

Nano-Contact Problem with Surface Effects on Triangle Distribution Loading

Liyuan Wang*, Wei Han, Shanlin Wang, Lihong Wang, Yiping Xin

Regional Circular Economy key Laboratory of Gansu Higher Institutions; Department of Mathematics, Longqiao College of Lanzhou University of Finance and Economics, Lanzhou, China

Email: *wly811220@163.com, talitha@sohu.com, wangshanl554517@163.com, wlh232x@sina.com, x.yip1987@163.com

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Abstract

This work presents a theoretical study of contact problem. The Fourier integral transform method based on the surface elasticity theory is adopted to derive the fundamental solution for the contact problem with surface effects, in which both the surface tension and the surface elasticity are considered. As a special case, the deformation induced by a triangle distribution force is discussed in detail. The results are compared with those of the classical contact problem. At nano-scale, the contributions of the surface tension and the surface elasticity to the stress and displacement are not equal at the contact surface. The surface tension plays a major role to the normal stress, whereas the shear stress is mainly affected by the surface elasticity. In addition, the hardness of material depends strongly on the surface effects. This study is helpful to characterize and measure the mechanical properties of soft materials through nanoindentation.

Keywords

Surface Tension, Surface Elasticity, Nano-Contact Problem, Fourier Integral Transforms Method, Triangle Distribution Force

1. Introduction

Nowadays, nanometer material and technology have been widely used in industrial and engineering fields. Many new nano-materials have been developed by utilizing the fact that materials begin to exhibit unique mechanical properties at nano-scale, which significantly differ from those at larger scale.

Nano-indentation tests have been widely used to measure such mechanical properties of materials. For micro-nano solids with large surface-to-bulk ratio the significance of surfaces is likely to be important. Form the viewpoint of continuum mechanics, this

difference can be described by such concepts as surface effects [1]. This is especially true for nano-scale materials or structures. In such cases, the surface tension and the surface elasticity play a critical role and thus have been adding its appeal to many researchers. For example, Miller and Shenoy [2] probed the size-dependent elastic properties of nano-plates and beams. Hang *et al.* [3] explained the size dependent phenomenon by the strain gradient continuum theory. Dingreville *et al.* [4] investigated the surface free energy and its effect on elastic behavior of the nano-sized particles, wires, and films. Yang [5] studied the size-dependent effective modulus of spherical nanocavities at dilute concentrations. Gao *et al.* [6] built a simple model to describe the influence of surface stress on the nanoscale adhesive contact. There are a lot of work regarding the surface/interface energy effects on the nanostructures and solids, and we can only include a small part of them here. For more recent developments in this field, the readers can refer to a review article by Wang *et al.* [7].

To study the mechanical behavior of nano-materials, the most celebrated continuum-based surface/interface model was first established by Gurtin, Murdoch and co-workers [8] [9] [10], which known as the theory of surface elasticity. In the study of nano-scale problems, all material constants appearing in that constitutive model were commonly calibrated with data obtained from either experimental measurements [11] or atomistic simulations [2] [12]. Therefore, the surface effect has been widely adopted to investigate the mechanical phenomena at nano-scale. Cammarata *et al.* [13] considered the size-dependent deformation in thin film with surface effects. Gao *et al.* [14] developed a finite-element method to account for the effect of surface elasticity. Wang *et al.* [15] studied to a half-plane subjected to normal pressures with surface tension. Zhao and Rajapakse [16] proposed a continuum-based model to study the influence of surface stresses on mechanical responses of an elastic half-space compressed by an axisymmetric, rigid, frictionless nano-indenter. Long and Wang [17] studied the effect of the residual surface stress on the two dimensional Hertz contact problem, and later Long *et al.* [18] generalized their work to the three dimensional case. Wang [19] derived the general analytical solution of nano-contact problem with surface effects by using the complex variable function method. Gao *et al.* [20] [21] derived the influence of the surface stress on the JKR adhesive contact, which is investigated by employing the non-classical Boussinesq fundamental solutions. In this paper, Fourier integral transform method is used to solve the non-classical boundary value problems with surface effects.

2. Problem Description

Now we consider a material occupying the upper half-plane $z > 0$, we refer to a Cartesian coordinate system $(o-xyz)$, as shown in **Figure 1**, where the x axis is along the surface and the z axis perpendicular to the surface. It is assumed that the material is subjected to triangle distribution force $p(x)$ and $q(x)$ over the region $|x| \leq a$. While the normal and shear force form zero (O_1 and O_2) uniformly increased to maximum p_0 and q_0 at the point O . The plane-strain conditions are assumed to $\varepsilon_{2i} = 0$, and

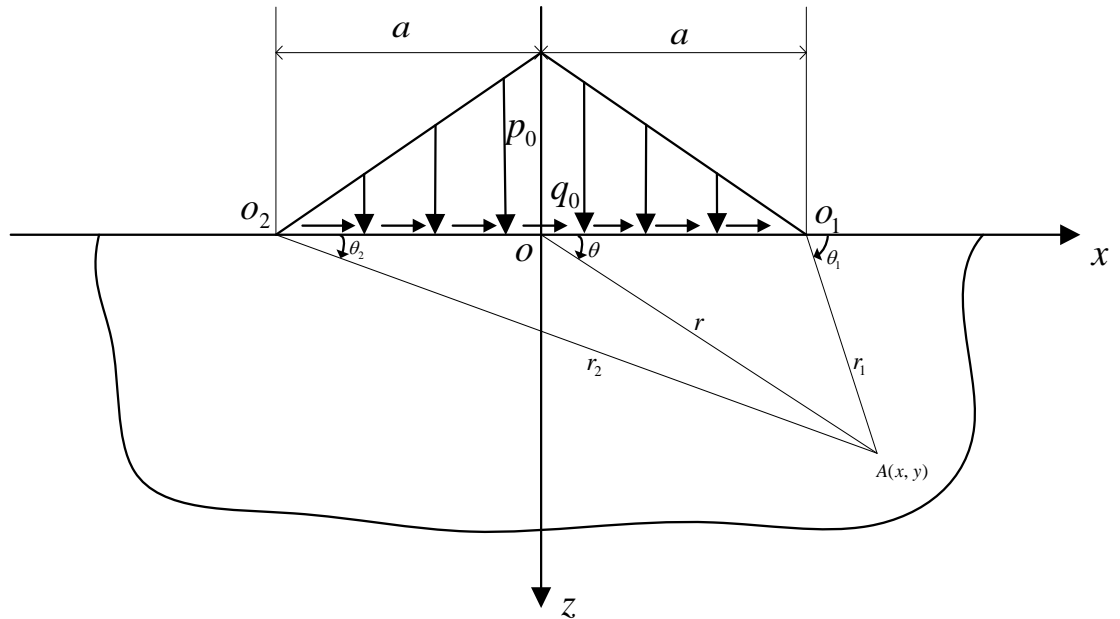


Figure 1. Schematic of contact problem under triangle distribution loads.

the contact is assumed to be frictionless.

