\right)^4} \\ & \times\left(\exp\left(6k\left(x+my\right)\right)-23\exp\left(4k\left(x+my\right)\right)+23\exp\left(2k\left(x+my\right)\right)-1\right)t^3+\cdots. \end{aligned} \tag{16}$$

Using Taylor series, one can obtain the exact solution

$$u(x,y,t)=\frac{2k\sqrt{6(1+m^2)}\exp\left(k\left(x+my-k^2(1+m^2)t\right)\right)}{\exp\left(2k\left(x+my-k^2(1+m^2)t\right)\right)+1}. \tag{17}$$

Figure 1 shows the single soliton (17) for $k=1$, $m=1$, $-4\le x\le 4$, $-4\le y\le 4$ and $t=0$, from which one can find Equation (17) is a single-soliton solution.

3.2. Two Solitary Waves Solution

In this case, we take

$$f(x,y)=\frac{4\sqrt{6(1+m^2)}\exp(x+my)}{\exp(2(x+my))+1} \tag{18}$$

Then from the above section, one can have

$$u_0(x,y,t)=\frac{4\sqrt{6(1+m^2)}\exp(x+my)}{\exp(2(x+my))+1}, \tag{19}$$

$$\begin{aligned}u_1(x,y,t)=&\frac{4(1+m^2)\sqrt{6(1+m^2)}\exp(x+my)}{\left(\exp(2(x+my))+1\right)^4}\left(\exp(6(x+my))\right.\\&\left.+73\exp(4(x+my))-73\exp(2(x+my))-1\right)t,\end{aligned} \tag{20}$$

$$\begin{aligned}u_2(x,y,t)=&\frac{2(1+m^2)^2\sqrt{6(1+m^2)}\exp(x+my)}{\left(\exp(2(x+my))+1\right)^7}\left(\exp(12(x+my))\right.\\&+2158\exp(10(x+my))+2863\exp(8(x+my))\\&-26236\exp(6(x+my))+2863\exp(4(x+my))\\&\left.+2158\exp(2(x+my))+1\right)t^2,\end{aligned} \tag{21}$$

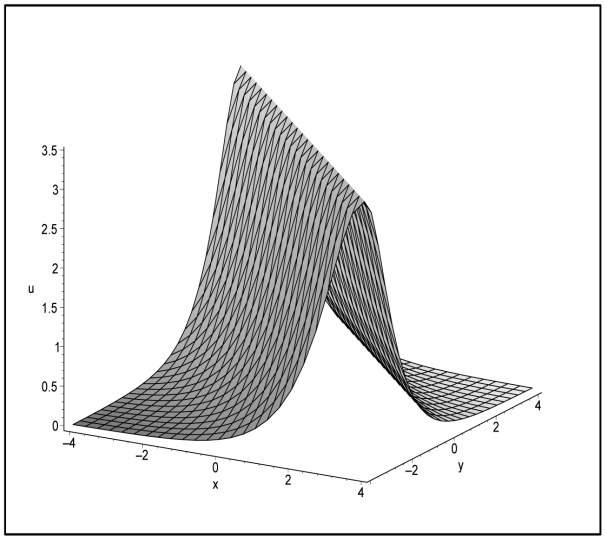

Figure 1. 3D plot of solution (17) for $k=-1$.

$$u_3(x,y,t)=\frac{2\left(1+m^2\right)^3\sqrt{6\left(1+m^2\right)}\exp(x+my)}{3\left(\exp\left(2(x+my)\right)+1\right)^{10}}\left(\exp\left(18(x+my)\right)\right.$$
$$+58951\exp\left(16(x+my)\right)+225620\exp\left(14(x+my)\right)$$
$$-1999268\exp\left(12(x+my)\right)-6147250\exp\left(10(x+my)\right) \quad (22)$$
$$+6147250\exp\left(8(x+my)\right)+1999268\exp\left(6(x+my)\right)$$
$$\left.-225620\exp\left(4(x+my)\right)-58951\exp\left(2(x+my)\right)-1\right)t^3,$$

