Journal of Applied Mathematics and Physics, 2016, 4, 1705-1713
http://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Robust Finite-Time H_∞ Filtering for Itô Stochastic Systems

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How to cite this paper: Zhang, A.Q. (2016) Robust Finite-Time H_∞ Filtering for Itô Stochastic Systems. *Journal of Applied Mathematics and Physics*, **4**, 1705-1713.
http://dx.doi.org/10.4236/jamp.2016.49179

Received: August 17, 2016
Accepted: September 17, 2016
Published: September 20, 2016

Abstract

This paper investigates the problem of robust finite-time H_∞ filter design for Itô stochastic systems. Based on linear matrix inequalities (LMIS) techniques and stability theory of stochastic differential equations, stochastic Lyapunov function method is adopted to design a finite-time H_∞ filter such that, for all admissible uncertainties, the filtering error system is stochastic finite-time stable (SFTS). A sufficient condition for the existence of a finite-time H_∞ filter for the stochastic system under consideration is achieved in terms of LMIS. Moreover, the explicit expression of the desired filter parameters is given. A numerical example is provided to illustrate the effectiveness of the proposed method.

Keywords

Stochastic Systems, H_∞ Filter, Finite-Time Stability, Linear Matrix Inequalities (LMIS)

1. Introduction

Since stochastic systems play an important role in many branches of science and engineering applications, there has been a rapidly growing interest in stochastic systems. In the past few years, much attention has been focused on the robust H_∞ filtering problems of stochastic systems; many contributions have been reported in the literature [1]-[6]. In [1], a H_∞ filter was designed for nonlinear stochastic systems. From the dissipation point of view, a H_∞ filtering theory and a H_∞-type theory for a class of stochastic nonlinear systems were established in [2] [3]. H_∞ filtering problems for discrete-time nonlinear stochastic systems were addressed in [4]. The H_∞ filtering problems for uncertain stochastic systems with delays were studied in [5]. A robust fuzzy filter for a class of nonlinear stochastic systems was designed in [6].

The previously mentioned literature was based on Lyapunov asymptotic stability which focuses on the steady-state behavior of plants over an infinite-time interval. However, in many practical applications, the goal is to keep the state trajectories within some prescribed bounds during a fixed time interval. In these cases, we need to guarantee that the system states remain within the given bounds, which is called finite-time stability. Recently, finite-time stability or short time stability and control problems for many types of dynamic systems were studied widely in [7]-[12]. The problem of finite-time stability and stabilization for a class of linear systems with time delay was addressed in [7]. In [8], the sufficient conditions were achieved for the finite-time stability of linear time-varying systems with jumps. The authors provided the sufficient conditions of finite-time stability for stochastic nonlinear systems in [9]. The problem of robust finite-time stabilization for impulsive dynamical linear systems was investigated in [10]. In [11] fuzzy control method was adopted to solve finite-time stabilization of a class of stochastic system. A robust finite-time filter was established for singular discrete-time stochastic system in [12]. It can be pointed out that all the FTS-related works for finite-time problems mentioned above were discussed for stochastic systems. To the best of the author's knowledge, the problem of robust finite-time filtering for stochastic systems has not been fully investigated. This motivates us to investigate the present study. One application of these new results could be used to detect generation of residuals for fault diagnosis problems.

This paper is organized as follows. Some preliminaries and the problem formulation are introduced in Section 2. In Section 3, a sufficient condition for SFTS of the corresponding filtering error system is established and the method to design a finite-time filter is presented. Section 4 presents a numerical example to demonstrate the affectivity of the mentioned methodology. Some conclusions are drawn in Section 5.

We use R^n to denote the n-dimensional Euclidean space. The notation $X > Y$ (respectively, $X \geq Y$, where X and Y are real symmetric matrices), means that the matrix $X - Y$ is positive definite (respectively, positive semi-definite). I and 0 denote the identity and zero matrices with appropriate dimensions. $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ denote the maximum and the minimum of the eigenvalues of a real symmetric matrix Q. The superscript T denotes the transpose for vectors or matrices. The symbol * in a matrix denotes a term that is defined by symmetry of the matrix.

