Journal of Applied Mathematics and Physics, 2016, 4, 233-248

http://dx.doi.org/10.4236/jamp.2016.42031

Magneto Convective Flow of a Non-Newtonian Fluid through Non-Homogeneous Porous Medium past a Vertical Porous Plate with Variable Suction

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Received 29 December 2015; accepted 19 February 2016; published 22 February 2016

Abstract

Radiation absorption and chemical reaction effects on unsteady MHD free convective flow of a visco-elastic fluid past a vertical porous plate in the presence of variable suction and heat source is considered. A uniform magnetic field is assumed to be applied in the transverse direction of the flow. The set of non-linear partial differential equations is transformed into a set of ordinary differential equations by super imposing a solution with steady and unsteady part. The set of ordinary differential equations is solved by using regular perturbation scheme. The expressions for velocity, temperature and species concentration fields are obtained and the expressions for Skin friction, Nusselt number and Sherwood number are also derived. The effects of numerous physical parameters on the above flow quantities are studied with the help of graphs and tables.

Keywords

MHD, Visco-Elastic Fluid, Unsteady Flow, Chemical Reaction, Radiation Absorption, Suction

1. Introduction

MHD free convection fluid flows frequently occur in natural world. Fluid passes through porous medium are of great interest nowadays and many researchers attract towards the applications in the fields of science and technology namely in the area of agriculture engineering to know about the ground water resources, in fuel technol-

How to cite this paper: Reddy, S.H., Raju, M.C. and Reddy, E.K. (2016) Magneto Convective Flow of a Non-Newtonian Fluid through Non-Homogeneous Porous Medium past a Vertical Porous Plate with Variable Suction. *Journal of Applied Mathematics and Physics*, **4**, 233-248. http://dx.doi.org/10.4236/jamp.2016.42031

ogy to study the moment of natural gas, oil, and water through the oil reservoirs. Chaudhary *et al.* [1] considered Hall effect on MHD mixed convection flow of a visco-elastic fluid past an infinite vertical porous plate with mass transfer and radiation. Satyanarayana *et al.* [2] discussed MHD free convective heat and mass transfer past a vertical porous plate with variable temperature. Kesavaiah *et al.* [3] studied and presented effects of chemical reaction and radiation absorption on unsteady MHD convection heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in porous medium with heat source and suction. Reddy *et al.* [4] considered effects of chemical reaction and radiation absorption on unsteady magnetohydrodynamic double diffusive convective flow of viscous fluid past a semi-infinite porous plate. Raju *et al.* [5] investigated radiation and mass transfer effects on a free convection flow through porous medium bounded by a vertical surface. Muthukumaraswamy *et al.* [6] analyzed first order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Das *et al.* [7] studied the effects of mass transfer on a flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Kandaswamy *et al.* [8] focused on the problem of chemical reaction, heat and mass transfer on magnetohydrodynamic flow over a vertical stretching surface with heat source and thermal stratification effects. Effects of chemical reaction and thermophoresis on MHD mixed convective heat and mass transfer flow along an inclined plate in the presence of heat generation/absorption with viscous dissipation and joule heating was considered by Alam *et al.* [9]. Muthucumaraswamy *et al.* [10] analyzed the effects of chemical reaction on moving infinite vertical plate with uniform heat flux and variable mass diffusion. Mahapatra *et al.* [11] investigated the effect of chemical reaction on free convection flow through a porous medium bounded by a vertical surface. Mishra *et al.* [12] considered the effect of mass and heat transfer on magnetohydrodynamic flow of a visco-elastic fluid through porous medium with oscillatory suction and heat source. Beget *et al.* [13] had established computational fluid dynamics modeling of bouncy induced visco-elastic flow in a porous medium with magnetic field effect. Soundalgekar *et al.* [14] investigated effects of mass transfer and natural convection effects on MHD stokes problem for a vertical plate. Kandasamy *et al.* [15] studied the effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of source or injection. Gurmindersingh *et al.* [16] analyzed the mass transfer with chemical reaction in MHD mixed convection flow along a vertical stretching sheet. Radiation effects on MHD free convection flow over a vertical plate with heat and mass flux was considered by Sivaiah *et al.* [17]. Sahim *et al.* [18] considered Laplace technique on MHD radiating and chemically reacting fluid over an infinite vertical surface. Singh *et al.* [19] investigated heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Reddy *et al.* [20] had presented thermal radiation and chemical reaction effects on magnetohydrodynamic mixed convective boundary layer slip flow in a porous medium with heat source and Ohmic heating. Rout *et al.* [21] studied effect of radiation and chemical reaction on free convective MHD flow through a porous medium with double diffusion. Effects of chemical reaction and radiation absorption on MHD flow of dusty visco-elastic fluid were considered by Prakash *et al.* [22]. Damala *et al.* [23] discussed radiation absorption, chemical reaction and magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Raju *et al.* [24] studied unsteady MHD free convection and chemically reactive flow past an infinite vertical porous plate. Umamaheswar *et al.* [25] investigated unsteady MHD free convective visco-elastic fluid flow bounded by an infinite inclined porous plate in the presence of heat source, viscous dissipation and Ohmic heating. Reedy *et al.* [26] considered chemical reaction and radiation effects on unsteady MHD free convection flow near a moving vertical plate. Rajesh *et al.* [27] studied radiation effects on MHD flow through a porous medium with variable temperature or variable mass diffusion. Kesavaiah *et al.* [28] investigated radiation and mass transfer effects on moving vertical plate with variable temperature and viscous dissipation. Recently Ravikumar *et al.* [29] investigated combined effects of heat absorption and MHD on convective Rivlin-Ericksenflow past a semi-infinite vertical porous plate. And Venkateswarlu *et al.* [30] had presented chemical reaction and radiation absorption effects on the flow and heat transfer of a Nano fluid in a rotating system.

