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To the Solution of Three-Dimensional Problems of Oscillations in the Theory of Elasticity in Thick Plates of Variable Thickness

Makhamatali K. Usarov

Institute of Mechanics and Seismic Stability of Structures, the Academy of Sciences, Tashkent, Uzbekistan

Email: umakhamatali@mail.ru

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Abstract

A technique for solving the three-dimensional problem of bending in the theory of elasticity in orthotropic plates of variable thickness is developed in the paper. On the basis of the method of expansion of displacements into an infinite series, the problem has been reduced to the solutions of two independent problems, which are described by two independent systems of two-dimensional infinite equations.

Keywords

Plate, Orthotropy, Infinite Series, Equation of Motion, Boundary Conditions, Bending and Oscillations

1. Introduction

Published papers on the calculation of plates of variable thickness are realized within the framework of classical theory of plates, developed with a number of simplifying hypotheses. It should be noted that in the places where the thickness of a plate of variable thickness is the greatest, it is not advisable to apply any simplifying hypotheses. Scientific researches have shown that in calculations of thick plates, it is necessary to take into account not only the moments and forces, but the bimoments as well. The account of bimoments in the sections of the plate is based on the application of the method of expansion of displacements into an infinite series along one of spatial coordinates directed along the normal.

In published literature, there are a number of papers devoted to the investigation of this problem. The law of thickness variation of plates and shells is taken in the form of linear, quadratic and other functions. Various problems of the behavior of thin-walled structural elements under the action of static and dynamic loads are considered.

A wide survey of papers devoted to plates of variable thickness is given in [1] [2]. In the monograph [1], static problems of bending of plates of variable thickness of different outlines under different types of loading are considered. The author's dissertation [2] is devoted to the dynamic and static problems of plates of constant and variable thickness.

In [3], dynamic problems of oscillation of plates of variable thickness are considered. The law of thickness variation is taken in the form of a quadratic function. The paper [4] is devoted to the study of free vibrations of plates of variable thickness of arbitrary shape in plan. The equations of oscillation of a plate of variable thickness are derived from the Kirchhoff's hypothesis. A methodology for the numerical solution of the problem is developed. Numerical results are obtained and compared with the results of other authors.

In [5], oscillations of super-elliptical plates of variable thickness with a fixed and freely supported edge are investigated. An analytical study of acoustic radiation from thin circular plates of linearly variable thickness is carried out in [6]. A review given in [5] [6], allows us to evaluate the state-of-the-art of the investigated problem. In [7], the stability problems of a thin-walled cylindrical shell of variable thickness are considered. The law of thickness variation is taken as a linear function.

In [8] [9] [10] [11], geometrically nonlinear problems of plates and shells of variable thickness with different systems of approximating functions are considered. In [8] [9], the problems of studying vibrations of viscoelastic plates of variable thickness are considered. The problems set on the basis of the Bubnov-Galerkin method are reduced to the solution of ordinary differential equations.

Monographs [10] [11] are devoted to the development of the methods for solving geometrically nonlinear problems of plates and shells of variable thickness under various loading options. In [12], geometrically nonlinear problems of determining the stress-strain state of plates of variable thickness with different systems of approximating functions are considered.

2. Statement of the Problem

As an object of investigation, a thick orthotropic elastic plate of variable thickness has been chosen; the plate is located between two asymmetric face surfaces $z=h_1(x_1,x_2)$, $z=-h_2(x_1,x_2)$. Let $h_1(x_1,x_2)\ge h_2(x_1,x_2)\ge 0$, then the thickness of the plate is $H=h_1(x_1,x_2)+h_2(x_1,x_2)$.

