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Electromagnetic Self-Force Mechanisms and Origin of R^{-1} Term

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Abstract

An accelerating charged particle exerts a force upon itself. If we model the particle as a spherical shell of radius R, and calculate the force of one piece of this shell on another and eventually integrate over the whole particle, there will be a net force on the particle due to the breakdown of Newton's third law. This symmetry breaking mechanism relies on the finite size of the particle; thus, as Feynman has stated, conceptually this mechanism doesn't make good sense for point particles. Nonetheless, in the point particle limit, two terms survive in the self-force series: R^0 and R^{-1} terms. The R^0 term can alternatively be attributed to the well-known radiation reaction but the origin of R^{-1} term is not clear. In this study, we will show that this new term can be accounted for by an inductive mechanism in which the changing magnetic field induces an inductive force on the particle. Using this inductive mechanism, we derive R^{-1} term in an extremely easy way.

Keywords

Electromagnetic Self-Force, Self-Force Mechanisms, Radiation Reaction, Faraday's Law of Induction

1. Introduction

As is well-known for more than a century, an accelerating charged particle exerts a force upon itself [1]. This self-force suffers from two notorious problems of runaway solutions and preacceleration [2] [3] [4] [5] [6] which have threatened the consistency of Maxwellian electrodynamics to the point that the some physicists have even tried to modify it [7]. For most practical situations, this self-force can be ignored [8] but from both theoretical and conceptual point of view, it has been a thorn in classical physics. The electromagnetic self-force has

not normally been part of standard undergraduate curriculum and even in graduate textbooks such as Jackson's [8], its discussion has been postponed to the last chapter for the reasons mentioned above. In addition, a quick look at most reviews on self-force reveals that their most focus has been on its calculations and its problems, and less attention has been given to its underlying mechanisms [2] [6]. This study is an attempt to fill this gap, in addition to discussing the conventional mechanisms attributed to electromagnetic self-force in a pedagogical manner and showing how they are used to calculate it, we will also introduce an inductive mechanism for the divergent term of the self-force *i.e.* R^{-1} term. This mechanism, as it will turn out, not only gives a conceptually reasonable basis for self-force but also provides an easy and quick way for calculation of this term.

2. Symmetry Breaking Mechanism

Origin of electromagnetic self-force can be traced back to the breakdown of Newton's third law for an accelerating charged particle [1]. To see this better, picture the particle as a spherical shell with charge uniformly spread over its surface. When it is moving with uniform velocity, the electric force of one part on another part gets canceled due to symmetry of the forces (**Figure 1(a)**) but when it accelerates, this symmetry of forces (Newton's third law) or electric fields breaks down (**Figure 1(b)**) resulting in a net force. This mechanism makes good conceptual sense for finite-sized objects but in the point particle limit as Feynman stated [7] "it seems silly to allow the possibility of a point particle acting on itself". As we will see in the next section, the self-force for a point particle can be more easily understood using radiation and induction mechanisms.

Using this symmetry breaking mechanism, we can calculate the self-force. The electromagnetic self-force for a continuous charge distribution is calculated by the integral form of Lorentz force law as follows:

$$\boldsymbol{F}_{self} = \int \rho\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)\mathrm{d}v \tag{1}$$

where ρ is the charge density, $\boldsymbol{v}$ is the velocity of the charged body, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields caused by the body itself, and the integration is over the extent of the charge body. The calculations of the self-force are pretty involved and the results are usually approximated and presented as series. For an accessible calculation of simple cases of self-force, refer to [9] [10], for a
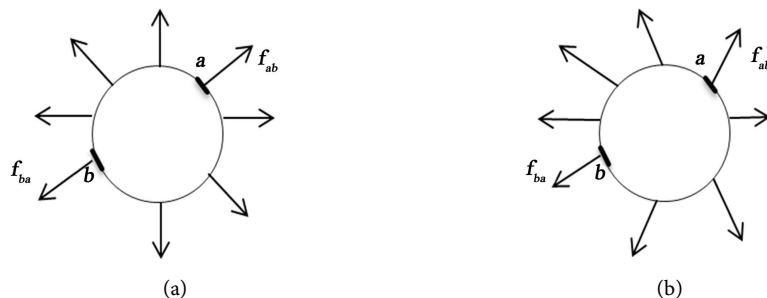

| (a) | (b) |

Figure 1. A charged particle moving with uniform velocity (a) and with acceleration (b).