2. Systems Descriptions and Problem Formulation

Consider an uncertain Itô stochastic system, which can be described as follows:

$$\mathrm{d}x(t) = \left\{\left[A + \Delta A(t)\right]x(t) + A_1 v(t)\right\}\mathrm{d}t + \left\{\left[D + \Delta D(t)\right]x(t) + D_1 v(t)\right\}\mathrm{d}w(t); \quad (1)$$

$$\mathrm{d}y(t) = \left\{\left[C + \Delta C(t)\right]x(t) + C_1 v(t)\right\}\mathrm{d}t + \left\{\left[E + \Delta E(t)\right]x(t) + E_1 v(t)\right\}\mathrm{d}w(t); \quad (2)$$

$$z(t) = Lx(t),\ x(0) = x_0. \quad (3)$$

where $x(t) \in R^n$, $y(t) \in R^m$, $v(t) \in R^p$, $z(t) \in R^q$ are state vector, measurement, disturbance input, and controlled output respectively, where $v(t) \in L_2[0,\infty)$, and

$w(t) \in R$ is a standard Wiener process. A, D, C, C_1, E, E_1, L are known constant matrices of appropriate dimensions and $\Delta A(t), \Delta D(t), \Delta C(t), \Delta E(t)$ are unknown matrices that represent the time-varying parameter uncertainties and are assumed to be of the form

$$\left[\Delta A(t)\ \ D(t)\ \ \Delta C(t)\ \ \Delta E(t)\right] = HF(t)\left[G_1\ \ G_2\ \ G_3\ \ G_4\right].$$

where $F(t) \in R^{\phi_k \times \varphi_k}$ is an unknown matrix with Lebesgue measurable elements satisfying $F^{\mathrm{T}}(t)F(t) \le I$. H, G_1, G_2, G_3, G_4 are known constant matrices with appropriate dimensions.

We now consider the following filter for system (1)-(3):

$$\begin{aligned} \mathrm{d}\hat{x}(t) &= A_f \hat{x}(t)\mathrm{d}t + B_f \mathrm{d}y(t); \\ \hat{z}(t) &= C_f \hat{x}(t),\ \hat{x}(0) = x_0 \end{aligned} \tag{4}$$

where $\hat{x}(t) \in R^n$ is the filter state, A_f, B_f, C_f are the filter parameters with compatible dimensions to be determined.

Define $\xi^{\mathrm{T}}(t) = \left[x^{\mathrm{T}}(t)\ \hat{x}^{\mathrm{T}}(t)\right]$ and $e(t) = z(t) - \hat{z}(t)$, then we can obtain the following filtering error system:

$$\mathrm{d}\xi(t) = \left[\bar{A}\xi(t) + \bar{B}v(t)\right]\mathrm{d}t + \left[\bar{D}\xi(t) + \bar{D}_1 v(t)\right]\mathrm{d}w(t); \tag{5}$$

$$e(t) = \bar{L}\xi(t),\quad \xi(0) = \xi_0 \tag{6}$$

where

$$\bar{A} = \begin{bmatrix} A + \Delta A(t) & 0 \\ B_f\left[C + \Delta C(t)\right] & A_f \end{bmatrix},\ \bar{B} = \begin{bmatrix} A_1 \\ B_f C_1 \end{bmatrix}$$

$$\bar{D} = \begin{bmatrix} D + \Delta D(t) & 0 \\ B_f\left[E + \Delta E(t)\right] & 0 \end{bmatrix},\ \bar{D}_1 = \begin{bmatrix} D_1 \\ B_f E_1 \end{bmatrix},\ \bar{L} = \begin{bmatrix} L & -C_f \end{bmatrix}$$

We introduce the following definitions and lemmas, which will be useful in the succeeding discussion.

Definition 1 [13]: The filtering error system (5) (6) is said to be stochastic finite-time stable (SSFTS) with respect to (c_1, c_2, T, R), where $R > 0, 0 < c_1 < c_2$ if for a given time-constant $T > 0$, the following relation holds: $\Xi\left\{x_0^{\mathrm{T}} R x_0\right\} < c_1 \Rightarrow \Xi\left\{x_t^{\mathrm{T}} R x_t\right\} < c_2$ $\forall t \in [0, T]$.