In contrast to classical theory of plates, the components of the displacement vector are defined as functions of three spatial coordinates and times $u_1(x_1,x_2,z,t)$, $u_2(x_1,x_2,z,t)$, $u_3(x_1,x_2,z,t)$. The components of the strain tensor are determined by the Cauchy relations. The plate is considered as a three-dimensional orthotropic body [13] [14] [15] [16], the material of which obeys the generalized

Hooke's law:

$$\begin{aligned}&\sigma_{11}=E_{11}\varepsilon_{11}+E_{12}\varepsilon_{22}+E_{13}\varepsilon_{33},\\&\sigma_{22}=E_{21}\varepsilon_{11}+E_{22}\varepsilon_{22}+E_{23}\varepsilon_{33},\\&\sigma_{12}=2G_{12}\varepsilon_{12},\ \sigma_{13}=2G_{13}\varepsilon_{13},\ \sigma_{23}=2G_{23}\varepsilon_{23},\end{aligned}\tag{1}$$

where $E_{11},E_{12},\cdots,E_{33}$ —are the elastic constants determined by Poisson's coefficients and elastic moduli, given in [15] [16], G_{12},G_{13},G_{23} —are the shear moduli of plate material.

For the equations of motion of the plate we would use the three-dimensional equations of motion of the theory of elasticity:

$$\frac{\partial\sigma_{i1}}{\partial x_1}+\frac{\partial\sigma_{i2}}{\partial x_2}+\frac{\partial\sigma_{i3}}{\partial z}=\rho\ddot{u}_i,\ (i=1,2,3).\tag{2}$$

Here ρ —is the density of plate material.

The boundary conditions on the lower and upper surfaces of the plate $z=h_1(x_1,x_2)$ and $z=-h_2(x_1,x_2)$ are written relative to the generalized external forces in the form:

$$\begin{aligned}&\sigma_{33}=q_3^{(+)},\quad\sigma_{31}=q_1^{(+)},\ \sigma_{32}=q_2^{((+))},\quad\text{at } z=h_1(x_1,x_2),\\&\sigma_{33}=q_3^{(-)},\quad\sigma_{31}=q_1^{(-)},\ \sigma_{32}=q_2^{(-)},\quad\text{at } z=-h_2(x_1,x_2).\end{aligned}\tag{3}$$

3. Method of Solution

The solution of the set problem of the theory of elasticity for thick plates of variable thicknesses (1), (2), and (3) is built by the method of expansion of the components of the displacement vector into the Maclaurin series [13] [14] [15] [16]:

$$\begin{aligned}&u_k=B_0^{(k)}+B_1^{(k)}\frac{z}{h_1}+B_2^{(k)}\left(\frac{z}{h_1}\right)^2+B_3^{(k)}\left(\frac{z}{h_1}\right)^3+\cdots+B_m^{(k)}\left(\frac{z}{h_1}\right)^m+\cdots,\ (k=1,2)\\&u_3=A_0+A_1\frac{z}{h_1}+A_2\left(\frac{z}{h_1}\right)^2+A_3\left(\frac{z}{h_1}\right)^3+\cdots+A_m\left(\frac{z}{h_1}\right)^m+\cdots.\end{aligned}\tag{4}$$

Here $B_m^{(k)},A_m$ —are the unknown functions of two spatial coordinates and time:

$$\begin{aligned}&B_m^{(k)}=B_m^{(k)}(x_1,x_2,t)=\frac{h_1^m}{m!}\left(\frac{\partial^m u_k}{\partial z^m}\right)_{z=0},(k=1,2),\\&A_m=A_m(x_1,x_2,t)=\frac{h_1^m}{m!}\left(\frac{\partial^m u_3}{\partial z^m}\right)_{z=0}\end{aligned}.\tag{5}$$