The objective of the present paper is to analyze radiation absorption and chemical reaction on MHD visco-elastic free convection flow through porous medium bounded by a vertical surface with constant heat and mass flux in the presence of homogeneous chemical reaction. The dimensionless equations of continuity, linear momentum, energy and diffusion which governed the flow field were solved using perturbation technique. The behavior of velocity, temperature, concentration and skin friction coefficient was discussed for various parameters involved in the governing equations the applicable criteria that follow.

2. Formulation of the Problem

The unsteady free convection, viscous incompressible electrically conducting flow of a radiation absorption, chemically reacting and visco-elastic (Walters B*) fluid past asemi-infinite vertical porous plate in a porous medium with variable suction as well as permeability in presence of a transverse magnetic field is considered. Let x*-axis be along the plate in the direction of the fluid flow and y*-axis perpendicular to it. It is assumed that, magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison with the applied transverse magnetic field. The basic flow in the medium is therefore, entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. This assumed that initially, at t$^* \le 0$,the plate as well as fluids are at the same temperature and concentration. As the concentration of the species is very low so that the Soret and Dofour effects are neglected. When t$^* > 0$, the temperature of the plate is instantaneously raised to T_w^* and the concentration of the species is set to C_w^* (see **Figure 1**).

It is considered that the permeability of the porous medium in the following form

$$K^*\left(t^*\right) = K_p^*\left(1+\varepsilon \mathrm{e}^{n^* t^*}\right) \tag{1}$$

where $K^*\left(t^*\right)$ is K^*_p is porosity, ω^* is frequency of oscillation, t^* is time, ε is a small positive constant.

The suction velocity is assumed to be time varying and it takes the following form

$$v\left(t^*\right) = -v_0\left(1+\varepsilon \mathrm{e}^{n^* t^*}\right) \tag{2}$$

Here $v_0 > 0$ and $\varepsilon \ll 1$ are positive constants. Under the above assumptions with usual Boussineq's approximation (Mishra *et al.* [31], Raju and Varma [32] [33]), the governing equations and boundary conditions are given by

$$\begin{aligned}\frac{\partial u^*}{\partial t^*} + v\frac{\partial u^*}{\partial y^*} &= \upsilon\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta\left(T^* - T_\infty\right) + g\beta^*\left(C^* - C_\infty\right) - \frac{\sigma B_0^2}{\rho}u^* \\ &\quad - \frac{\upsilon u^*}{\kappa_p^*\left(1+\varepsilon \mathrm{e}^{iw^* t^*}\right)} - \frac{k_0}{\rho}\left(\frac{\partial^3 u^*}{\partial t^* \partial {y^*}^2} + v\frac{\partial^3 u^*}{\partial {y^*}^3}\right)\end{aligned} \tag{3}$$

$$\frac{\partial T^*}{\partial t^*} + v\frac{\partial T^*}{\partial y^*} = \kappa\frac{\partial^2 T^*}{\partial y^{*2}} + s^*\left(T^* - T_\alpha\right) - R^*\left(C^* - C_\alpha\right) \tag{4}$$

$$\frac{\partial C^*}{\partial t^*} + v\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - k^*\left(C^* - C_\alpha\right) \tag{5}$$