$$u(x,y,t)=\frac{4\sqrt{6\left(1+m^2\right)}\exp(x+my)}{\exp\left(2(x+my)\right)+1}+\frac{4\left(1+m^2\right)\sqrt{6\left(1+m^2\right)}\exp(x+my)}{\left(\exp\left(2(x+my)\right)+1\right)^4}$$
$$\times\left(\exp\left(6(x+my)\right)+73\exp\left(4(x+my)\right)-73\exp\left(2(x+my)\right)-1\right)t$$
$$+\frac{2\left(1+m^2\right)^2\sqrt{6\left(1+m^2\right)}\exp(x+my)}{\left(\exp\left(2(x+my)\right)+1\right)^7}\left(\exp\left(12(x+my)\right)\right.$$
$$+2158\exp\left(10(x+my)\right)+2863\exp\left(8(x+my)\right)-26236\exp\left(6(x+my)\right)$$
$$\left.+2863\exp\left(4(x+my)\right)+2158\exp\left(2(x+my)\right)+1\right)t^2$$

$$+\frac{2\left(1+m^2\right)^3\sqrt{6\left(1+m^2\right)}\exp(x+my)}{3\left(\exp\left(2(x+my)\right)+1\right)^{10}}\left(\exp\left(18(x+my)\right)\right.$$
$$+58951\exp\left(16(x+my)\right)+225620\exp\left(14(x+my)\right)$$
$$-1999268\exp\left(12(x+my)\right)-6147250\exp\left(10(x+my)\right) \quad (23)$$
$$+6147250\exp\left(8(x+my)\right)+1999268\exp\left(6(x+my)\right)$$
$$\left.-225620\exp\left(4(x+my)\right)-58951\exp\left(2(x+my)\right)-1\right)t^3+\cdots.$$

Using Taylor series, one can obtain the exact solution

$$u(x,y,t)$$
$$=\frac{4\sqrt{6\left(1+m^2\right)}\left(\exp(\xi-\eta)+3\exp(27\xi-3\eta)+3\exp(29\xi-5\eta)+\exp(55\xi-7\eta)\right)}{1+4\exp(2\xi-2\eta)+6\exp(28\xi-4\eta)+4\exp(54\xi-6\eta)+\exp(56\xi-8\eta)}, \quad (24)$$

where $\xi=\left(1+m^2\right)t$ and $\eta=x+my$.

Figure 2 shows the two-soliton solution (24) for $m=1$, $-10\le x\le 5$, $-10\le y\le 5$ and $t=-0.2$, from which one can find Equation (24) is a two-soliton solution. Figure 3 shows the two-soliton solution (24) for $m=1$, $-8\le x\le 8$ $-8\le y\le 8$ and $t=-0.1$. Figure 4 shows the two-soliton solution (24) for $m=1$, $-10\le x\le 10$ $-10\le y\le 10$ and $t=0$. Figure 5 shows the two-soliton solution (24) for $m=1$, $-8\le x\le 8$ $-8\le y\le 8$ and $t=0.1$. Figure 6 shows the two-soliton solution (24) for $m=1$, $-5\le x\le 10$ $-5\le y\le 10$ and $t=0.2$. Figures 2-6 show the velocities of the two solitons are different.

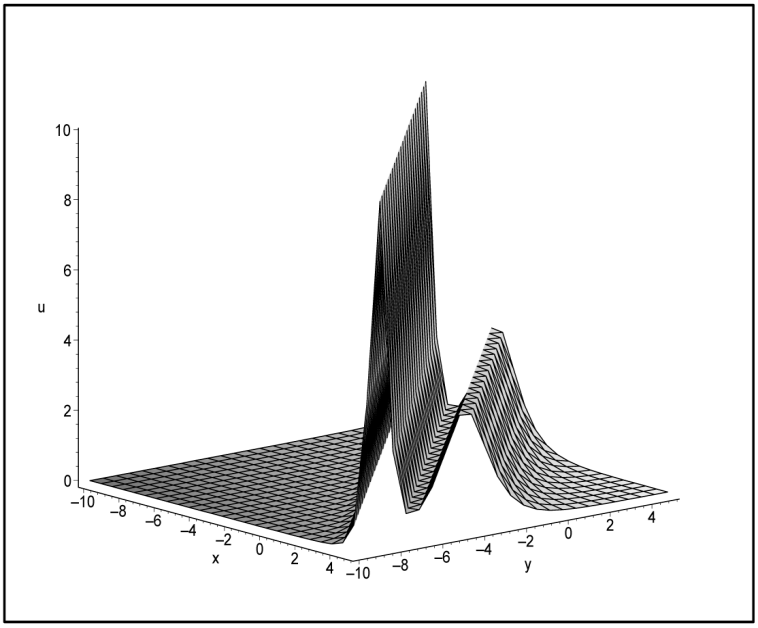

Figure 2. 3D plot of solution (24) for $m=1$ and $t=-0.2$.