On the basis of expansion (4), the components of the strain and stress tensor also expand into the Maclaurin series in the form:

$$\varepsilon_{ij}=\varepsilon_{ij}^{(0)}+\varepsilon_{ij}^{(1)}\frac{z}{h_1}+\varepsilon_{ij}^{(2)}\left(\frac{z}{h_1}\right)^2+\varepsilon_{ij}^{(3)}\left(\frac{z}{h_1}\right)^3+\cdots+\varepsilon_{ij}^{(m)}\left(\frac{z}{h_1}\right)^m+\cdots\quad(i,j=1,2,3),\tag{6}$$

$$\sigma_{ij}=\sigma_{ij}^{(0)}+\sigma_{ij}^{(1)}\frac{z}{h_1}+\sigma_{ij}^{(2)}\left(\frac{z}{h_1}\right)^2+\sigma_{ij}^{(3)}\left(\frac{z}{h_1}\right)^3+\cdots+\sigma_{ij}^{(m)}\left(\frac{z}{h_1}\right)^m+\cdots\quad(i,j=1,2,3).\tag{7}$$

Here, the expansion coefficients are defined as:

$$\varepsilon_{ij}^{(m)} = \varepsilon_{ij}^{(m)}\left(x_1, x_2, t\right) = \frac{h_1^m}{m!}\left(\frac{\partial^m \varepsilon_{ij}}{\partial z^m}\right)_{z=0},$$

$$\sigma_{ij}^{(m)} = \sigma_{ij}^{(m)}\left(x_1, x_2, t\right) = \frac{h_1^m}{m!}\left(\frac{\partial^m \sigma_{ij}}{\partial z^m}\right)_{z=0} \quad \left(m = 1, 2, 3, \cdots\right).$$

Based on the Cauchy relation and expansion (4), we obtain the expressions for the expansion of strain (6) of thick plates of variable thickness. The components of elongation strain are:

$$\varepsilon_{11}^{(m)} = \frac{\partial B_m^{(1)}}{\partial x_1} - B_m^{(1)} \frac{m}{h_1}\frac{\partial h_1}{\partial x_1}, \tag{8a}$$

$$\varepsilon_{22}^{(m)} = \frac{\partial B_m^{(2)}}{\partial x_2} - B_m^{(2)} \frac{m}{h_1}\frac{\partial h_1}{\partial x_2}, \tag{8b}$$

$$\varepsilon_{33}^{(m)} = \frac{\left(m+1\right) A_{m+1}}{h_1}. \tag{8c}$$

Components of angle strain are written in the form:

$$\varepsilon_{12}^{(m)} = \frac{1}{2}\left(\frac{\partial B_m^{(1)}}{\partial x_2} + \frac{\partial B_m^{(2)}}{\partial x_1} - \frac{m}{h_1}\frac{\partial h_1}{\partial x_2} B_m^{(1)} - \frac{m}{h_1}\frac{\partial h_1}{\partial x_1} B_m^{(2)}\right), \tag{9a}$$

$$\varepsilon_{13}^{(m)} = \frac{1}{2}\left(\left(m+1\right)\frac{B_{m+1}^{(1)}}{h_1} + \frac{\partial A_m}{\partial x_1} - \frac{A_m}{h_1}\frac{\partial h_1}{\partial x_1}\right), \tag{9b}$$

$$\varepsilon_{23}^{(m)} = \frac{1}{2}\left(\left(m+1\right)\frac{B_{m+1}^{(2)}}{h_1} + \frac{\partial A_m}{\partial x_2} - \frac{A_m}{h_1}\frac{\partial h_1}{\partial x_2}\right). \tag{9c}$$

Based on Hooke's law (1) and expansions (5) and (7), we obtain the expressions for the coefficients of stress expansion (7). The coefficients of expansion of normal stresses are defined as:

$$\begin{aligned}
\sigma_{11}^{(m)} &= E_{11}\varepsilon_{11}^{(m)} + E_{12}\varepsilon_{22}^{(m)} + E_{13}\varepsilon_{33}^{(m)}, \\
\sigma_{22}^{(m)} &= E_{21}\varepsilon_{11}^{(m)} + E_{22}\varepsilon_{22}^{(m)} + E_{23}\varepsilon_{33}^{(m)}, \\
\sigma_{33}^{(m)} &= E_{31}\varepsilon_{11}^{(m)} + E_{32}\varepsilon_{22}^{(m)} + E_{33}\varepsilon_{33}^{(m)}.
\end{aligned} \tag{10a}$$

The coefficients of the expansion of tangential stresses are defined as:

$$\sigma_{12}^{(m)} = 2G_{12}\varepsilon_{12}^{(m)}, \quad \sigma_{13}^{(m)} = 2G_{13}\varepsilon_{13}^{(m)}, \quad \sigma_{23}^{(m)} = 2G_{23}\varepsilon_{23}^{(m)}, \tag{10b}$$

where $m = 0, 1, 2, 3, \cdots$.

