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Generalization of the Force Approach to Radiation Reaction

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Abstract

A new approach to radiation reaction for the correction of the linear and circular motion of a charged particle takes into account the emission of electromagnetic radiation due to its acceleration. This new formulation was based on expressing the radiation reaction force in terms of the external force rather than the acceleration of the charge. In this paper, a generalization of the radiation reaction force in terms of the external force approach is formulated for any arbitrary motion of the charged particle. This generalization includes the linear and circular acceleration cases previously investigated.

Keywords

Synchrotron Radiation, Radiation Reaction Force, Relativistic Charged Particle

1. Introduction

There is an old standing problem of radiation reaction [1] [2] [3] [4] consisting mainly in the that the usual approach, based on expressing the radiation reaction force in terms of the third time differentiation of the position of the charged particle, brings about of what is called a pre-acceleration of the charged particle even if there is not external force at all. However, experimentally one sees that as soon the external force is zero, the acceleration of the charged particle is zero too, and emission of electromagnetic radiation stops. This experimental fact led to propose an alternative approach [5] to the radiation reaction force. This new approach is based on expressing the radiation reaction force in terms of the external force acting on the charged particle, responsible for its acceleration. Although, this approach seems to point in the right direction, experimental verification of this theoretical idea is required. In reference [5], the approach was implemented for linear and circular acceleration of a charged particle. In this paper, a generalization of the idea is carried out, using the same method presented in that paper, and the generalization takes the same form as Equatioin (1), but with the additional radiation reaction force term.

2. Acceleration and Force Relation

The relativistic equation to describe the motion of a charge particle under an external force $\boldsymbol{F}$ is given by [6]

$$\frac{\mathrm{d}\gamma m\boldsymbol{v}}{\mathrm{d}t}=\boldsymbol{F}, \tag{1}$$

where m and $\boldsymbol{v}$ are the mass and the velocity of the charged particle, and γ is the usual relativistic time dilation factor

$$\gamma=\left(1-\beta^2\right)^{-1/2}, \tag{2}$$

being β the speed of the charged particle in units of the speed of light "c",

$$\beta=\frac{v}{c}=\frac{1}{c}\sqrt{v_x^2+v_y^2+v_z^2}. \tag{3}$$

Making the differentiation in (1), this equation can be written as

$$m\gamma^3\begin{pmatrix}1-\beta_y^2-\beta_z^2 & \beta_x\beta_y & \beta_x\beta_z\\ \beta_y\beta_x & 1-\beta_x^2-\beta_z^2 & \beta_y\beta_z\\ \beta_z\beta_x & \beta_y\beta_y & 1-\beta_x^2-\beta_y^2\end{pmatrix}\begin{pmatrix}\dot{v}_x\\ \dot{v}_y\\ \dot{v}_x\end{pmatrix}=\boldsymbol{F}, \tag{4}$$

where $\beta_i=v_i/c \quad i=x,y,z$. Inverting this matrix and making some rearrangements, it follows that

$$\dot{\boldsymbol{\beta}}=\frac{1}{mc\gamma}\aleph\boldsymbol{F}, \tag{5}$$

where $\aleph$ is the matrix

$$\aleph=\begin{pmatrix}1-\beta_x^2 & -\beta_x\beta_y & -\beta_x\beta_z\\ -\beta_y\beta_x & 1-\beta_y^2 & -\beta_y\beta_z\\ -\beta_z\beta_x & -\beta_z\beta_y & 1-\beta_z^2\end{pmatrix}. \tag{6}$$

This is the expression with makes the relation between the normalized acceleration with the external force.