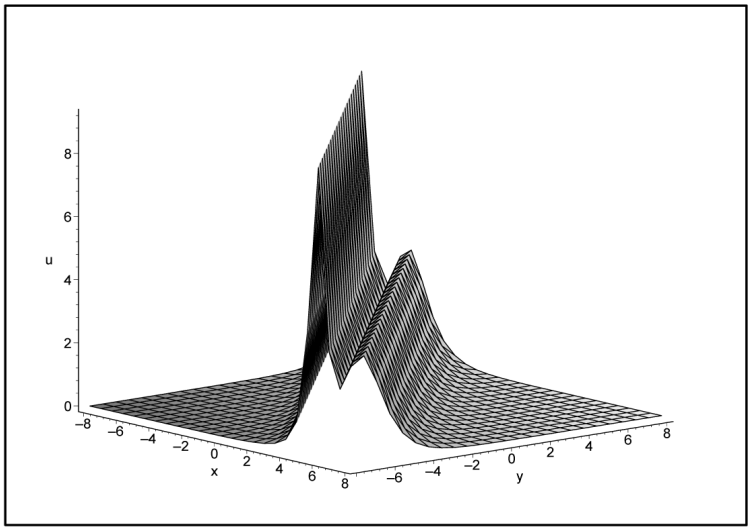

Figure 3. 3D plot of solution (24) for $m=1$ and $t=-0.1$.

3.3. Rational Solution

Here, our goal is to find the rational solution of mZK equation. To do this, we consider the form of the initial value as follows:

$$f(x,y)=\frac{2I\sqrt{6\left(1+m^2\right)}}{x+my-a}. \tag{25}$$

Due to the above section, it is obtained

$$u_0(x,y,t)=\frac{2I\sqrt{6\left(1+m^2\right)}}{x+my-a}, \tag{26}$$

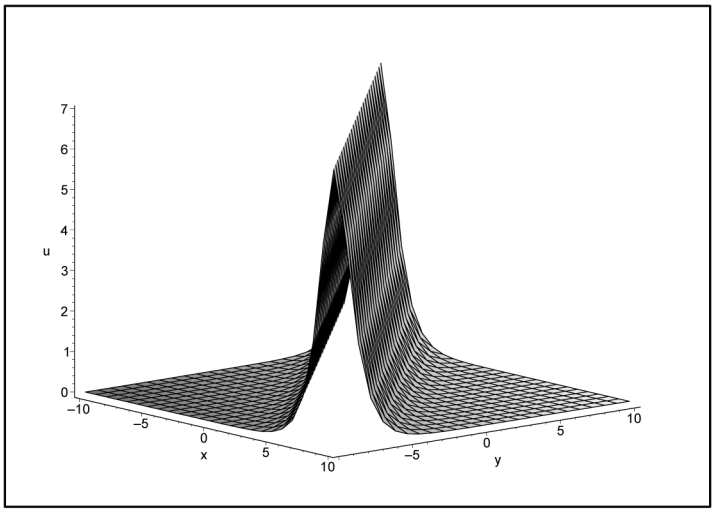

Figure 4. 3D plot of solution (24) for $m=1$ and $t=0$.

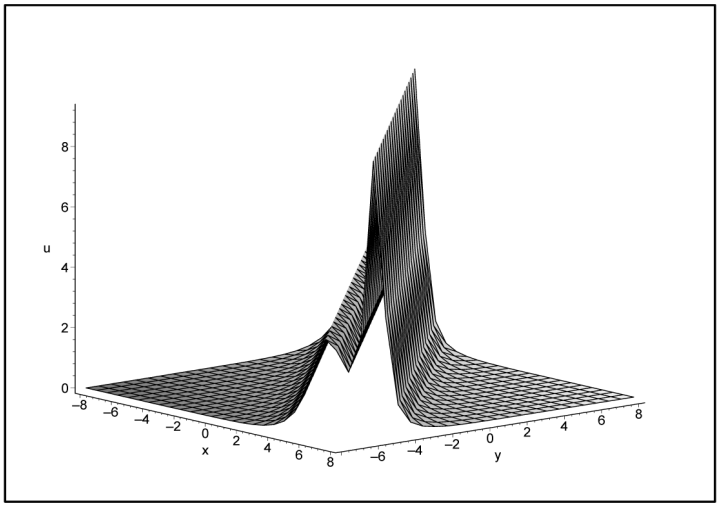

Figure 5. 3D plot of solution (24) for $m=1$ and $t=0.1$.