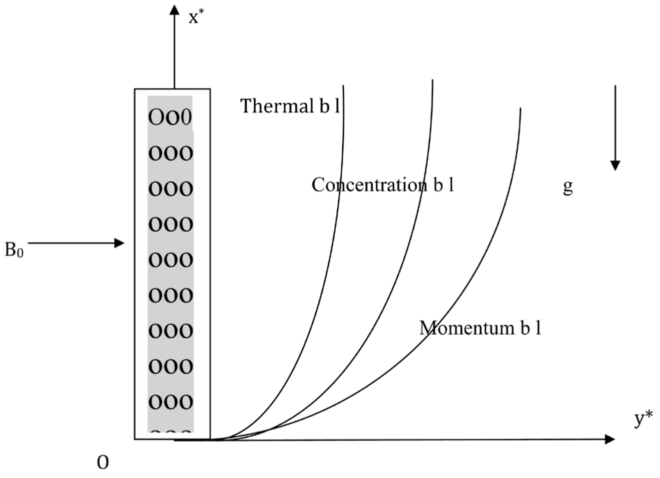

Figure 1. Physical configuration and coordinate geometry.

$$\begin{aligned} &u = 0, T^* = T_\infty + \varepsilon\left(T_\omega - T_\infty\right)\mathrm{e}^{n^* t^*}, C^* = C_\infty + \varepsilon\left(C_\omega - C_\infty\right)\mathrm{e}^{n^* t^*} \quad \text{at} \quad y = 0, \\ &u \to 0, T^* \to T_\infty, C^* \to C_\infty \quad \text{as} \quad y \to \infty \end{aligned} \tag{6}$$

Introducing the non-dimensional quantities,

$$\begin{aligned} &y = \frac{v_0 y^*}{\upsilon}, t = \frac{v_0^2 t^*}{4\upsilon}, \omega = \frac{4\upsilon\omega^*}{v_0^2}, u = \frac{u^*}{v_0}, T = \frac{T^* - T_\alpha}{T_w - T_\infty}, C = \frac{C^* - C_\infty}{C_w - C_\infty}, \\ &S = \frac{\upsilon S^*}{v_0^2}, \kappa_p = \frac{v_0^2 k_p^*}{\upsilon^2}, M^2 = \frac{\sigma B_0^2}{\rho}\frac{\upsilon}{v_0^2}, P_r = \frac{\upsilon}{k}, G_c = \frac{\upsilon g \beta^*\left(C_w - C_\infty\right)}{v_0^3}, \\ &G_r = \frac{\upsilon g \beta\left(T_w - T_\infty\right)}{v_0^3}, S_c = \frac{\upsilon}{D}, R_c = \frac{k_0 v_0^2}{\rho\upsilon^2}, \kappa_r = \frac{k^*\upsilon}{v_0^2}, R_1 = \frac{\upsilon\left(C_w - C_\infty\right)R^*}{v_0^2\left(T_w - T_\infty\right)} \end{aligned} \tag{7}$$

The Equations (3) to (5) are reduced to the following dimensionless equations

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1 + \varepsilon\mathrm{e}^{nt}\right)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r T + CG_c - M^2 u - \frac{u}{K_p\left(1 + \varepsilon\mathrm{e}^{nt}\right)} - R_c\left(\frac{1}{4}\frac{\partial^3 u}{\partial t \partial y^2} - \left(1 + \varepsilon\mathrm{e}^{nt}\right)\frac{\partial^3 u}{\partial y^3}\right) \tag{8}$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - \left(1 + \varepsilon\mathrm{e}^{nt}\right)\frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} + ST - R_1 C \tag{9}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \left(1 + \varepsilon\mathrm{e}^{nt}\right)\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - C\kappa_r \tag{10}$$

$$\begin{aligned} &u = 0, T = 1 + \varepsilon\mathrm{e}^{nt}, C = 1 + \varepsilon\mathrm{e}^{nt} \quad \text{at} \quad y = 0 \\ &u \to 0, T \to 0, C \to 0 \quad \text{as} \quad y \to \infty \end{aligned} \tag{11}$$

3. Method of Solution

In view of transient suction, temperature and concentration at the plate let us assume the velocity, temperature, concentration in the neighborhood of the plate.

$$u\left(y,t\right) = u_0\left(y\right) + \varepsilon u_1\left(y\right)\mathrm{e}^{nt} + o\left(\varepsilon^2\right) \tag{12}$$

$$T\left(y,t\right) = T_0\left(y\right) + \varepsilon T_1\left(y\right)\mathrm{e}^{nt} + o\left(\varepsilon^2\right) \tag{13}$$

$$C\left(y,t\right) = C_0\left(y\right) + \varepsilon C_1\left(y\right)\mathrm{e}^{nt} + o\left(\varepsilon^2\right) \tag{14}$$