3. Radiation Reaction Force

It is well known that the power radiated per solid angle of an accelerated charged particle of charge "q" is [7]

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega}=\frac{q^2}{4\pi c}\frac{\left|\hat{\boldsymbol{R}}\times\left[\left(\hat{\boldsymbol{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1-\hat{\boldsymbol{R}}\cdot\boldsymbol{\beta}\right)^5}, \tag{7}$$

where $\hat{\boldsymbol{R}}$ is the unitary vector that goes from the position of the charge at the retarded time ($t'=t+R/c$) to the observer position. So, using (5) on this expression, it follows that

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega}=\frac{q^2}{4\pi m^2c^3\gamma^2}\frac{\left|\hat{\boldsymbol{R}}\times\left[\left(\hat{\boldsymbol{R}}-\boldsymbol{\beta}\right)\times\aleph\boldsymbol{F}\right]\right|^2}{\left(1-\hat{\boldsymbol{R}}\cdot\boldsymbol{\beta}\right)^5}, \tag{8}$$

Integrating with respect the solid angle and with respect the time in the interval $[0,t]$, the energy radiated by the charged particle during this time is

$$U(t)=\frac{q^2}{4\pi m^2c^3}\int_0^t\frac{\mathrm{d}t}{\gamma^2}\int_\Omega\frac{\left|\hat{\boldsymbol{R}}\times\left[\left(\hat{\boldsymbol{R}}-\boldsymbol{\beta}\right)\times\aleph\boldsymbol{F}\right]\right|^2}{\left(1-\hat{\boldsymbol{R}}\cdot\boldsymbol{\beta}\right)^5}\mathrm{d}\Omega. \tag{9}$$

which can be written in terms of angles as

$$U(t)=\frac{q^2}{4\pi m^2c^3}\int_0^t\frac{|\aleph\boldsymbol{F}|^2\,\mathrm{d}t}{\gamma^2}\int_\Omega\frac{|e_1\sin\theta_1-e_2\beta\sin\theta_2|^2}{(1-\beta\cos\theta)^5}\mathrm{d}\Omega, \tag{10}$$

where e_1 is the unitary vector in the direction $\hat{\boldsymbol{R}}\times\left(\hat{\boldsymbol{R}}\times\aleph\boldsymbol{F}\right)$ and e_2 is the unitary vector in the direction $\hat{\boldsymbol{R}}\times(\boldsymbol{\beta}\times\aleph\boldsymbol{F})$, θ_1 is the angle between the vectors $\hat{\boldsymbol{R}}$ and $\hat{\boldsymbol{R}}\times\aleph\boldsymbol{F}$, and θ_2 is the angle between the vectors $\hat{\boldsymbol{R}}$ and $\boldsymbol{\beta}\times\aleph\boldsymbol{F}$. One can choose our reference system such that θ be the angle related with the solid angle coordinates. The angles θ_1 and θ_2 depend on the solid angle coordinates (θ,ϕ).

Assume now that this energy lost is due to the work done by a nonconservative radiation reaction force, to move the charged particle from the position $\boldsymbol{x}_0$ a the time $t=0$, to the position $\boldsymbol{x}$ at the time t. So, one would have

$$U(t)=\int_{\boldsymbol{x}_0}^{\boldsymbol{x}}\boldsymbol{F}_{rad}\cdot\mathrm{d}\boldsymbol{x}. \tag{11}$$

However, one has that $\mathrm{d}\boldsymbol{x}=\boldsymbol{v}\mathrm{d}t$. Then, it follows that

$$U(t)=\int_0^t\boldsymbol{F}_{rad}\cdot\boldsymbol{v}\mathrm{d}t. \tag{12}$$

Equaling (10) and (12), and since the resulting expression is valid for any time intervale $[0,t]$ on the real line, one obtains

$$\boldsymbol{F}_{rad}=\frac{q^2|\aleph\boldsymbol{F}|^2}{4\pi m^2c^3\gamma^2v\cos\varphi}\int_\Omega\frac{|e_1\sin\theta_1-e_2\beta\sin\theta_2|^2}{(1-\beta\cos\theta)^5}\mathrm{d}\Omega, \tag{13}$$

where v is the charged particle speed and φ is the angle between the vectors $\boldsymbol{F}_{rad}$ and $\boldsymbol{v}$. Since $\boldsymbol{F}_{rad}$ must represent a force causing damping on the motion of the charged particle, $\boldsymbol{F}_{rad}$ must point on the $-\hat{\boldsymbol{n}}$ direction, where $\hat{\boldsymbol{n}}=\boldsymbol{v}/v$, meaning the the angle between $\boldsymbol{F}_{rad}$ and $\boldsymbol{v}$ must be $\varphi=\pi$. Therefore, one gets the following expression for radiation reaction force