f_{ab} denotes the force of part b on part a. In (a) $f_{ab} = f_{ba}$ but in (b) $f_{ab} \neq f_{ba}$.

more rigorous and general case [6] and for a comparison of different calculating methods [11]. Here we just report the results for a particle modeled as spherical shell of radius R. For the nonrelativistic case $(v \ll c)$, this force can be written in a neat form as follows [12]:

$$F_{self} = \frac{\mu_0 q^2 c}{12\pi R^2}\left[v\left(t - \frac{2R}{c}\right) - v(t) \right] \tag{2}$$

where μ_0 is the vacuum permeability, c is the speed of light, q is the charge of the particle and also SI units have been used throughout the whole paper. The term $\frac{2R}{c}$ in the velocity's argument is the time it takes for light to cross the shell's diameter so $v\left(t - \frac{2R}{c}\right)$ is the velocity of the particle this time earlier and considering the fact that the Equation(2) is the difference between velocities at an earlier time and now, we can infer that self-force is always opposite the acceleration. For a small R, we can expand this around $R = 0$ as follows:

$$F_{self} = \frac{\mu_0 q^2 c}{12\pi R^2}\left[v(t) - \frac{2R}{c}\dot{v}(t) + \frac{1}{2}\left(\frac{2R}{c}\right)^2 \ddot{v}(t) + \cdots - v(t) \right] \tag{3}$$

The relativistic self-force can be most easily obtained by using Lorentz transformation of acceleration and its derivative from S frame, in which Equation (3) is derived, to $\acute{S}$ frame in which the body is moving with velocity $\acute{v}$. These transformations will yield the correct answer for the electromagnetic self-force, for a proof of this see [6]. Denoting $\acute{\gamma} = \left(1 - \beta^2\right)^{-\frac{1}{2}}$ with $\beta = \frac{\acute{v}}{c}$, the transformations are:

$$a_\parallel = \acute{\gamma}^3 \acute{a}_\parallel \tag{4a}$$

$$a_\perp = \acute{\gamma}^2 \acute{a}_\perp \tag{4b}$$

$$\dot{a}_\parallel = \acute{\gamma}^4 \acute{\dot{a}}_\parallel + \frac{3\acute{\gamma}^6}{c^2}\left|\acute{a}_\parallel\right|^2 \acute{v} \tag{4c}$$

$$\dot{a}_\perp = \acute{\gamma}^3 \acute{\dot{a}}_\perp + \frac{3\acute{\gamma}^5}{c^2}\left(\acute{v} \cdot \acute{a}\right)\acute{a}_\perp \tag{4d}$$

where $\parallel$ and $\perp$ symbols show the components parallel and perpendicular to velocity respectively, the dot shows the time derivative and a is the acceleration. Dividing Equation (3) into parallel and perpendicular components and substituting them with Equations (4) and finally combining the parallel and perpendicular components and removing their primes, we will have:

$$F_{self} = -\frac{\mu_0 q^2}{6\pi R}\gamma^3 a$$
$$+ \frac{\mu_0 q^2 \gamma^2}{6\pi c}\left\{ \dot{a} + \frac{3\gamma^2}{c^2}(v \cdot a)a + \frac{\gamma^2}{c^2}\left[v \cdot \dot{a} + \frac{3\gamma^2}{c^2}(v \cdot a)^2 \right]v \right\} + O(R) \tag{5}$$

We have written the terms proportional to R^n as $O(R^n)$ $(n = 1, 2, 3, \cdots)$.