Definition 2: Given a disturbance attenuation level $\gamma > 0$, the filtering error system (5) (6) is said to be robustly stochastic finite-time stable (SFTS) with respect to (c_1, c_2, T, R) with a prescribed H_∞ disturbance attenuation level γ, if it is robustly stochastic finite-time stable in the sense of Definition 1 and $\Xi\left\|e(t)\right\|_2^2 \le \gamma^2 \left\|v(t)\right\|_2^2$ for all nonzero $v(t) \in L_2[0, \infty)$ and all admissible uncertainties.

Lemma 1 [14]: Let M, N and F be matrices of appropriate dimensions, and $F^{\mathrm{T}}F \le I$, then for any scalar $\varepsilon > 0$,

$$MFN + N^{\mathrm{T}}F^{\mathrm{T}}M^{\mathrm{T}} \le \varepsilon MM^{\mathrm{T}} + \varepsilon^{-1}N^{\mathrm{T}}N. \tag{7}$$

Lemma 2 [15]: Let A, B, C, F and S be matrices of appropriate dimensions such that $S > 0$, and $F^{\mathrm{T}}F \le I$. Then for any scalar $\varepsilon > 0$, such that $S - \varepsilon BB^{\mathrm{T}} > 0$, the following inequality holds

$$(A + BFC)^{\mathrm{T}} S^{-1} (A + BFC) \le A^{\mathrm{T}} \left(S - \varepsilon BB^{\mathrm{T}}\right)^{-1} A + \varepsilon^{-1} C^{\mathrm{T}} C . \tag{8}$$

Lemma 3 [16] (Gronwall inequality): Let $v(t)$ be a nonnegative function such that

$$v(t) \le a + b\int_0^t v(s)\,\mathrm{d}s,\ 0 < t < T$$

for some constants $a, b \ge 0$, then we have $v(t) \le a\exp(bt), 0 < t < T$.

Lemma 4 [17] [18] (Schur complement): Given a symmetric matrix $\phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$ the following three conditions are equivalent to each other:

1) $\phi < 0$;
2) $\phi_{11} < 0$ and $\phi_{22} - \phi_{12}^{\mathrm{T}} \phi_{11}^{-1} \phi_{12} < 0$;
3) $\phi_{22} < 0$ and $\phi_{11} - \phi_{12} \phi_{22}^{-1} \phi_{12}^{\mathrm{T}} < 0$.

3. Robust Finite-Time H_∞ Filter Design

Theorem 1: Suppose that the filter parameters A_f, B_f, C_f in (4) are given. The filtering error system (5) (6) is robustly SSFTS with respect to (c_1, c_2, T, R), if there exist scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \alpha > 0$ and symmetric positive definite matrix P satisfying

$$p = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} = R^{\frac{1}{2}} Q R^{\frac{1}{2}}$$

$$P_1^{-1} - \varepsilon_3 HH^{\mathrm{T}} > 0,\ P_2^{-1} - \varepsilon_4 B_f HH^{\mathrm{T}} B_f^{\mathrm{T}} > 0 ,$$

such that the following LMIs hold

$$\Pi < 0, \tag{9}$$

$$\Pi = \begin{bmatrix} \Pi_{11} & C^{\mathrm{T}} B_f^{\mathrm{T}} P_2 & D^{\mathrm{T}} & (B_f E)^{\mathrm{T}} & G_1^{\mathrm{T}} & G_3^{\mathrm{T}} & G_2^{\mathrm{T}} & G_4^{\mathrm{T}} \\ * & \Pi_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_4 I \end{bmatrix} \tag{10}$$

and

$$\frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)} c_1 \mathrm{e}^{\alpha T} \le c_2 \tag{11}$$

$$\Pi_{11} = P_1^{\mathrm{T}} A + A^{\mathrm{T}} P_1 + \varepsilon_1 P_1^{\mathrm{T}} HH^{\mathrm{T}} P_1 - \alpha P_1 ,$$

$$\Pi_{22} = P_2^{\mathrm{T}} A_f + A_f^{\mathrm{T}} P_2 + \varepsilon_2 P_2^{\mathrm{T}} B_f HH^{\mathrm{T}} B_f^{\mathrm{T}} P_2 - \alpha P_2 ,$$

$$\Pi_{33} = \varepsilon_3 HH^{\mathrm{T}} - P_1^{-1},\quad \Pi_{44} = \varepsilon_4 B_f HH^{\mathrm{T}} B_f^{\mathrm{T}} - P_2^{-1} .$$

where "*" denotes the transposed elements in the symmetric positions.