$$\boldsymbol{F}_{rad}=-\frac{q^2|\aleph\boldsymbol{F}|^2\boldsymbol{v}}{4\pi m^2c^3\gamma^2v^2}\int_\Omega\frac{|e_1\sin\theta_1-e_2\beta\sin\theta_2|^2}{(1-\beta\cos\theta)^5}\mathrm{d}\Omega, \tag{14}$$

In this way, the modified relativistic equation of motion of a charged particle under an arbitrary external force $\boldsymbol{F}$ is

$$\frac{\mathrm{d}\gamma m\boldsymbol{v}}{\mathrm{d}t}=\boldsymbol{F}+\boldsymbol{F}_{rad}, \tag{15}$$

where the radiation reaction force term $\boldsymbol{F}_{rad}$ has been added to the expression

(1), or it can be written, using (14), as

$$\frac{\mathrm{d}\gamma m\boldsymbol{v}}{\mathrm{d}t}=\boldsymbol{F}-\frac{q^2\left|\aleph\boldsymbol{F}\right|^2\boldsymbol{v}}{4\pi m^2c^3\gamma^2v^2}\int_\Omega\frac{\left|e_1\sin\theta_1-e_2\beta\sin\theta_2\right|^2}{\left(1-\beta\cos\theta\right)^5}\mathrm{d}\Omega. \tag{16}$$

One must point out that if the external force $\boldsymbol{F}$ is zero, the radiation reaction force $\boldsymbol{F}_{rad}$ is also zero, and noncausal preacceleration is absent since the charged particle will have constant velocity, as the experiments indicate so far.

4. Special Cases

As one can see from expression (14), the integration over the solid angle is not in general a trivial matter, and numerical integration may be required. However, there are two cases where this integration can be done without any difficulty, and these cases are presented below.

a) Linear acceleration case: In this case one has that $\aleph\boldsymbol{F}$ is parallel to $\boldsymbol{\beta}$ ($e_2=0$) and there is not dependence on ϕ, this integration is well known [7], and choosing $\boldsymbol{\beta}=\left(0,0,\beta_z\right)$ and $\boldsymbol{F}=\left(0,0,F\right)$, the resulting equation is

$$\frac{\mathrm{d}\gamma mv}{\mathrm{d}t}=F-\frac{\lambda_0F^2}{v}, \tag{17}$$

where $\gamma=\left(1-\beta_z^2\right)^{-1/2}$, and λ_0 is defined as

$$\lambda_0=\frac{2q^2}{3m^2c^3}. \tag{18}$$

This equation is the same as that proposed for this case in reference [5].

b) Circular acceleration case: In this case one has that $\aleph\boldsymbol{F}$ and $\boldsymbol{\beta}$ are orthogonal. The integration is also well known [7], and choosing $\boldsymbol{\beta}=\left(\beta_x,\beta_y,0\right)$ and $\boldsymbol{F}=\left(F_x,F_y,0\right)$, one arrives to the following equation

$$\frac{\mathrm{d}\gamma m\boldsymbol{v}}{\mathrm{d}t}=\boldsymbol{F}-\frac{\lambda_0F^2}{v^2\gamma^2}\boldsymbol{v}, \tag{19}$$

where $\gamma=\left(1-\beta_x^2-\beta_y^2\right)^{-1/2}$, $v^2=v_x^2+v_y^2$, and the motion is reduced to the plane (x,y). This expression is the same as that one proposed on reference [5].

5. Conclusion

Following the same approach of reference [5], a generalization has been formulated for the force approach to radiation reaction, expression (16). This generalization has the same main property of the force approach, wherever the external force is zero, the acceleration of the charged particle is zero, and the radiation reaction force is zero too, and non pre-acceleration concept will appear here.

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