The problem statement is to determine the triangle distribution force exerted by the elastic field (e.g., displacement and stresses) with the half-plane for the influence of surface effects.

3. Basic Equations of Surface Elasticity and General Solutions

In surface elasticity theory, the equilibrium and constitutive equations in the bulk of material are the same as those in classical elastic theory, but the presence of surface stresses gives rise to a non-classical boundary condition.

3.1. Basic Equations

In the absence of body force, the equilibrium equations, constitutive law, and geometry relations in the bulk are as follows

$$\begin{aligned}\sigma_{ij,j} &= 0 \\ \sigma_{ij} &= 2G \left(\varepsilon_{ij} + \frac{\mu}{1-2\mu} \varepsilon_{kk} \delta_{ij} \right)\end{aligned}\quad (1)$$

where G and μ are the shear modulus and Poisson's ratio of the bulk material, σ_{ij} and ε_{ij} are the stress tensor and strain tensor in the bulk material, respectively.

The strain tensor is related to the displacement vector u_i by

$$\varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (2)$$

On the surface, the generalized Young-Laplace equation [22], surface constitutive relation and strain-displacement relationship can be expressed as

$$\sigma_{\beta\alpha} n_\beta + \sigma_{\beta\alpha,\beta}^s = 0 \quad (3)$$

$$\sigma_{ij} n_i n_j = \sigma_{\beta\alpha}^s \kappa_{\beta\alpha}$$

$$\sigma_{\beta\alpha}^s = \tau^s \delta_{\beta\alpha} + 2(\mu^s - \tau^s) \varepsilon_{\beta\alpha} + (\lambda^s + \tau^s) \varepsilon_{\gamma\gamma} \delta_{\beta\alpha} + \tau^s u_{\beta,\alpha}^s \quad (4)$$

where n_i denotes the normal to the surface, $\kappa_{\beta\alpha}$ the curvature tensor of the surface, $\sigma_{\alpha\beta}^s$ is the 2×2 surface stress tensor on the surface, σ_{ij} is the stress tensor in the elastic material, and $\varepsilon_{\alpha\beta}^s$ is the 2×2 surface strain tensor which is equal to the tangential components of the strain tensor of the elastic material on the surface, τ^s is the residual surface tension under unstrained conditions, μ^s and λ^s are surface Lamé constants which can be determined by atom simulations or experiments [2].

3.2. General Solutions

Based on previous work by Wang [15] and Wang [23], Fourier integral transformation method is adopted to solve the stress and displacement components by

$$\begin{aligned} \sigma_{xx} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [A(\xi) + (z - 2|\xi|)B(\xi)] \xi e^{-ix\xi - z|\xi|} d\xi \\ \sigma_{zz} &= -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [A(\xi) + zB(\xi)] \xi^2 e^{-ix\xi - z|\xi|} d\xi \\ \sigma_{xz} &= \frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \xi [(1 - |\xi|z)B(\xi) - |\xi|A(\xi)] e^{-ix\xi - z|\xi|} d\xi \\ u(x, z) &= \frac{i}{2G\sqrt{2\pi}} \int_{-\infty}^{\infty} [(2 - \nu)|\xi|A(\xi) + (z|\xi| - 2(1 - \nu))B(\xi)] e^{-ix\xi - z|\xi|} d\xi + C_1 \\ w(x, z) &= \frac{1}{2G\sqrt{2\pi}} \int_{-\infty}^{\infty} [|\xi|A(\xi) + (1 - 2\nu + z|\xi|)B(\xi)] e^{-ix\xi - z|\xi|} d\xi + C_2 \end{aligned} \quad (5)$$

where A and B are generally functions of ξ as yet to be determined by boundary conditions.

4. Elastic Solution under Normal Triangle Distribution Force

As a particular example, let us consider the effect of a normal triangle distribution force $p(x)$ over the region $|x| \leq a$, and the normal force from zero (O_1 and O_2) uniformly increased to maximum p_0 (O), while remainder of the boundary $z = 0$ being unstressed as shown in **Figure 1**.

$$p(x) = \frac{p_0}{a}(a - |x|), \quad |x| \leq a \quad (6)$$

Due to the surface tension mostly influences the normal stress [20]. Therefore, we keep only the first term in Equation (4). Then, the surface stresses are given by

$$\sigma_{\beta\alpha}^s = \tau^s \delta_{\beta\alpha} \quad (7)$$

On the surface, the boundary conditions (3) can be written by

$$\begin{aligned} \sigma_{xz}(x) &= 0 \\ p(x) + \sigma_{zz}(x) &= -\frac{\tau^s}{R(x)} \end{aligned} \quad (8)$$

Substituting Equation (8) into Equation (5), one obtains

$$B = A|\xi| \quad (9)$$

Due to deformation the radius of curvature of the surface is given by

$$\frac{1}{R(x)} = \frac{\partial^2 w(x,0)}{\partial x^2} \quad (10)$$

By substituting Equations (9) and (10) into the surface condition Equation (8), $A(\xi)$ is determined by

$$A(\xi) = \frac{\tilde{p}(\xi)}{(1+s|\xi|)\xi^2} \quad (11)$$

where

$$s = \frac{\tau^s(1-\nu)}{G} \quad (12)$$

$$\tilde{p}(\xi) = \frac{p_0}{a} \sqrt{\frac{2}{\pi}} \frac{1 - \cos(a\xi)}{\xi^2} \quad (13)$$

where s is a length parameter depending on the surface property and material elastic constants. It should be pointed out that this parameter indicates the thickness size of the zone where the surface effect is significant, and plays a critical role in the surface elasticity. For metals, s is estimated on the order of nanometers.