Proof: Consider a stochastic Lyapunov function candidate defined as follows:

$$V(\xi(t)) = \xi^{\mathrm{T}}(t) P \xi(t), \tag{12}$$

By Itô formula, we have the stochastic differential $\mathrm{d}V(\xi(t))$ along the trajectories of system (5) (6) with $v(t)=0$ as follows:

$$\mathrm{d}V(\xi(t)) = LV(\xi(t)) + 2\xi^{\mathrm{T}}(t) P\left[\bar{D}(t)\xi(t)\right]\mathrm{d}w(t)$$

where

$$\begin{aligned}
LV(\xi(t)) &= \xi^{\mathrm{T}}(t) P^{\mathrm{T}} \bar{A}(t)\xi(t) + \left[\bar{A}(t)\xi(t)\right]^{\mathrm{T}} P\xi(t) + \left[\bar{D}(t)\xi(t)\right]^{\mathrm{T}} P\bar{D}(t)\xi(t) \\
&= \xi^{\mathrm{T}}(t)\left\{\begin{bmatrix} P_1^{\mathrm{T}}A + A^{\mathrm{T}}P_1 & C^{\mathrm{T}}B_f^{\mathrm{T}}P_2 \\ P_2^{\mathrm{T}}B_f C & P_2^{\mathrm{T}}A_f + A_f^{\mathrm{T}}P_2 \end{bmatrix} + \begin{bmatrix} P_1^{\mathrm{T}}H \\ 0 \end{bmatrix} F(t)\begin{bmatrix} G_1 & 0 \end{bmatrix}\right. \\
&\quad + \left(\begin{bmatrix} P_1^{\mathrm{T}}H \\ 0 \end{bmatrix} F(t)\begin{bmatrix} G_1 & 0 \end{bmatrix}\right)^{\mathrm{T}} + \begin{bmatrix} 0 \\ P_2^{\mathrm{T}}B_f H \end{bmatrix} F(t)\begin{bmatrix} G_3 & 0 \end{bmatrix} + \left(\begin{bmatrix} 0 \\ P_2^{\mathrm{T}}B_f H \end{bmatrix} F(t)\begin{bmatrix} G_3 & 0 \end{bmatrix}\right)^{\mathrm{T}} \\
&\quad + \left(\begin{bmatrix} D & 0 \end{bmatrix} + HF(t)\begin{bmatrix} G_2 & 0 \end{bmatrix}\right)^{\mathrm{T}} P_1\left(\begin{bmatrix} D & 0 \end{bmatrix} + HF(t)\begin{bmatrix} G_2 & 0 \end{bmatrix}\right) \\
&\quad \left. + \left(\begin{bmatrix} B_f E & 0 \end{bmatrix} + B_f HF(t)\begin{bmatrix} G_4 & 0 \end{bmatrix}\right)^{\mathrm{T}} P_2\left[\begin{bmatrix} B_f E & 0 \end{bmatrix} + B_f HF(t)\begin{bmatrix} G_4 & 0 \end{bmatrix}\right]\right\}\xi(t)
\end{aligned}$$