Substituting above Equations (12)-(14) into the Equations (8)-(10) and equating the ε^0 coefficient and coefficient of ε^1, we get the following equations.

$$R_c u_0^{111} + u_0^{11} + u_0^1 - \left(M^2 + \frac{1}{\kappa_p}\right)u_0 = -CG_c - G_r T_0 \tag{15}$$

$$R_c u_1^{111} + \left(1 - \frac{nR_c}{4}\right)u_1^{11} + u_1^1 - \left(M^2 + \frac{1}{\kappa_p} + \frac{n}{4}\right)u_1 = -u_0^1 - G_r T_1 - C_1 G_c - \frac{u_0}{\kappa_p} - R_c u_0^{111} \tag{16}$$

$$T_0^{11} + P_r T_0^1 + SP_r T_0 = R_1 P_r C_0 \tag{17}$$

$$T_1^{11} + P_r T_1^1 + P_r\left(S - \frac{n}{4}\right)T_1 = P_r\left(R_1 C_1 - T_0^1\right) \tag{18}$$

$$C_0^{11} + S_c C_0^1 - S_c \kappa_r C_0 = 0 \tag{19}$$

$$C_1^{11} + S_c C_1^1 - S_c\left(\kappa_r + \frac{n}{4}\right)C_1 = -S_c C_0^1 \tag{20}$$

Now the boundary conditions are reduced to the following forms

$$\begin{aligned} &u_0 = 0, u_1 = 0, T_0 = 1, T_1 = 1, C_0 = 1, C_1 = 1 \quad \text{as} \quad y = 0 \\ &u_0 \to 0, u \to 0, T_0 = T_1 \to 0, C_0 = C_1 \to 0 \quad \text{as} \quad y \to \infty \end{aligned} \tag{21}$$

The Equations (15) and (16) are not solvable by using the given boundary conditions (21). Hence the perturbation method has been applied using R_c ($R_c < 1$), the elastic parameter as the perturbation parameter.

$$\begin{aligned} u_0 &= u_{00}(y) + R_c u_{01}(y) + o\left(R_c^2\right) \\ u_1 &= u_{10}(y) + R_c u_{11}(y) + o\left(R_c^2\right) \end{aligned} \tag{22}$$

Substituting Equation (22) into Equations (15) and (16), equating the coefficients of R_c^0 and R_C^1 to zero, we get the following set of equations.

Zeroth order equations

$$u_{00}^{11} + u_{00}^{1} - \left(M^2 + \frac{1}{\kappa_p}\right) u_{00} = -G_c C_0 - G_r T_0 \tag{23}$$

$$u_{01}^{11} + u_{01}^{1} - \left(M^2 + \frac{1}{\kappa_p}\right) u_{01} = -u_{00}^{111} \tag{24}$$

First order equations

$$u_{10}^{11} + u_{10}^{1} - \left(M^2 + \frac{1}{\kappa_p} + \frac{n}{4}\right) u_{10} = -u_{00}^{1} - G_r T_1 - G_c C_1 - \frac{u_{00}}{\kappa_p} \tag{25}$$

$$u_{11}^{11} + u_{11}^{1} - \left(M^2 + \frac{1}{\kappa_p} + \frac{n}{4}\right) u_{11} = -u_{01}^{1} - \frac{u_{01}}{\kappa_p} - u_{00}^{111} + \frac{n}{4} u_{10}^{11} - u_{10}^{111} \tag{26}$$

Using the perturbation the boundary conditions are reduced as follows:

$$\begin{aligned} &u_{00} = 0, u_{01} = 0, u_{10} = 0, u_{11} = 0 \quad \text{as} \quad y = 0 \\ &u_{00} = 0, u_{01} = 0, u_{10} = 0, u_{11} = 0 \quad \text{as} \quad y = \infty \end{aligned} \tag{27}$$

Solving these differential equations by using the boundary conditions we get the following results (**Appendix**)

$$\begin{aligned} u = {} & A_{44} \mathrm{e}^{-m_{11} y} + A_{45} \mathrm{e}^{-m_9 y} + A_{46} \mathrm{e}^{-m_5 y} + A_{47} \mathrm{e}^{-m_1 y} \\ & + \varepsilon \left(A_{48} \mathrm{e}^{-m_{15} y} + A_{49} \mathrm{e}^{-m_{13} y} + A_{50} \mathrm{e}^{-m_{11} y} + A_{51} \mathrm{e}^{-m_9 y} + A_{52} \mathrm{e}^{-m_7 y} + A_{53} \mathrm{e}^{-m_5 y} + A_{54} \mathrm{e}^{-m_3 y} + A_{55} \mathrm{e}^{-m_1 y} \right) \mathrm{e}^{nt} \end{aligned}$$