Therefore $A(\xi)$ is given by

$$A(\xi) = \frac{p_0}{a} \sqrt{\frac{2}{\pi}} \frac{1 - \cos(a\xi)}{(1+s|\xi|)\xi^4} \quad (14)$$

Substituting Equation (14) into Equation (5), the stresses component and displacements component are obtained as

$$\begin{aligned} \sigma_{xx} &= \frac{2q_0}{\pi a} \int_0^\infty \left(\frac{z\xi - 1}{1+s\xi} \right) \frac{\cos(x\xi)}{\xi^2} [1 - \cos(a\xi)] e^{-z\xi} d\xi \\ \sigma_{zz} &= -\frac{2q_0}{\pi a} \int_0^\infty \left(\frac{z\xi + 1}{1+b\xi} \right) \frac{\cos(x\xi)}{\xi^2} [1 - \cos(a\xi)] e^{-z\xi} d\xi \\ \sigma_{xz} &= -\frac{2q_0}{\pi a} \int_0^\infty \left(\frac{z}{1+s\xi} \right) \frac{\sin(x\xi)}{\xi} [1 - \cos(a\xi)] e^{-z\xi} d\xi \\ u(x, z) &= \frac{p_0}{\pi G a} \int_0^\infty \left[\frac{z\xi + 2\nu - 1}{\xi^3 (1+s\xi)} \right] \sin(x\xi) [1 - \cos(a\xi)] e^{-z\xi} d\xi + C_1 \\ w(x, z) &= \frac{p_0}{\pi G a} \int_0^\infty \frac{z\xi + 2(1-\nu)}{\xi^3 (1+b\xi)} \cos(x\xi) [1 - \cos(a\xi)] e^{-z\xi} d\xi + C_2 \end{aligned} \quad (15)$$

It is seen, when $s = 0$, that is, the surface influence is ignored in Equation (15), the stresses of the half-plane are consistent with those in the classical elastic contact results which is the same result [23], respectively.

$$\begin{aligned}
\sigma_{xx} &= \frac{p_0}{\pi a} \left[(x-a)\theta_1 + (x+a)\theta_2 - 2x\theta + 2z \ln(r_1 r_2 / r^2) \right] \\
\sigma_{zz} &= \frac{p_0}{\pi a} \left[(x-a)\theta_1 + (x+a)\theta_2 - 2x\theta \right] \\
\sigma_{xz} &= -\frac{z p_0}{\pi a} \left[\theta_1 + \theta_2 - 2\theta \right]
\end{aligned} \tag{16}$$

where

$$r_1^2 = (x-a)^2 + z^2, r_2^2 = (x+a)^2 + z^2, r^2 = x^2 + z^2, \tan \theta_1 = \frac{z}{x-a}, \tan \theta_2 = \frac{z}{x+a}, \tan \theta = \frac{z}{x}.$$

On the contact surface ($z = 0$), the normal stress is given by

$$\sigma_{zz}(x, 0) = -\frac{2p_0}{\pi} \int_0^\infty \left(\frac{1 - \cos t}{t^2} \right) \left(1 + \frac{s}{a} t \right)^{-1} \cos\left(\frac{x}{a} t\right) dt \tag{17}$$

According to the Saint-Venant's Principle, we assume that the normal displacement is w specified to be zero at a distance r_0 on the contact surface, that is, $w(r_0, 0) = 0$, the displacement on the surface is derived as

$$w(x, 0) = \frac{2p_0(1-\nu)a}{\pi G} \int_0^\infty \left(\frac{1 - \cos(t)}{t^3} \right) \left(\frac{s}{a} + 1 \right)^{-1} \left[\cos\left(\frac{x}{a} t\right) - \cos\left(\frac{r_0}{a} t\right) \right] dt \tag{18}$$

Assuming that the origin has no displacement in the x direction, that is, $u(0, 0) = 0$ one obtains

$$u(x, 0) = -\frac{p_0(1-2\nu)a}{\pi G} \int_0^\infty \left(\frac{1 - \cos(t)}{t^3} \right) \left(\frac{s}{a} + 1 \right)^{-1} \sin\left(\frac{x}{a} t\right) dt \tag{19}$$

As show in **Figure 2**, the results indicated that the normal stress σ_{zz} is a smooth distribution when the surface tension is considered by at the loading boundary ($x = \pm a$), However, it is just the reverse with classical results that is the surface tension is ignored ($s/a = 0$). In addition, the actual normal stress σ_{zz} is smaller than the classical value in the loading zone and is larger outside of the zone.

Due to the different surface tension value, the horizontal displacement is displayed in **Figure 3**, where we set $r_0 = 5a$, and $K_1 = (1-2\nu)/\pi G$. It is seen that the horizontal displacement is continuous everywhere on the deformed surface. However, the classical elasticity theory predicted unreasonably that the horizontal displacement is discontinuously at the load boundary $x = \pm a$, as seen from the curve of $s/a = 0$. The indent depth is plotted in **Figure 4** with $K_2 = 2(1-\nu)/\pi G$, which also shows that the slope of the deformed surface is continuous everywhere. It is also found the indent depth decreases with the increase of surface tension.

5. Elastic Solution under Tangential Triangle Distribution Force

Now, let us consider the effect of a tangential triangle distribution force $q(x)$ over the region $|x| \leq a$, while the shears force form zero (O_1 and O_2) uniformly increased to maximum q_0 at the point O , while remainder of the boundary $z = 0$ being unstressed as shown in **Figure 1**.