We prove

$$LV(\xi(t)) < \alpha V(\xi(t)) \tag{13}$$

By Lemma 1 and Lemma 2, we have

$$\begin{aligned}
LV(\xi(t)) - \alpha V(\xi(t)) \le \xi^{\mathrm{T}}(t) &\left\{\begin{bmatrix} P_1^{\mathrm{T}}A + A^{\mathrm{T}}P_1 - \alpha P_1 & C^{\mathrm{T}}B_f^{\mathrm{T}}P_2 \\ P_2^{\mathrm{T}}B_f C & P_2^{\mathrm{T}}A_f + A_f^{\mathrm{T}}P_2 - \alpha P_2 \end{bmatrix}\right. \\
&+ \varepsilon_1\begin{bmatrix} P_1^{\mathrm{T}}H \\ 0 \end{bmatrix}\begin{bmatrix} H^{\mathrm{T}}P_1 & 0 \end{bmatrix}\xi(t) + \varepsilon_1^{-1}\begin{bmatrix} G_1^{\mathrm{T}} \\ 0 \end{bmatrix}\begin{bmatrix} G_1 & 0 \end{bmatrix} \\
&+ \varepsilon_2\begin{bmatrix} 0 \\ P_2^{\mathrm{T}}B_f H \end{bmatrix}\begin{bmatrix} 0 & H^{\mathrm{T}}B_f^{\mathrm{T}}P_2 \end{bmatrix} + \varepsilon_2^{-1}\begin{bmatrix} G_3^{\mathrm{T}} \\ 0 \end{bmatrix}\begin{bmatrix} G_3 & 0 \end{bmatrix} \\
&+ \begin{bmatrix} D^{\mathrm{T}} \\ 0 \end{bmatrix}\left(P_1^{-1} - \varepsilon_3 HH^{\mathrm{T}}\right)^{-1}\begin{bmatrix} D & 0 \end{bmatrix} + \varepsilon_3^{-1}\begin{bmatrix} G_2^{\mathrm{T}} \\ 0 \end{bmatrix}\begin{bmatrix} G_2 & 0 \end{bmatrix} \\
&\left. + \begin{bmatrix} B_f E \\ 0 \end{bmatrix}^{\mathrm{T}}\left(P_2^{-1} - \varepsilon_4 B_f HH^{\mathrm{T}}B_f^{\mathrm{T}}\right)^{-1}\begin{bmatrix} B_f E & 0 \end{bmatrix} + \varepsilon_4^{-1}\right\}\begin{bmatrix} G_4^{\mathrm{T}} \\ 0 \end{bmatrix}\begin{bmatrix} G_4 & 0 \end{bmatrix}\xi(t)
\end{aligned}$$

By Lemma 4 and (7) (8), it follows that

$$LV(\xi(t)) - \alpha V(\xi(t)) \le \xi^{\mathrm{T}}(t)\Pi\xi(t) < 0,$$

Integrating both sides of (13) from 0 to t with $t \in [0, T]$, and taking expectation, we have

$$\Xi\{V(\xi(t))\} < V(\xi(0)) + \alpha\int_0^T \Xi\{V(\xi(s))\}\mathrm{d}s$$

By Lemma 3, it follows $\Xi\{V(\xi(t))\} < V(\xi(0))\mathrm{e}^{\alpha t}$

$$\Xi\left\{V\left(\xi(t)\right)\right\}=\Xi\left\{\xi^{\mathrm{T}}(t)R^{\frac{1}{2}}QR^{\frac{1}{2}}\xi(t)\right\}\geq\lambda_{\min}(Q)\Xi\left\{\xi^{\mathrm{T}}(t)R\xi(t)\right\}\tag{14}$$

$$\begin{aligned}V\left(\xi(0)\right)\mathrm{e}^{\alpha t}&=\xi^{\mathrm{T}}(0)R^{\frac{1}{2}}QR^{\frac{1}{2}}\xi(0)\mathrm{e}^{\alpha t}\\&\leq\lambda_{\max}(Q)R\xi^{\mathrm{T}}(0)\xi(0)\mathrm{e}^{\alpha t}\\&\leq\lambda_{\max}(Q)c_1\mathrm{e}^{\alpha T}\end{aligned}$$

From (12) and (13), we obtain

$$\Xi\left\{\xi^{\mathrm{T}}(t)R\xi(t)\right\}\leq\frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)}c_1\mathrm{e}^{\alpha t}$$

It implies $\Xi\left\{\xi^{\mathrm{T}}(t)R\xi(t)\right\}\leq c_2$, $t\in[0,T]$.