We can recover (3) from (5) in $\beta \ll 1$ limit by keeping the terms first order in β. This is the self-force for relativistically rigid spherical of shell meaning that the shell is spherical in its instantaneous rest frame but contracts in the velocities' direction in an inertial frame. Simplifying the Equation (3), we will have:

$$\boldsymbol{F}_{self} = -\frac{\mu_0 q^2}{6\pi R}\boldsymbol{a} + \frac{\mu_0 q^2}{6\pi c}\dot{\boldsymbol{a}} + O(R) + O(R^2) \tag{6}$$

In the point particle limit $\to 0$, only the first two terms remain: the $\frac{1}{R}$ term which goes to infinity and the second term which is independent of the shape of the body. Unfortunately, both these terms have troubled many physicists for the past century. Writing the equation of motion in the point particle limit we will have:

$$\boldsymbol{F}_{ext} - \frac{\mu_0 q^2}{6\pi R}\boldsymbol{a} + \frac{\mu_0 q^2}{6\pi c}\dot{\boldsymbol{a}} = m_0\boldsymbol{a} \tag{7}$$

In which m_0 is the bare mass and $\boldsymbol{F}_{ext}$ the external force. The first term of the self-force which goes to infinity in the point particle limit is normally put to the right side and absorbed into the bare mass and together they are known as the physical mass. This is known as renormalization process *i.e.* although $\frac{1}{R}$ term becomes infinite but the physical mass is kept finite by some mechanism, for example, this infinity can get canceled by the negative bare mass of attractive gravitational force [10]. The renormalization was first considered by Dirac more than 70 years ago [13] serving as an inspiration for this process in quantum field theory. Renormalizing the mass we will have:

$$\boldsymbol{F}_{ext} + \frac{\mu_0 q^2}{6\pi c}\dot{\boldsymbol{a}} = m\boldsymbol{a} \tag{8}$$

In which $m = m_0 + -\frac{\mu_0 q^2}{6\pi R}$ is the physical (observed) mass. This equation suffers from two problems: runaway solutions and preacceleration, for an account of these problems see [6] [14]. These difficulties also remain in the relativistic form of self-force. To just shortly explain these problems, if $\boldsymbol{F}_{ext} = 0$, it admits solutions in which velocity starts increasing exponentially (it runs away so the term runaway solutions) violating the conservation of energy. This can be avoided by the appropriate initial conditions (for example by putting initial acceleration zero) unfortunately by doing so another problem arises; the particle starts to accelerate even before the force is applied. This is the so-called preacceleration problem which violates the causality. Many attempts have been made to clarify and solve these problems. For example, these problems will disappear if we replace this equation with an approximate or alternative one (see [12] for argument on which one is better) known as Landau–Lifshitz equation which basically expresses the acceleration perturbatively in terms of external force [3] and also another way to get rid of these problems is to throw away point particle model and put a lower bound on its radius. For example, if electron's radius is bigger than electron classical radius $(R > R_{class})$ [4], these problems will disap-

pear, but the electron radius is known to be smaller than this [15].

3. Radiation Reaction Mechanism

In the point particle limit $R \to 0$, only the first two terms of the electromagnetic self-force (6) survive: R^{-1} and R^0 terms. The mechanism responsible for the R^0 term is well-known. When a particle accelerates, it emits radiation. For nonrelativistic velocities the total power radiated is given by Larmor formula $P = \frac{\mu_0 q^2 a^2}{6\pi c}$. This radiation in turn causes a recoil force on the particle much like a gun shooting a bullet. This radiation reaction force for nonrelativistic case is known as the Abraham-Lorentz force *i.e.* R^0 term. The reason to believe that this radiation mechanism is responsible for the R^0 term is that it can be obtained independently using Larmor formula and conservation of energy. The derivation can be found in any standard electrodynamics textbook. The energy lost by radiation is equal to the work done by radiation reaction force. But to include the effects of the non-radiated field we calculate this energy for system which has the same state at t_1 and t_2 (to see why refer to [10]) as follows:

$$\int_{t_1}^{t_2} \boldsymbol{F}_{rad} \cdot \boldsymbol{v} \mathrm{d}t = -\int_{t_1}^{t_2} \frac{\mu_0 q^2 a^2}{6\pi c} \mathrm{d}t \tag{9}$$

Integrating this by parts and dropping the boundary term we will have:

$$\int_{t_1}^{t_2} \left(\boldsymbol{F}_{rad} - \frac{\mu_0 q^2 \dot{a}}{6\pi c} \right) \cdot \boldsymbol{v} \mathrm{d}t = 0 \tag{10}$$

This is always true if the expression in the parentheses is zero which is equivalent to the R^0 term and using the Equations (4c) and (4d), we can easily obtain the relativistic form of this radiation reaction force.