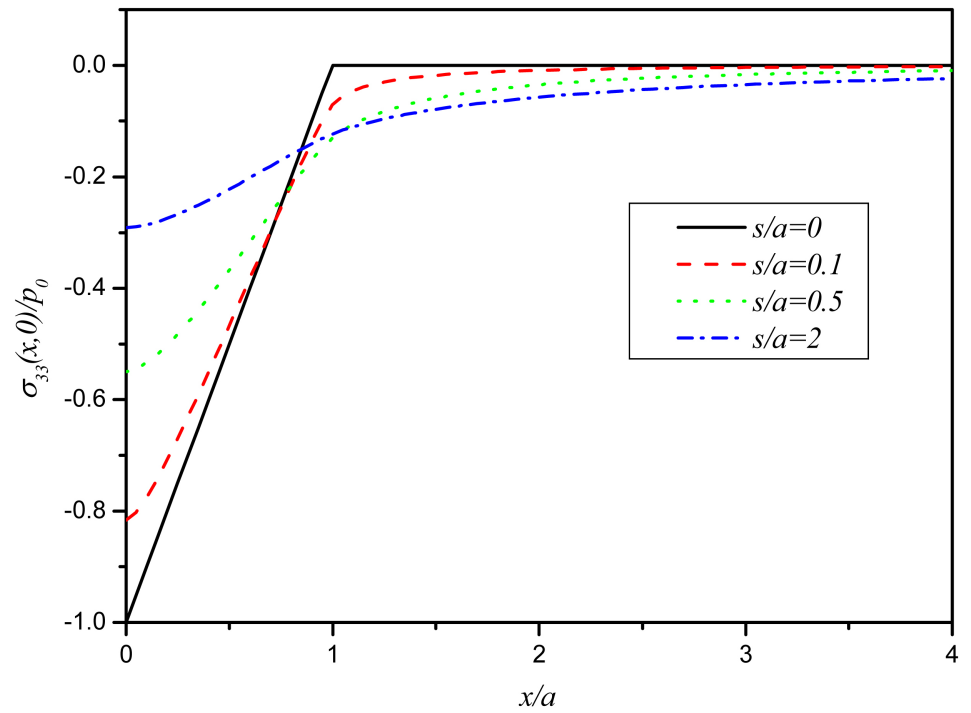


Figure 2. The distribution of contact normal stress σ_{zz} under normal triangle distribution load.

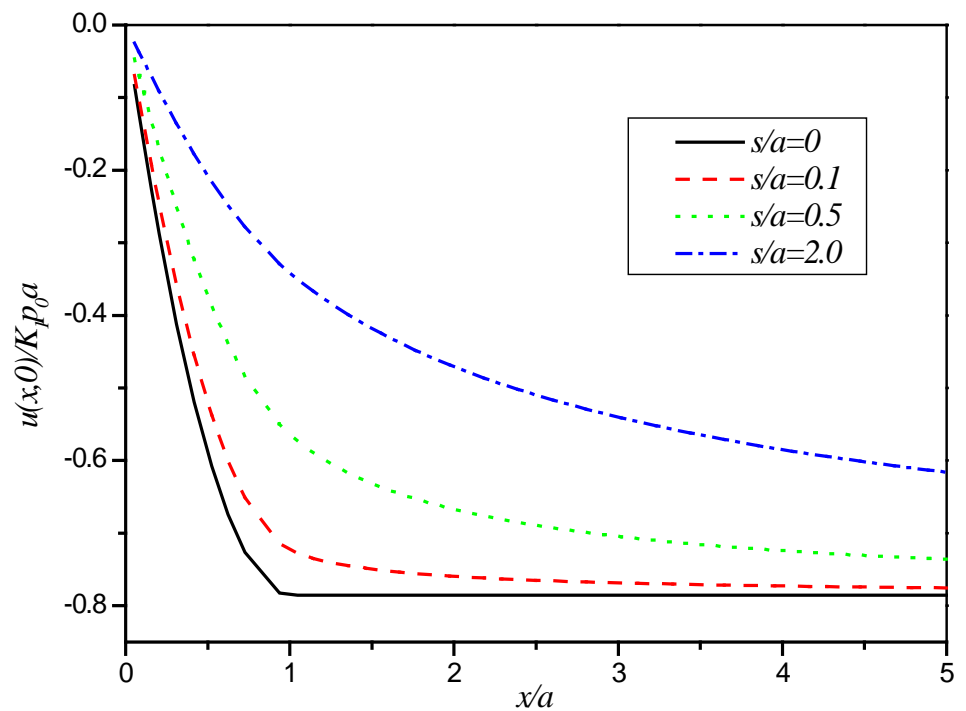


Figure 3. The distribution of surface displacement u under normal triangle distribution load.

At this moment, the boundary conditions (3) on the contact surface ($z = 0$) are simplified to

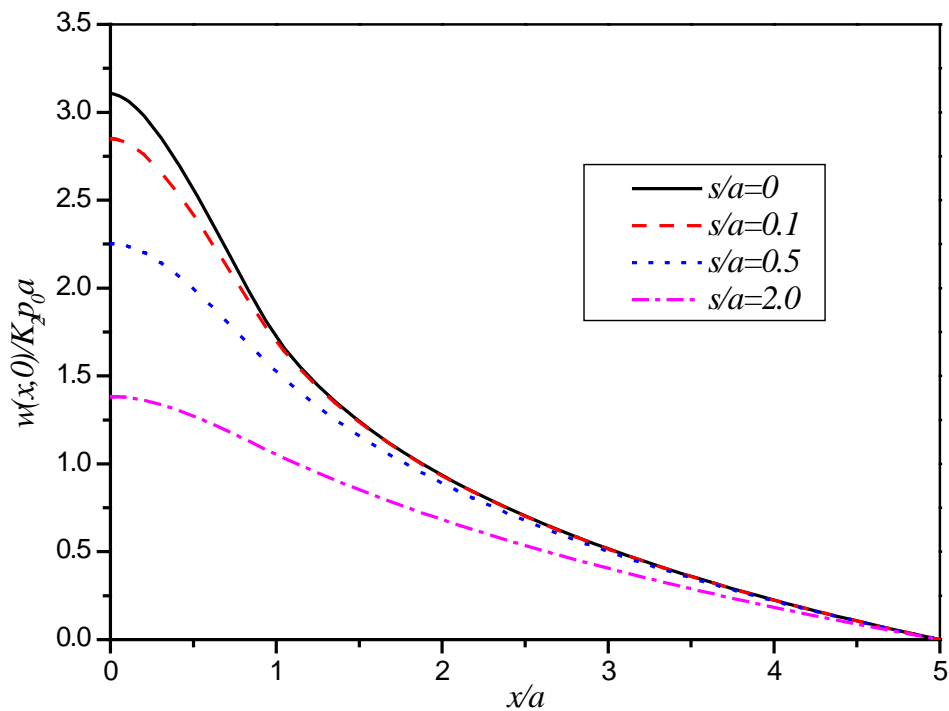


Figure 4. The distribution of surface indentation w under normal triangle distribution load.