Theorem 2: For given $\gamma>0$. The filtering error system (5) (6) is robustly SSFTS with respect to (c_1,c_2,T,R), with a prescribed H_∞ disturbance attenuation level γ, if there exist scalars $\varepsilon_5,\varepsilon_6$, $\alpha>0$ and matrices $\Theta_1,\Theta_2,\Theta_3$ and the same matrix P as theorem 1, satisfying

$$P^{-1}-\varepsilon_5\tilde{H}\tilde{H}^{\mathrm{T}}>0,$$

such that (11) and the following LMIs hold

$$\Omega<0.\tag{15}$$

$$\Omega=\begin{bmatrix}\tilde{A}P^{-1}+P^{-\mathrm{T}}\tilde{A}^{\mathrm{T}}-\alpha P & \bar{B}P^{-1} & \bar{H}P^{-1} & P^{-\mathrm{T}}\bar{G}^{T}P & P^{-\mathrm{T}}\bar{L}^{\mathrm{T}}P^{-1} & P^{-\mathrm{T}}\tilde{D}^{\mathrm{T}}P^{-1} & P^{-\mathrm{T}}\tilde{G}^{\mathrm{T}}P^{-1}\\ * & -\gamma^2I & 0 & 0 & 0 & 0 & 0\\ * & * & -\varepsilon_6^{-1}I & 0 & 0 & 0 & 0\\ * & * & * & -\varepsilon_6I & 0 & 0 & 0\\ * & * & * & * & -I & 0 & 0\\ * & * & * & * & * & \left(\varepsilon_5\tilde{H}\tilde{H}^{\mathrm{T}}-P^{-1}\right) & 0\\ * & * & * & * & * & * & -\varepsilon_5I\end{bmatrix}$$

In this case, the suitable filter parameters A_f,B_f,C_f in system (4) can be given by $A_f=P_2^{-\mathrm{T}}\Theta_1,B_f=P_2^{-\mathrm{T}}\Theta_2,C_f=P_2^{-\mathrm{T}}\Theta_2P_2^{-1}$.

Proof: It follows from Theorem 1 and Schur complements lemma that the filtering error system (5) (6) is robustly SFTS with respect to (c_1,c_2,T,R) and (13) is followed.

Next, we shall show that the system (5) (6)) satisfies

$$\Xi\left\|e(t)\right\|_2^2\leq\gamma^2\left\|v(t)\right\|_2^2.\tag{16}$$

where the Lyapunov function candidate $V\left(t,\xi(t)\right)$ is given in (12)

By Itô formula, we have the stochastic differential as

$$\mathrm{d}V\left(t,\xi(t)\right)=LV\left(t,\xi(t)\right)+2\xi^{\mathrm{T}}(t)P\left[\bar{D}(t)\xi(t)+\bar{D}_1\right]\mathrm{d}w(t),$$

where

$$\begin{aligned}LV\left(t,\xi(t)\right)=&\,\xi^{\mathrm{T}}(t)P^{\mathrm{T}}\left[\bar{A}(t)\xi(t)+\bar{B}v(t)\right]+\left[\bar{A}(t)\xi(t)+\bar{B}v(t)\right]^{\mathrm{T}}P\xi(t)\\&+\left[\bar{D}(t)\xi(t)+\bar{D}_1v(t)\right]^{\mathrm{T}}P\left[\bar{D}(t)\xi(t)+\bar{D}_1v(t)\right]\end{aligned}$$

and $\bar{A},\bar{B},\bar{D},\bar{D}_1$ are defined in part 2.