$$T = (1 - A_2) \mathrm{e}^{-m_5 y} + A_2 \mathrm{e}^{-m_1 y} + \varepsilon \left\{ \left[1 - \left(A_6 + A_7 + A_8 \right) \right) \mathrm{e}^{-m_7 y} + A_6 \mathrm{e}^{-m_3 y} + A_7 \mathrm{e}^{-m_1 y} + A_8 \mathrm{e}^{-m_5 y} \right\} \mathrm{e}^{nt}$$

$$C = \mathrm{e}^{-m_1 y} + \varepsilon \left\{ (1 - A_1) \mathrm{e}^{-m_3 y} + A_1 \mathrm{e}^{-m_1 y} \right\} \mathrm{e}^{nt}$$

The skin friction, Nusselt number and Sherwood number at the plate are defined as follows:

$$\tau = -\frac{\partial u}{\partial y} \quad \text{at} \quad y = 0$$

$$Nu = \frac{\partial T}{\partial y} \quad \text{at} \quad y = 0$$

$$Sh = \frac{\partial C}{\partial y} \quad \text{at} \quad y = 0$$

4. Results and Discussion

The present study considers the effects of radiation absorption and chemical reaction effect on transient free convection flow of a non-Newtonian fluid through non-homogeneous porous medium past a vertical porous plate with magnetic field and variable suction. Solutions for velocity, temperature and concentration field are obtained by using perturbation technique. The effects of various parameters like Grashof number for heat and mass transfer *Gr* and *Gc*, chemical reaction *Kr*, Radiation absorption R_1,Prandtl number *Pr*, on velocity, temperature and concentration have been studied analytically and effects are executed with the help of **Figures 2-15**. Also the behavior of skin friction, rate of heat transfer and rate of mass transfer with respect to various parameters have been studied and results were presented in **Tables 1-10**.

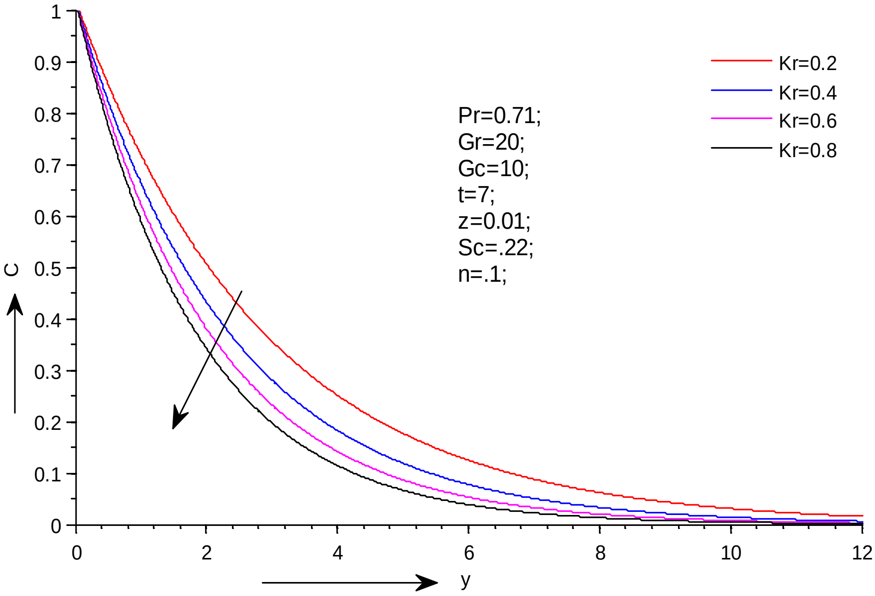

Figure 2. Effect of *Kr* on concentration.