$$\begin{aligned} \sigma_{zz} &= 0 \\ \sigma_{xz} + q(x) &= -\left(\frac{d\tau^s}{dx} + k^s \frac{d^2u}{dx^2} \right) \end{aligned} \tag{20}$$

where $q(x)$ is the tangential triangle distribution force applied on the materials surface, and $k^s = 2\mu^s + \lambda^s$ is a surface constant.

$$q(x) = \frac{q_0}{a}(a - |x|), \quad |x| \leq a \tag{21}$$

Substituting Equation (21) into Equation (20), one can be obtained

$$\begin{aligned} A(\xi) &= 0 \\ B(\xi) &= i \frac{\tilde{q}(\xi)}{\xi} \frac{1}{1 + b\xi} \end{aligned} \tag{22}$$

where

$$b = \frac{k^s(1 - \nu)}{G} \tag{23}$$

$$\tilde{q}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} q(x) e^{ix\xi} dx \tag{24}$$

where b is a length parameter depending on the material surface property. It should be pointed out that this parameter plays a critical role in the surface elasticity.

Substituting Equation (24) into the surface condition Equations (12) and (13), the solution of stresses and displaces under pure shear load were obtained

$$\begin{aligned}
\sigma_{xx} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \xi (z - 2|\xi|) B(\xi) e^{-ix\xi - z|\xi|} d\xi \\
\sigma_{zz} &= -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \xi^2 z B(\xi) e^{-ix\xi - z|\xi|} d\xi \\
\sigma_{xz} &= \frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \xi (1 - |\xi|z) B(\xi) e^{-ix\xi - z|\xi|} d\xi \\
u(x, z) &= \frac{i}{2G\sqrt{2\pi}} \int_{-\infty}^{\infty} (z|\xi| - 2(1-\nu)) B(\xi) e^{-ix\xi - z|\xi|} d\xi + C_1 \\
w(x, z) &= \frac{1}{2G\sqrt{2\pi}} \int_{-\infty}^{\infty} (1 - 2\nu + z|\xi|) B(\xi) e^{-ix\xi - z|\xi|} d\xi + C_2
\end{aligned} \tag{25}$$

Substituting Equation (24) into Equations (22), one obtains

$$\tilde{q}(\xi) = \frac{q_0}{a} \sqrt{\frac{2}{\pi}} \frac{1 - \cos(a\xi)}{\xi^2} \tag{26}$$

Therefore $B(\xi)$ is given by

$$B(\xi) = \frac{iq_0}{a} \sqrt{\frac{2}{\pi}} \frac{1 - \cos(a\xi)}{(1 + b\xi)\xi^3} \tag{27}$$

Substituting Equation (27) into Equations (25), the stresses component and displaces component are obtained as

$$\begin{aligned}
\sigma_{xx} &= \frac{2q_0}{\pi a} \int_0^{\infty} \left(\frac{z\xi - 2}{1 + b\xi} \right) \frac{\sin(x\xi)}{\xi^2} [1 - \cos(a\xi)] e^{-z\xi} d\xi \\
\sigma_{zz} &= -\frac{2q_0}{\pi a} \int_0^{\infty} \left(\frac{z}{1 + b\xi} \right) \frac{\sin(x\xi)}{\xi} [1 - \cos(a\xi)] e^{-z\xi} d\xi \\
\sigma_{xz} &= -\frac{2q_0}{\pi a} \int_0^{\infty} \left(\frac{1 - z\xi}{1 + b\xi} \right) \frac{\cos(x\xi)}{\xi^2} [1 - \cos(a\xi)] e^{-z\xi} d\xi \\
u(x, z) &= -\frac{q_0}{\pi G a} \int_0^{\infty} \left[\frac{z\xi - 2(1-\nu)}{\xi^3 (1 + b\xi)} \right] \cos(x\xi) [1 - \cos(a\xi)] e^{-z\xi} d\xi + C_1 \\
w(x, z) &= \frac{q_0}{\pi G a} \int_0^{\infty} \frac{1 - 2\nu + z\xi}{\xi^3 (1 + b\xi)} \sin(x\xi) [1 - \cos(a\xi)] e^{-z\xi} d\xi
\end{aligned} \tag{28}$$

It is seen, when $b = 0$, that is, the surface influence is ignored in Equation (28), the stresses and displacements of the half-plane are consistent with those in the classical elastic results [24], respectively.

$$\begin{aligned}
\sigma_{xx} &= \frac{2q_0}{\pi a} \left[2x \ln \left(\frac{r_1 r_2}{r^2} \right) + 2a \ln \left(\frac{r_2}{r_1} \right) - 3z(\theta_1 + \theta_2 - 2\theta) \right] \\
\sigma_{zz} &= -\frac{zq_0}{\pi a} (\theta_1 + \theta_2 - 2\theta) \\
\sigma_{xz} &= \frac{q_0}{\pi a} \left[(x - a)\theta_1 + (x + a)\theta_2 - 2x\theta + 2z \ln \left(\frac{r_1 r_2}{r^2} \right) \right]
\end{aligned} \tag{29}$$

On the contact surface $z = 0$, the stresses is given by

$$\begin{aligned} \sigma_{xx}(x, 0) &= -\frac{4q_0}{\pi} \int_0^\infty \left(\frac{1-\cos t}{t^2}\right) \left(1+\frac{b}{a}t\right)^{-1} \sin\left(\frac{x}{a}t\right) dt \\ \sigma_{xz}(x, 0) &= -\frac{2q_0}{\pi} \int_0^\infty \left(\frac{1-\cos t}{t^2}\right) \left(1+\frac{b}{a}t\right)^{-1} \cos\left(\frac{x}{a}t\right) dt \end{aligned} \tag{30}$$