Assume $\Xi\left(\int_0^t LV\left(x(s)\right)\mathrm{d}s\right)\geq 0.$

By (13), we have that

$$\begin{aligned}J(t)&=\Xi\left\{\int_0^t\left[e^{\mathrm{T}}(s)e(s)-\gamma^2v^{\mathrm{T}}(s)v(s)\right]\mathrm{d}s\right\}\\&\leq\Xi\left\{\int_0^t\left[LV\left(\xi(s)\right)+e^{\mathrm{T}}(s)e(s)-\gamma^2v^{\mathrm{T}}(s)v(s)\right]\mathrm{d}s\right\}\\&<\Xi\left[\alpha V\left(\xi(t)\right)\right]\end{aligned}$$

Observe that

$$\begin{aligned}&LV\left(\xi(t)\right)+e^{\mathrm{T}}(t)e(t)-\gamma^2v^{\mathrm{T}}(t)v(t)-\alpha V\left(\xi(t)\right)\\&=\begin{bmatrix}\xi(t)\\v(t)\end{bmatrix}^{\mathrm{T}}\left\{\begin{bmatrix}P^{\mathrm{T}}\bar{A}+\bar{A}^{\mathrm{T}}P+\bar{L}^{\mathrm{T}}\bar{L}-\alpha P & P^{\mathrm{T}}\bar{B}\\ \bar{B}^{\mathrm{T}}P & -\gamma^2 I\end{bmatrix}+\begin{bmatrix}\bar{D}^{\mathrm{T}}\\ \bar{D}_1^{\mathrm{T}}\end{bmatrix}P\begin{bmatrix}\bar{D} & \bar{D}\end{bmatrix}\right\}\begin{bmatrix}\xi(t)\\v(t)\end{bmatrix}\end{aligned}$$

By Lemma 1 and Lemma 2, we have

$$\begin{aligned}\begin{bmatrix}\bar{D}^{\mathrm{T}}\\ \bar{D}_1^{\mathrm{T}}\end{bmatrix}P\begin{bmatrix}\bar{D}(t) & \bar{D}\end{bmatrix}&=\left[\tilde{D}+\tilde{H}F(t)\tilde{G}\right]^{\mathrm{T}}P\left[\tilde{D}+\tilde{H}F(t)\tilde{G}\right]\\&\leq\tilde{D}^{\mathrm{T}}\left(P^{-1}-\varepsilon_5\tilde{H}\tilde{H}^{\mathrm{T}}\right)^{-1}\tilde{D}+\varepsilon_5^{-1}\tilde{G}^{\mathrm{T}}\tilde{G}\end{aligned}$$

where, $\tilde{G}=\begin{bmatrix}G_2 & G_4 & 0\end{bmatrix}$, $\tilde{D}=\begin{bmatrix}D & B_fE_1 & D_1\end{bmatrix}$, $\tilde{H}=\begin{bmatrix}H & B_fH & 0\end{bmatrix}$.

$$\begin{aligned}&\begin{bmatrix}P^{\mathrm{T}}\bar{A}+\bar{A}^{\mathrm{T}}P+\bar{L}^{\mathrm{T}}\bar{L}-\alpha P & P^{\mathrm{T}}\bar{B}\\ \bar{B}^{\mathrm{T}}P & -\gamma^2 I\end{bmatrix}\\&\leq\begin{bmatrix}\begin{array}{l}P^{\mathrm{T}}\tilde{A}+\tilde{A}^{\mathrm{T}}P+\bar{L}^{\mathrm{T}}\bar{L}-\alpha P+\\ \varepsilon_6P^{\mathrm{T}}\bar{H}\bar{H}^{\mathrm{T}}P+\varepsilon_6^{-1}\bar{G}^{\mathrm{T}}\bar{G}\end{array} & P^{\mathrm{T}}\bar{B}\\ \bar{B}^{\mathrm{T}}P & -\gamma^2 I\end{bmatrix}\end{aligned}$$

where $\tilde{A}=\begin{bmatrix}A & 0\\ B_fC & A_f\end{bmatrix},\bar{H}=\begin{bmatrix}H^{\mathrm{T}} & H^{\mathrm{T}}B_f^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},\bar{G}=\begin{bmatrix}G_1^{\mathrm{T}} & G_3^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$.