$$u_1(x,y,t)=-\frac{36I\sqrt{6\left(1+m^2\right)}\left(1+m^2\right)}{\left(x+my-a\right)^4}t, \tag{27}$$

$$u_2(x,y,t)=\frac{432I\sqrt{6\left(1+m^2\right)}\left(1+m^2\right)^2}{\left(x+my-a\right)^7}t^2, \tag{28}$$

$$u_3(x,y,t)=-\frac{5184I\sqrt{6\left(1+m^2\right)}\left(1+m^3\right)}{\left(x+my-a\right)^{10}}t^3, \tag{29}$$

$$\begin{aligned}u(x,y,t)=&\frac{2I\sqrt{6\left(1+m^2\right)}}{x+my-a}-\frac{36I\sqrt{6\left(1+m^2\right)}\left(1+m^2\right)}{\left(x+my-a\right)^4}t\\&+\frac{432I\sqrt{6\left(1+m^2\right)}\left(1+m^2\right)^2}{\left(x+my-a\right)^7}t^2-\frac{5184I\sqrt{6\left(1+m^2\right)}\left(1+m^3\right)}{\left(x+my-a\right)^{10}}t^3+\cdots.\end{aligned} \tag{30}$$

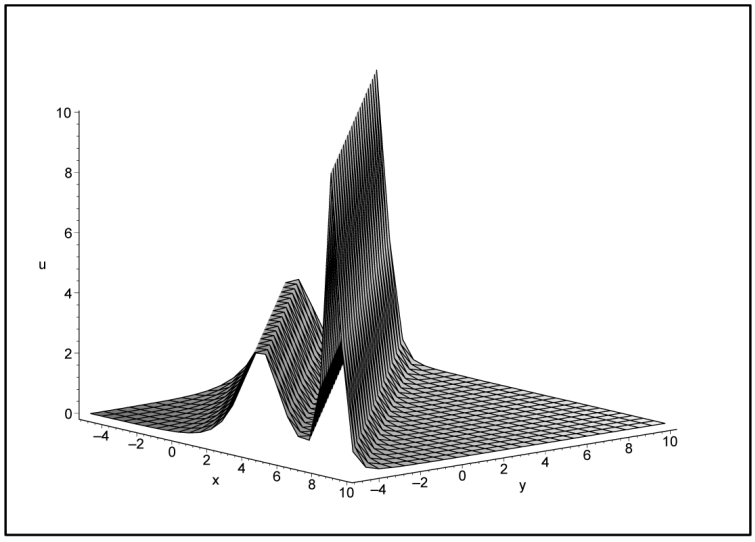

Figure 6. 3D plot of solution (24) for $m=1$ and $t=0.2$.

From the knowledge of Taylor series, one can get the exact solution

$$u(x,y,t)=\frac{2I\sqrt{6\left(1+m^2\right)}\left(\left(x+my-a\right)^3-6\left(1+m^2\right)t\right)}{\left(x+my-a\right)\left(\left(x+my-a\right)^3+12\left(1+m^2\right)t\right)}, \tag{31}$$

which is singular at $x+my=a$ or $\left(x+my-a\right)^3+12\left(1+m^2\right)t=0$.

4. Conclusion

In summary, we successfully apply homotopy perturbation method to the mZK equation with the initial value problem and obtain the analytical approximate solution of the mZK equation. Using the form of the initial value, the single solitary wave, two solitary waves and rational solutions of the mZK are obtained. Here, we get the two-soliton solution without using bilinear forms, Wronskian. In our later works, we will focus on the form of the initial value that can create the two solitary waves solutions.

Acknowledgements

This work was supported by the National Natural Science Foundation of China under Grant No. 11305048, the Science and Technology Research Key Project of Education Department of Henan Province under Grant No. 13A110329, the Basic and Frontier Research Program of Henan Province under Grant No. 132300410223, the Doctor Foundation of Henan Polytechnic University under Grant No. B2011-006, and the Key Teacher Foundation of Henan Polytechnic University (Grant 2014).

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