Based on the previous assumption, $u(r_0, 0) = 0$, the displacement on the surface component is derived as

$$\begin{aligned} u(x, 0) &= \frac{2(1-\nu)q_0a}{\pi G} \int_0^\infty \left(\frac{1-\cos t}{t^3}\right) \left(1+\frac{b}{a}t\right)^{-1} \left[\cos\left(\frac{x}{a}t\right) - \cos\left(\frac{r_0}{a}t\right)\right] dt \\ w(x, 0) &= \frac{(1-2\nu)q_0a}{\pi G} \int_0^\infty \left(\frac{1-\cos t}{t^3}\right) \left(1+\frac{b}{a}t\right)^{-1} \sin\left(\frac{x}{a}t\right) dt \end{aligned} \tag{31}$$

It is instructive to examine the influence of the surface elasticity on the stresses and displacements of the contact surface and compare them with those in classical contact problem. **Figure 5** and **Figure 6** show the distribution of the stresses σ_{xx} and σ_{xz} on the contact surface, where the solution of $b/a = 0$ is consistent with the classical elastic result.

It can be seen from **Figure 7** that the normal stress σ_{xx} transits continuously across the loading boundary $x = \pm a$, which is increasing monotonically with respect to x in the loading region ($|x/a| < 0.75$), and the inverse is observed outside the loading region ($|x/a| > 0.75$). It means that the analytical expressions can give approximate results in the region that is far from the loading boundary. It is also found **Figure 5** that shear stress changes smoothly across the loading boundary $x = \pm a$, which is different from a

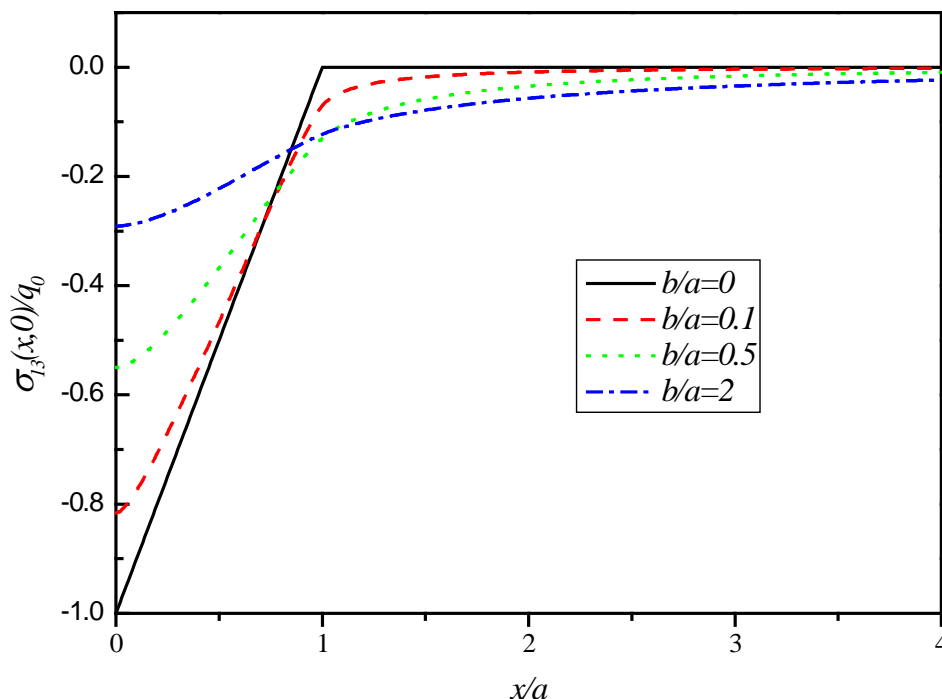


Figure 5. The distribution of the shear stress σ_{xz} under tangential triangle distribution load.

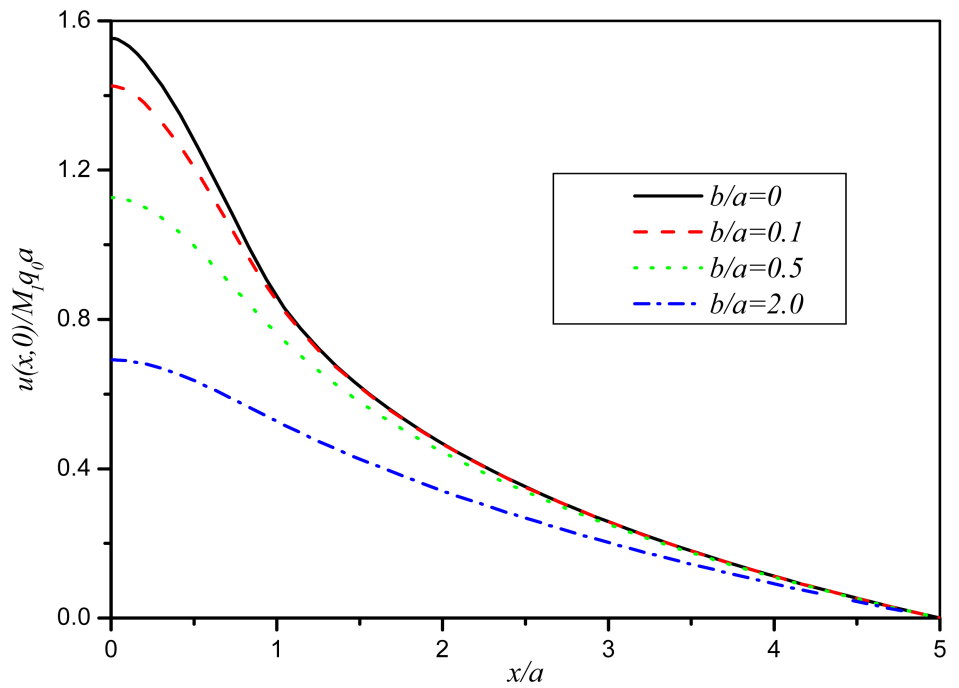


Figure 6. The distribution of surface displacement u under tangential triangle distribution load.