Based on expansion (4), we would demonstrate that the proposed elasticity problem in plates of variable thickness is described by two unrelated problems, each of which is formulated on the basis of a system of infinite recurrent two-dimensional equations with the corresponding boundary conditions. The first system of recurrent equations has the form:

$$\frac{\partial \sigma_{11}^{(2m)}}{\partial x_1} - \frac{2m\sigma_{11}^{(2m)}}{h_1}\frac{\partial h_1}{\partial x_1} + \frac{\partial \sigma_{12}^{(2m)}}{\partial x_2} - \frac{2m\sigma_{12}^{(2m)}}{h_1}\frac{\partial h_1}{\partial x_2} + \frac{\left(2m+1\right)\sigma_{13}^{(2m+1)}}{h_1} = \rho \ddot{B}_{2m}^{(1)}, \tag{11a}$$

$$\frac{\partial \sigma_{21}^{(2m)}}{\partial x_1} - \frac{2m\sigma_{21}^{(2m)}}{h_1}\frac{\partial h_1}{\partial x_1} + \frac{\partial \sigma_{22}^{(2m)}}{\partial x_2} - \frac{2m\sigma_{22}^{(2m)}}{h_1}\frac{\partial h_1}{\partial x_2} + \frac{(2m+1)\sigma_{23}^{(2m+1)}}{h_1} = \rho \ddot{B}_{2m}^{(2)}, \tag{11b}$$

$$\begin{aligned}&\frac{\partial \sigma_{31}^{(2m+1)}}{\partial x_1} - \frac{(2m+1)\sigma_{31}^{(2m+1)}}{h_1}\frac{\partial h_1}{\partial x_1} + \frac{\partial \sigma_{32}^{(2m+1)}}{\partial x_2} - \frac{(2m+1)\sigma_{32}^{(2m+1)}}{h_1}\frac{\partial h_1}{\partial x_2} + \\ &\frac{(2m+2)\sigma_{33}^{(2m+2)}}{h_1} = \rho \ddot{A}_{2m+1}\end{aligned}. \tag{11c}$$

Here $m = 0,1,2,3,\cdots$.

The second system of recurrent equations has the form:

$$\begin{aligned}&\frac{\partial \sigma_{11}^{(2m+1)}}{\partial x_1} - \frac{(2m+1)\sigma_{11}^{(2m+1)}}{h_1}\frac{\partial h_1}{\partial x_1} + \frac{\partial \sigma_{12}^{(2m+1)}}{\partial x_2} - \frac{(2m+1)\sigma_{12}^{(2m+1)}}{h_1}\frac{\partial h_1}{\partial x_2} + \\ &\frac{(2m+2)\sigma_{13}^{(2m+2)}}{h_1} = \rho \ddot{B}_{2m+1}^{(1)}\end{aligned}, \tag{12a}$$

$$\begin{aligned}&\frac{\partial \sigma_{12}^{(2m+1)}}{\partial x_1} - \frac{(2m+1)\sigma_{12}^{(2m+1)}}{h_1}\frac{\partial h_1}{\partial x_1} + \frac{\partial \sigma_{22}^{(2m+1)}}{\partial x_2} - \frac{(2m+1)\sigma_{22}^{(2m+1)}}{h_1}\frac{\partial h_1}{\partial x_2} + \\ &\frac{(2m+2)\sigma_{23}^{(2m+2)}}{h_1} = \rho \ddot{B}_{2m+1}^{(2)}\end{aligned}, \tag{12b}$$

$$\frac{\partial \sigma_{31}^{(2m)}}{\partial x_1} - \frac{2m\sigma_{31}^{(2m)}}{h_1}\frac{\partial h_1}{\partial x_1} + \frac{\partial \sigma_{32}^{(2m)}}{\partial x_2} - \frac{2m\sigma_{32}^{(2m)}}{h_1}\frac{\partial h_1}{\partial x_2} + \frac{(2m+1)\sigma_{33}^{(2m+1)}}{h_1} = \rho \ddot{A}_{2m}. \tag{12c}$$