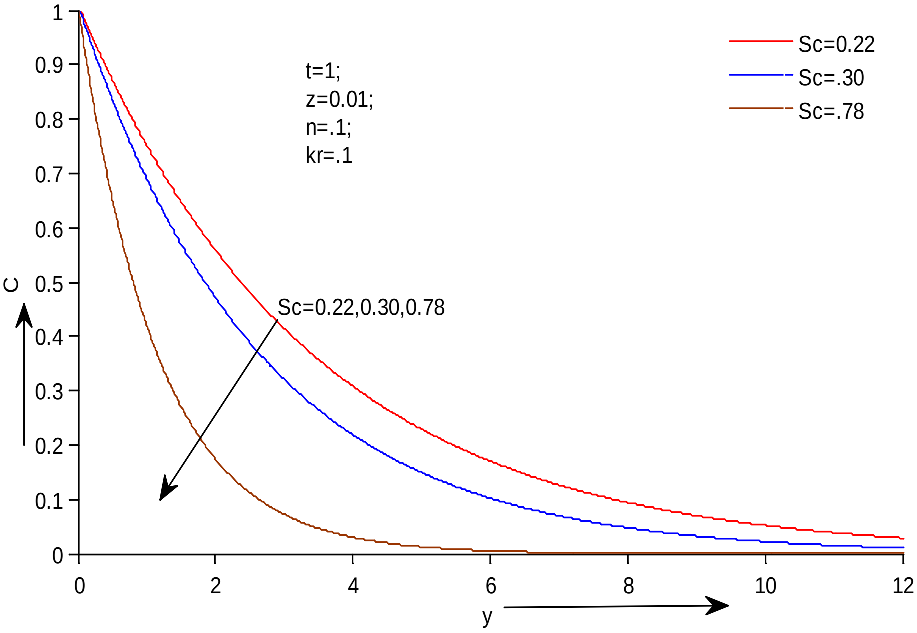

Figure 3. Effects of *Sc* on concentration.

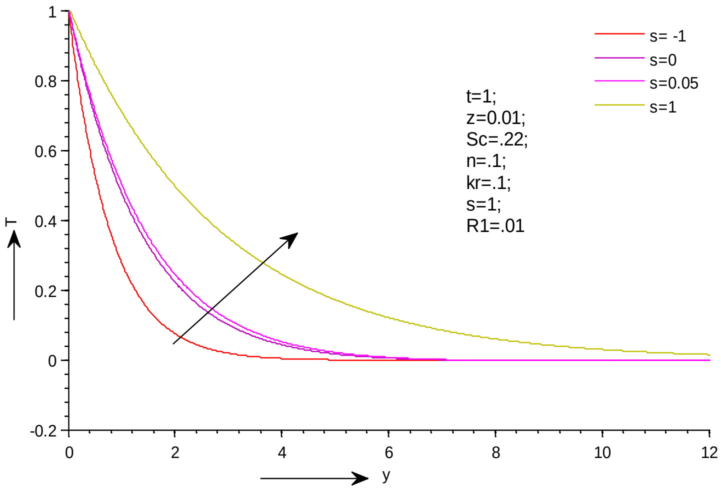

Figure 4. Effects of *S* on temperature.

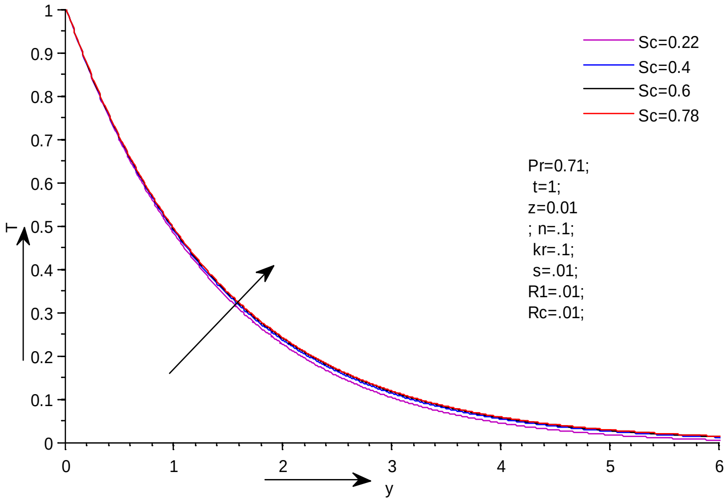

Figure 5. Effects of *Sc* on temperature.

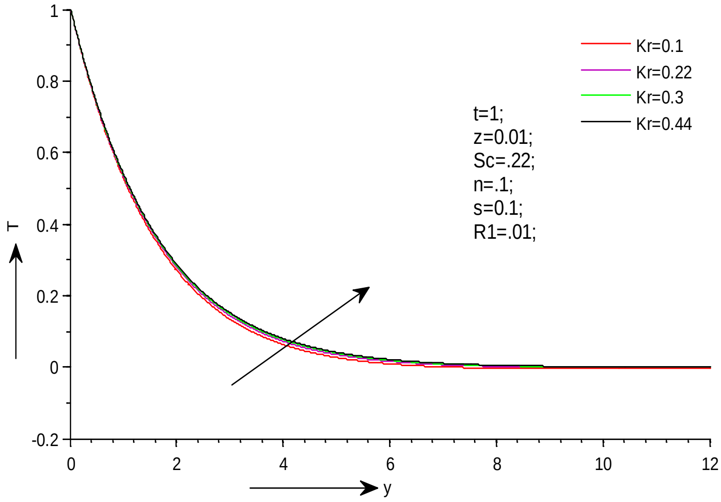

Figure 6. Effects of *Kr* on temperature.