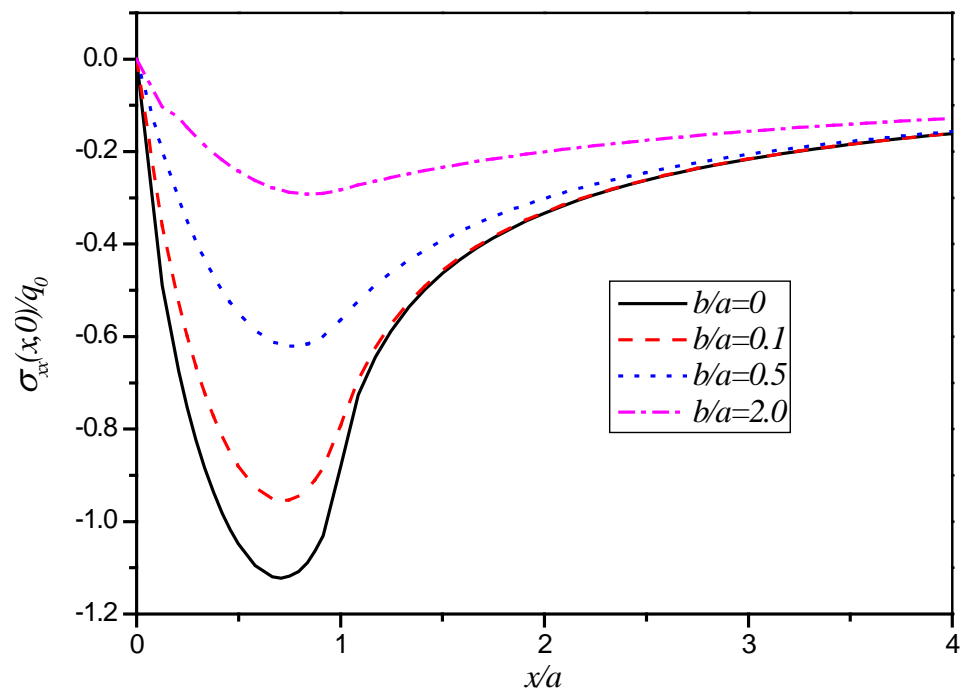


Figure 7. The distribution of the normal stress σ_{xx} under tangential triangle distribution load.

singularity predicted by classical elasticity.

Due to the different surface elasticity value, the horizontal displacement is displayed in **Figure 6**, where we set $r_0 = 5a$, and $M_1 = 2(1-\nu)/\pi G$. It is seen that the slope of

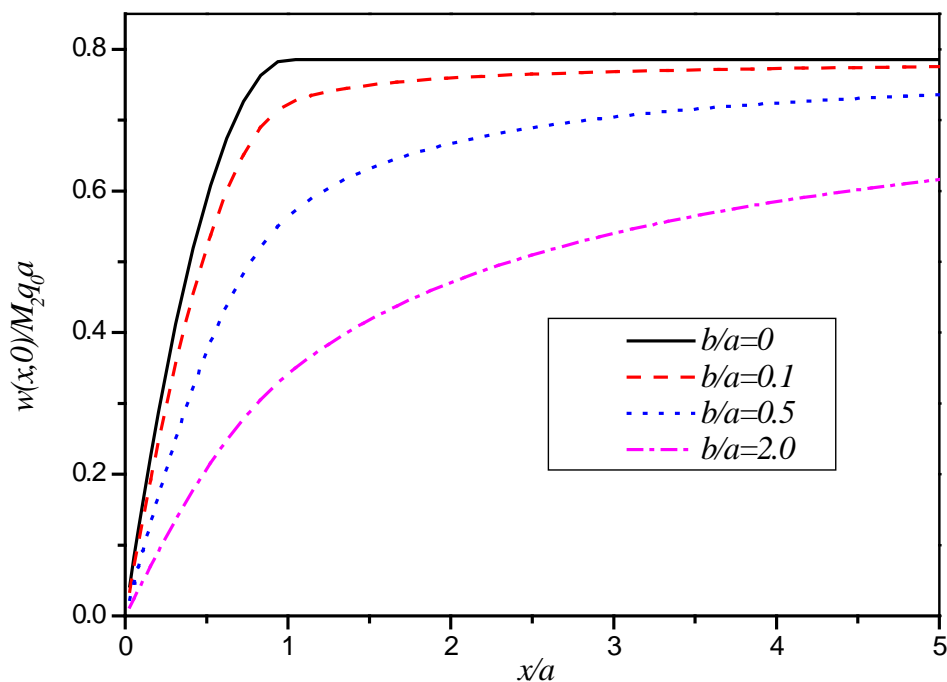


Figure 8. The distribution of surface indentation w under tangential triangle distribution load.

the deformed surface for $a > 0$ is continuous everywhere. It is also found the horizontal displacement decreases with the increase of surface elasticity. The indent depth is plotted in Figure 8 with $M_2 = (1 - 2\nu)/\pi G$, which also shows that the normal displacement is continuous everywhere on the deformed surface. In addition, the indent depth decreases continuously with the increase of surface elasticity.

6. Conclusion

In this paper, we consider the two-dimensional contact problem in the light of surface elasticity theory. Fourier integral transform method is adopted solving general analytical solution. For two particular loading cases of triangle distribution forces, the results are analyzed in detail and compared with the classical linear elastic solutions. A series of theoretical and numerical results show that the influences of the surface tension and the surface elasticity on the stresses and displacements are not always equal. It is found that the surface elasticity theory illuminates some interesting characteristics of contact problems at nano-scale, which are distinctly different from the classical solutions of elasticity without surface effects. Therefore, the influence of surface effects should be considered for nano-contact problems.

References

- [1] Cammarata, R.C. (1994) Surface and Interface Stress Effects in Thin Films. *Progress in Surface Science*, **46**, 1-38. [http://dx.doi.org/10.1016/0079-6816\(94\)90005-1](http://dx.doi.org/10.1016/0079-6816(94)90005-1)
- [2] Miller, R.E. and Shenoy, V.B. (2000) Size-Dependent Elastic Properties of Nanosized Structural Elements. *Nanotechnology*, **11**, 139-147.