4. Induction Mechanism

The mechanism behind the R^{-1} is different. When a charge particle accelerates, there seems to appear an induced electric field opposing the particle's acceleration. To see this better, consider a charged particle q moving in $\hat{z}$ direction with constant velocity $\vec{v}$ (Figure 2) and also consider an imaginary circle just above the particle. The direction of the magnetic fields is out of the page. If we apply a force on this particle in $\hat{z}$ direction increasing its speed, according to Biot-Savart law, the magnetic fields inside the circle will increase and in turn using the Faraday and Lenz's Laws, we can see that there will be an induced electric field opposite the direction of particle's movement just like a solenoid resisting the change in current. Because of the symmetry of the situation, the forces in other directions get cancelled and there will just remain a net force $-\hat{z}$ direction if the speed increases and $\hat{z}$ direction if it decreases.

We can derive this self-force which is applied by the particle's own induced electric field by calculating the induced electric field and multiplying it by q. The equations for induced electric field are $\nabla \cdot \boldsymbol{E} = 0$ and $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$. Comparing these with magnetostatics equations, we immediately find the answer [10]:
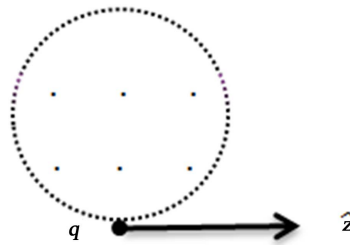
Figure 2. A particle moving with uniform velocity. The six dots represent the magnetic fields of this particle inside the imaginary dotted circle.

$$E = -\frac{1}{4\pi}\frac{\partial}{\partial t}\int \frac{\boldsymbol{B}(\acute{\boldsymbol{r}})\times(\hat{\boldsymbol{r}}-\acute{\hat{\boldsymbol{r}}})}{|\boldsymbol{r}-\acute{\boldsymbol{r}}|^2}\,\mathrm{d}\acute{v} \tag{11}$$

In which $\acute{\boldsymbol{r}}$ the source point and $\boldsymbol{r}$ is the field point. Note that in this integral the magnetic field *is* the source and due to the symmetry of the system only the $\hat{z}$ component of it survives. We solve this integral for a point particle which while moving with a constant velocity is acted upon by a force. For a nonrelativistic point particle we have $\boldsymbol{r}=0$ and $\vec{\boldsymbol{B}} = \frac{\mu_0 q}{4\pi}\frac{v\sin\acute{\theta}}{\acute{r}^2}\hat{\varphi}$ and using $\hat{\varphi}\times\hat{r} = -\sin\theta\hat{z}$ and dropping the prime we will have:

$$E_z = -\frac{1}{4\pi}\frac{\partial}{\partial t}\int \frac{\mu_0 q}{4\pi}\frac{v\sin^2\theta}{r^4}r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\varphi \tag{12}$$

Integrating and taking the time derivative we will have $E_z = -\frac{\mu_0 q a}{6\pi R}$ in which a is the magnitude of acceleration and $R \to 0$. Multiplying this by q, we have:

$$F = -\frac{\mu_0 q^2 a}{6\pi R}\hat{z} \tag{13}$$

This is the R^0 term easily derived by the induction mechanism, for a two-page derivation of this term refer to [6]. Using Equation (4a) the relativistic form for this particular motion can be easily derived:

$$F = -\frac{\mu_0 q^2}{6\pi R}\gamma^3 a\hat{z} \tag{14}$$

5. Conclusion

In this study, we have discussed the conventional symmetry breaking mechanism of the electromagnetic self-force in a pedagogical manner. We argued that this mechanism doesn't make good sense for a point particle limit and should disappear in this limit, but in this limit two terms remain of which one is attributed to radiation and the other one as we have shown can be accounted for by an induction mechanism. As shown above, this induction mechanism provides an extremely easy way to derive this force compared with other conventional methods. In addition, a better understanding of the underlying mechanisms of electromagnetic self-force may also be helpful for other self-forces e.g. gravitational self-force. Here we have only considered the point particle limit but further studies are needed to examine the effects of these mechanisms most espe-

cially induction mechanism for finite-sized particle model.

Acknowledgements

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