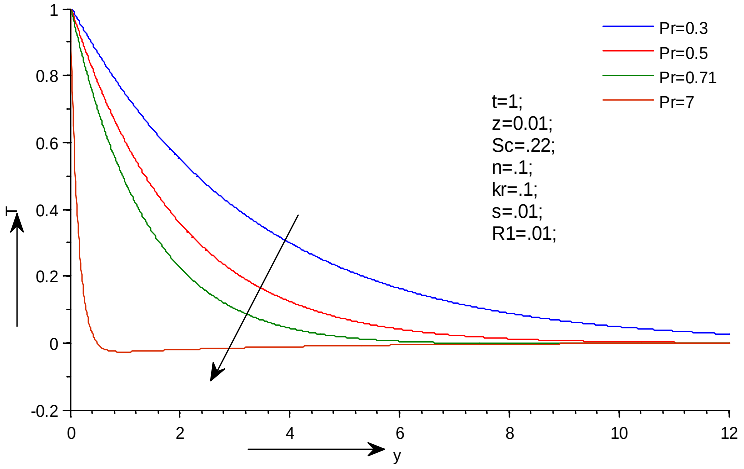

Figure 7. Effects of *Pr* on temperature.

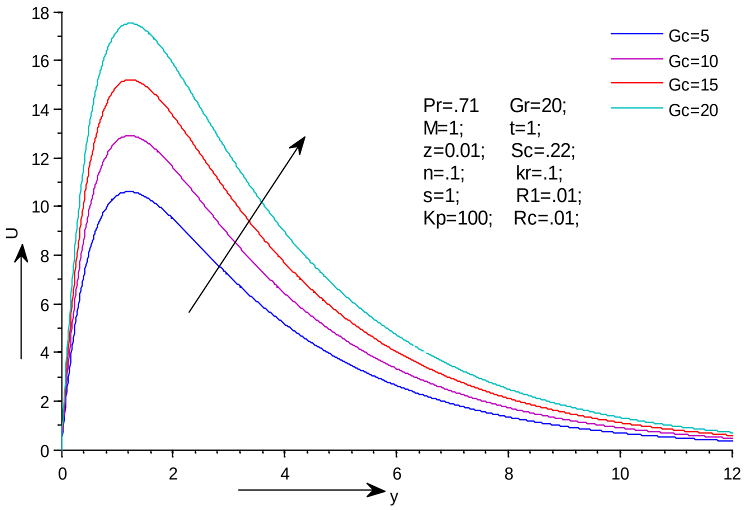

Figure 8. Effects of *Gc* on velocity.

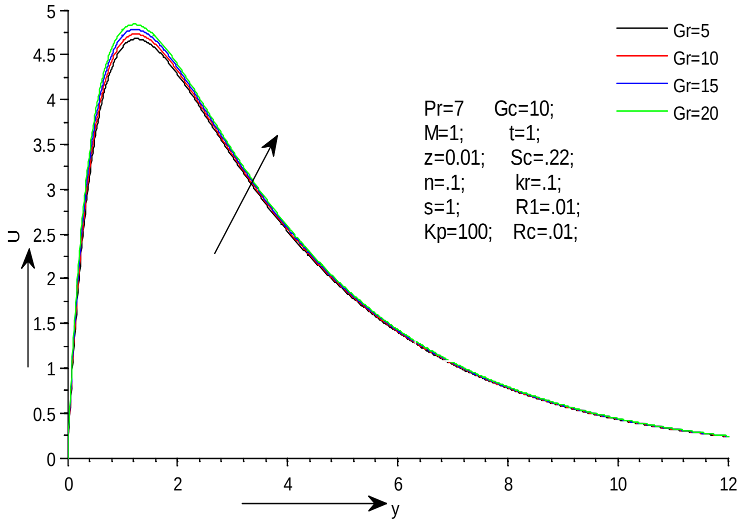

Figure 9. Effects of *Gr* on velocity.