- <http://dx.doi.org/10.1088/0957-4484/11/3/301>
- [3] Huang, Y., Zhang, F. and Hwang, K.C. (2006) A Model of Size Effects in Nano-Indentation. *Journal of the Mechanics & Physics of Solids*, **54**, 1668-1686. <http://dx.doi.org/10.1016/j.jmps.2006.02.002>
- [4] Dingreville, R., Qu, J. and Cherkaoui, M. (2005) Surface Free Energy and Its Effect on the Elastic Behavior of Nano-Sized Particles, Wires And films. *Journal of the Mechanics & Physics of Solids*, **53**, 1827-1854. <http://dx.doi.org/10.1016/j.jmps.2005.02.012>
- [5] Yang, F. (2004) Size-Dependent Effective Modulus of Elastic Composite Materials: Spherical Nanocavities at Dilute Concentrations. *Journal of Applied Physics*, **95**, 3516-3520. <http://dx.doi.org/10.1063/1.1664030>
- [6] Gao, X., Hao, F. and Huang, Z. (2014) Mechanics of Adhesive Contact at the Nanoscale: The Effect of Surface Stress. *International Journal of Solids & Structures*, **51**, 566-574. <http://dx.doi.org/10.1016/j.ijsolstr.2013.10.017>
- [7] Wang, J.X., Huang, Z.P., et al. (2011) Surface Stress Effect in Mechanics of Nanostructured Materials. *Acta Mechanica Solida Sinica*, **24**, 52-82. [http://dx.doi.org/10.1016/s0894-9166\(11\)60009-8](http://dx.doi.org/10.1016/s0894-9166(11)60009-8)
- [8] Gurtin, M.E. and Murdoch, A.I. (1975) A Continuum Theory of Elastic Material Surfaces. *Archive for Rational Mechanics & Analysis*, **57**, 291-323. <http://dx.doi.org/10.1007/BF00261375>
- [9] Gurtin, M.E. and Murdoch, A.I. (1976) Effect of Surface Stress on Wave Propagation in Solids. *Journal of Applied Physics*, **47**, 4414-4421. <http://dx.doi.org/10.1063/1.322403>
- [10] Gurtin, M.E. and Murdoch, A.I. (1978) Surface Stress in Solids. *International Journal of Solids & Structures*, **14**, 431-440. [http://dx.doi.org/10.1016/0020-7683\(78\)90008-2](http://dx.doi.org/10.1016/0020-7683(78)90008-2)
- [11] Jing, G.Y., Duan, H.L., Sun, X.M., et al. (2006) Surface Effects on Elastic Properties of Silver Nanowires: Contact Atomic-Force Microscopy. *Physical Review B*, **73**, 235409. <http://dx.doi.org/10.1103/physrevb.73.235409>
- [12] Shenoy, V.B. (2005) Atomistic Calculations of Elastic Properties of Metallic Fcc Crystal Surfaces. *Physical Review B*, **71**, 4104. <http://dx.doi.org/10.1103/PhysRevB.71.094104>
- [13] Cammarata, R.C., Sieradzki, K. and Spaepen, F. (2000) Simple Model for Interface Stresses with Application to Misfit Dislocation Generation in Epitaxial Thin Films. *Journal of Applied Physics*, **87**, 1227-1234. <http://dx.doi.org/10.1063/1.372001>
- [14] Wei, G., Shouwen, Y. and Ganyun, H. (2014) Finite Element Characterization of the Size-Dependent Mechanical Behaviour in Nanosystems. *Nanotechnology*, **17**, 1118-1122. <http://dx.doi.org/10.1088/0957-4484/17/4/045>
- [15] Wang, G.F. and Feng, X.Q. (2007) Effects of Surface Stresses on Contact Problems at Nanoscale. *Journal of Applied Physics*, **101**, 013510. <http://dx.doi.org/10.1063/1.2405127>
- [16] Zhao, X.J. and Rajapakse, R.K.N.D. (2009) Analytical Solutions for a Surface-Loaded Isotropic Elastic Layer with Surface Energy Effects. *International Journal of Engineering Science*, **47**, 1433-1444. <http://dx.doi.org/10.1016/j.ijengsci.2008.12.013>
- [17] Long, J.M., Wang, G.F. and Feng, X.Q. (2012) Two-Dimensional Hertzian Contact Problem with Surface Tension. *International Journal of Solids & Structures*, **49**, 1588-1594. <http://dx.doi.org/10.1016/j.ijsolstr.2012.03.017>
- [18] Long, J.M. and Wang, G.F. (2013) Effects of Surface Tension on Axisymmetric Hertzian Contact Problem. *Mechanics of Materials*, **56**, 65-70. <http://dx.doi.org/10.1016/j.mechmat.2012.09.003>
- [19] Lei, D.X., Wang, L.Y. and Ou, Z.Y. (2012) Elastic Analysis for Nanocontact Problem with

- Surface Stress Effects under Shear Load. *Journal of Nanomaterials*, **6**, 4531-4531.
<http://dx.doi.org/10.1155/2012/505034>
- [20] Gao, X., Hao, F., Fang, D., *et al.* (2013) Boussinesq Problem with the Surface Effect and Its Application to Contact Mechanics at the Nanoscale. *International Journal of Solids & Structures*, **50**, 2620-2630. <http://dx.doi.org/10.1016/j.ijsolstr.2013.04.007>
- [21] Gao, X., Hao, F., Fang, D., *et al.* (2014) Mechanics of Adhesive Contact at the Nanoscale: The Effect of Surface Stress. *International Journal of Solids & Structures*, **51**, 566-574.
<http://dx.doi.org/10.1016/j.ijsolstr.2013.10.017>
- [22] Povstenko, Y.Z. (1993) Theoretical Investigation of Phenomena Caused by Heterogeneous Surface Tension in Solids. *Journal of the Mechanics & Physics of Solids*, **41**, 1499-1514.
[http://dx.doi.org/10.1016/0022-5096\(93\)90037-G](http://dx.doi.org/10.1016/0022-5096(93)90037-G)
- [23] Wang, L.Y. (2015) Elastic Analysis for Contact Problems with Surface Effects under Normal Load. *Mathematics & Mechanics of Solids*. <http://dx.doi.org/10.1177/1081286514568585>
- [24] Johnson, K.L. (1985) Contact Mechanics. Cambridge University Press, London.
<http://dx.doi.org/10.1017/CBO9781139171731>



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