Here $m = 0,1,2,3,\cdots$.

On the basis of expansion (7), the boundary conditions on the surface of the plate $z = +h_1(x_1, x_2)$ (3a) are rewritten as:

$$\sigma_{31}^{(+)} = \sigma_{31}^{(0)} + \sigma_{31}^{(1)} + \sigma_{31}^{(2)} + \sigma_{31}^{(3)} + \sigma_{31}^{(4)} + \sigma_{31}^{(5)} + \cdots = q_1^{(+)} \tag{13a}$$

$$\sigma_{32}^{(+)} = \sigma_{32}^{(0)} + \sigma_{32}^{(1)} + \sigma_{32}^{(2)} + \sigma_{32}^{(3)} + \sigma_{32}^{(4)} + \sigma_{32}^{(5)} + \cdots = q_2^{(+)} \tag{13b}$$

$$\sigma_{33}^{(+)} = \sigma_{33}^{(0)} + \sigma_{33}^{(1)} + \sigma_{33}^{(2)} + \sigma_{33}^{(3)} + \sigma_{3k}^{(4)} + \sigma_{33}^{(5)} + \cdots = q_3^{(+)} \tag{13c}$$

On the basis of expansion (7), the boundary conditions on the surface of the plate $z = -h_2(x_1, x_2)$ (3b) are rewritten as:

$$\sigma_{31}^{(-)} = \sigma_{31}^{(0)} - \alpha\sigma_{31}^{(1)} + \alpha^2\sigma_{31}^{(2)} - \alpha^3\alpha_{31}^{(3)} + \alpha^4\sigma_{31}^{(4)} - \alpha^5\sigma_{31}^{(5)} + \cdots = q_1^{(-)}, \tag{14a}$$

$$\sigma_{32}^{(-)} = \sigma_{32}^{(0)} - \alpha\sigma_{32}^{(1)} + \alpha^2\sigma_{32}^{(2)} - \alpha^3\alpha_{32}^{(3)} + \alpha^4\sigma_{32}^{(4)} - \alpha^5\sigma_{32}^{(5)} + \cdots = q_2^{(-)}, \tag{14b}$$

$$\sigma_{33}^{(-)} = \sigma_{33}^{(0)} - \alpha\sigma_{33}^{(1)} + \alpha^2\sigma_{33}^{(2)} - \alpha^3\alpha_{33}^{(3)} + \alpha^4\sigma_{33}^{(4)} - \alpha^5\sigma_{33}^{(5)} + \cdots = q_3^{(-)}, \tag{14c}$$

where $\alpha = \dfrac{h_2(x_1, x_2)}{h_1(x_1, x_2)}$.

Two independent systems of boundary conditions relative to the expansion coefficients (4) and (7) correspond to each boundary condition at the edges of the plate.

If the displacements at the edge of the plate are equal to zero, then we get:

$$\begin{aligned}&B_0^{(1)} = 0,\ \ B_0^{(2)} = 0,\ \ B_2^{(1)} = 0,\ \ B_2^{(2)} = 0,\ B_4^{(1)} = 0,\ \ B_4^{(2)} = 0,\cdots \\ &A_1 = 0,\ \ A_3 = 0,\ \ A_5 = 0,\cdots\end{aligned}, \tag{15a}$$

$$\begin{aligned}&B_1^{(1)}=0,\ \ B_1^{(2)}=0,\ \ B_3^{(1)}=0,\ \ B_3^{(2)}=0,\ \ B_5^{(1)}=0,\ \ B_5^{(2)}=0,\cdots\\&A_0=0,\ \ A_2=0,\ \ A_4=0,\cdots\end{aligned}. \tag{15b}$$