Therefore, using Lemma 4, it follows that

$$\begin{aligned}&\Xi\left\{\int_0^t\left[LV\left(\xi(s)\right)+e^{\mathrm{T}}(s)e(s)-\gamma^2v^{T}(s)v(s)-\alpha V\left(\xi(s)\right)\right]\mathrm{d}s\right\}\\&\leq\Xi\left\{\int_0^t\left[\xi^{\mathrm{T}}(s)\quad v^{\mathrm{T}}(s)\right]\Theta\begin{bmatrix}\xi(s)\\v(s)\end{bmatrix}\mathrm{d}s\right\}\end{aligned}\tag{17}$$

where

$$\begin{aligned}\Theta=&\begin{bmatrix}P^{\mathrm{T}}\tilde{A}+\tilde{A}^{\mathrm{T}}P+\bar{L}^{\mathrm{T}}\bar{L}-\alpha P+\varepsilon_6P^{\mathrm{T}}\bar{H}\bar{H}^{\mathrm{T}}P+\varepsilon_6^{-1}\bar{G}^{\mathrm{T}}\bar{G} & P^{\mathrm{T}}\bar{B}\\ \bar{B}^{\mathrm{T}}P & -\gamma^2 I\end{bmatrix}\\&+\left[\tilde{D}^{\mathrm{T}}\left(P^{-1}-\varepsilon_5\tilde{H}\tilde{H}^{\mathrm{T}}\right)^{-1}\tilde{D}+\varepsilon_6^{-1}\tilde{G}^{\mathrm{T}}\tilde{G}\right]\end{aligned}$$

On the other hand, let

$$\Theta_1=P_2^{\mathrm{T}}A_f,\Theta_2=P_2^{\mathrm{T}}B_f,\Theta_3=P_2^{\mathrm{T}}C_fP_2,$$

pre- and post-multiply (15) by $diag\{P^{\mathrm{T}}, I, I, I, I, I, I, I\}$, $diag\{P, I, I, I, I, I, I, I\}$ respectively.

By Surch complement, (15) implies $\Theta < 0$.

It follows from (17) that

$J(t) \le 0$ (18)

for all $t > 0$. Then (16) follows immediately from (13) and (18).

4. Numerical Example

We now give a numerical example to illustrate the proposed approach. Suppose that we have a Itô stochastic system in the form of (1)-(3) with coefficients

$$A = \begin{bmatrix} -2 & 0.9 \\ 0.8 & -1 \end{bmatrix}, A_1 = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, D = \begin{bmatrix} -1 & 0.6 \\ 0.8 & -1.8 \end{bmatrix}, D_1 = \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 0.6 \\ 0.8 & -1.8 \end{bmatrix}, C_1 = \begin{bmatrix} -0.2 \\ 0.3 \end{bmatrix}, E = \begin{bmatrix} 0.3 & 0.2 \\ -0.3 & -1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.01 \\ -0.02 \end{bmatrix}, G_1 = \begin{bmatrix} -0.03 & -0.02 \end{bmatrix}, G_2 = \begin{bmatrix} -0.02 & -0.01 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} -0.03 & 0.01 \end{bmatrix}, G_4 = \begin{bmatrix} -0.01 & -0.01 \end{bmatrix}, L = \begin{bmatrix} 0.02 & -1 \end{bmatrix}.$$

In this example by setting

$\gamma = 2$, $\alpha = 0.01$, $c_1 = 0.6$, $c_2 = 3.2$, $T = 30$, $\varepsilon_3 = 0.1$, $\varepsilon_4 = 0.1$, $R = diag\{2, 2\}$ and applying Theorem 2, we find that LMIs (15) is feasible. Thus the system is stochastic finite-time stable with respect to $(0.6, 3.2, 30, R)$. Moreover, applying Theorem 2, we can obtain the corresponding filter parameters as follows:

$$A_f = \begin{bmatrix} 0.8316 & 0.1675 \\ 0.1675 & 0.3122 \end{bmatrix}, B_f = \begin{bmatrix} 1.6683 & -0.2736 \\ -0.2736 & 0.0164 \end{bmatrix}.$$

$$C_f = \begin{bmatrix} 0.0164 & -0.0419 \end{bmatrix}.$$

5. Conclusion

In this paper, the robust finite-time H_∞ filtering problem has been studied for Itô stochastic systems. Based on LMI technique, stochastic Lyapunov function method is adopted to obtain a sufficient condition for the existence of a finite-time H_∞ filter. The resulting filter satisfies prescribed performance constraint.

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