If the edge of the plate is free of supports, then the boundary conditions have the form:

$$\begin{aligned}&\sigma_{11}^{(0)}=0,\ \ \sigma_{12}^{(0)}=0,\ \ \sigma_{13}^{(1)}=0,\ \sigma_{11}^{(2)}=0,\ \ \sigma_{12}^{(2)}=0,\cdots\\&\sigma_{13}^{(3)}=0,\ \ \sigma_{11}^{(4)}=0,\ \ \sigma_{12}^{(4)}=0,\ \ \sigma_{13}^{(5)}=0,\cdots\end{aligned}, \tag{16a}$$

$$\begin{aligned}&\sigma_{11}^{(1)}=0,\ \ \sigma_{12}^{(1)}=0,\ \ \sigma_{13}^{(0)}=0,\ \sigma_{11}^{(3)}=0,\ \ \sigma_{12}^{(3)}=0,\cdots\\&\sigma_{13}^{(2)}=0,\ \ \sigma_{11}^{(5)}=0,\ \ \sigma_{12}^{(5)}=0,\ \ \sigma_{13}^{(4)}=0,\cdots\end{aligned}. \tag{16b}$$

If the edge of the plate is supported, then the boundary conditions have the form:

$$\begin{aligned}&\sigma_{11}^{(0)}=0,\ \ \sigma_{12}^{(0)}=0,\ \ A_1=0,\ \ \sigma_{11}^{(2)}=0,\ \ \sigma_{12}^{(2)}=0,\cdots\\&A_3=0,\ \ \sigma_{11}^{(4)}=0,\ \ \sigma_{12}^{(4)}=0,\ \ A_5=0,\cdots\end{aligned}, \tag{17a}$$

$$\begin{aligned}&\sigma_{11}^{(1)}=0,\ \ \sigma_{12}^{(1)}=0,\ \ A_0=0,\ \ \sigma_{11}^{(3)}=0,\ \ \sigma_{12}^{(3)}=0,\ldots\\&A_2=0,\ \ \sigma_{11}^{(5)}=0,\ \ \sigma_{12}^{(5)}=0,\ \ A_4=0,\ldots\end{aligned}. \tag{17b}$$

If the edge of the plate is supported and there is no displacement along the tangential direction to the plate contour, then the boundary conditions have the form:

$$\begin{aligned}&\sigma_{11}^{(0)}=0,\ \ B_0^{(2)}=0,\ \ A_1=0,\ \ \sigma_{11}^{(2)}=0,\ \ B_2^{(2)}=0,\ \ A_3=0,\cdots\\&\sigma_{11}^{(4)}=0,\ \ B_4^{(2)}=0,\ \ A_5=0,\cdots\end{aligned}, \tag{18a}$$

$$\begin{aligned}&\sigma_{11}^{(1)}=0,\ \ B_1^{(1)}=0,\ \ A_0=0,\ \ \sigma_{11}^{(3)}=0,\ \ B_3^{(1)}=0,\ \ A_2=0,\cdots\\&\sigma_{11}^{(5)}=0,\ \ B_5^{(1)}=0,\ \ A_4=0,\cdots\end{aligned}. \tag{18b}$$

4. Conclusion

Thus, a three-dimensional bending and elasticity problem for thick plates of variable thickness is set and reduced to the solutions of two independent two-dimensional problems, which are described by two dependent systems of infinite recurrent differential equations in partial derivatives. It should be noted that the equations of motion for the bimoment theory of plates of constant thickness developed in [13] [14] [15] [16] could be obtained from the constructed systems of equations. It could be shown that to obtain numerical results with satisfactory accuracy, it is sufficient to take into account eight terms of the series (4).

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