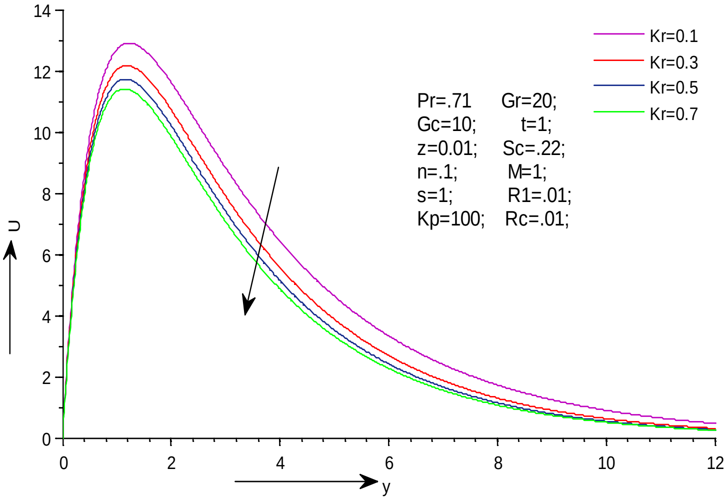

Figure 10. Effects of *Kr* on velocity.

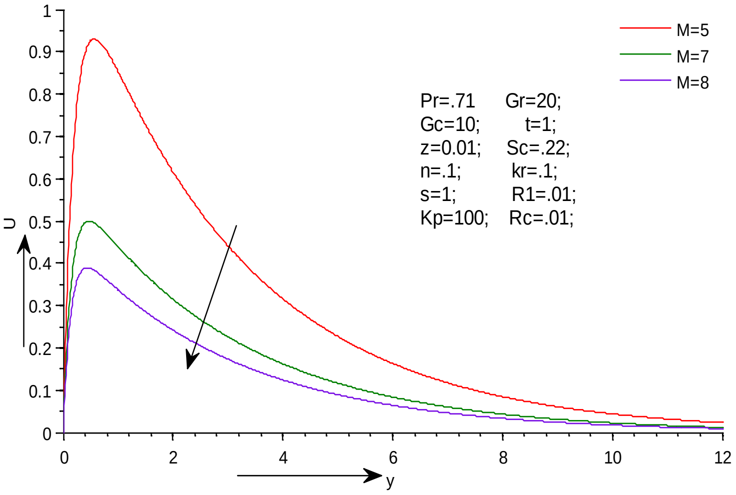

Figure 11. Effects of *M* on velocity.

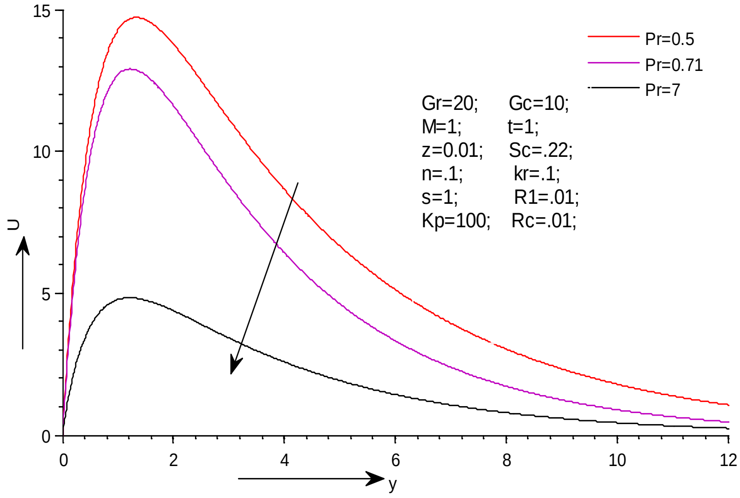

Figure 12. Effects of *Pr* on velocity.

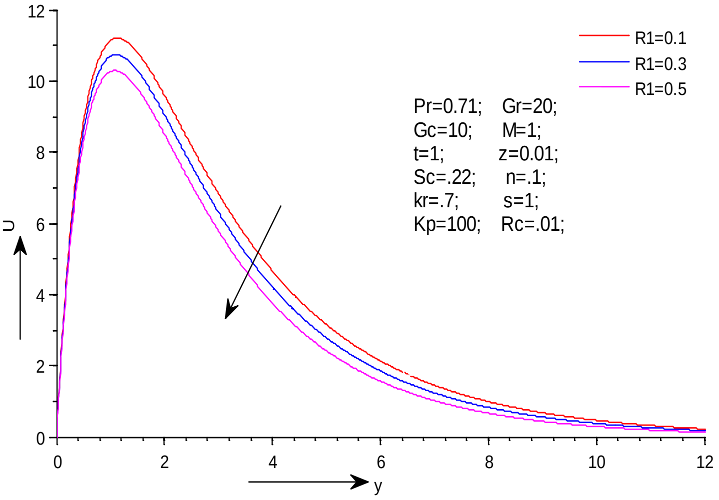

Figure 13. Effects of R_1 on velocity.

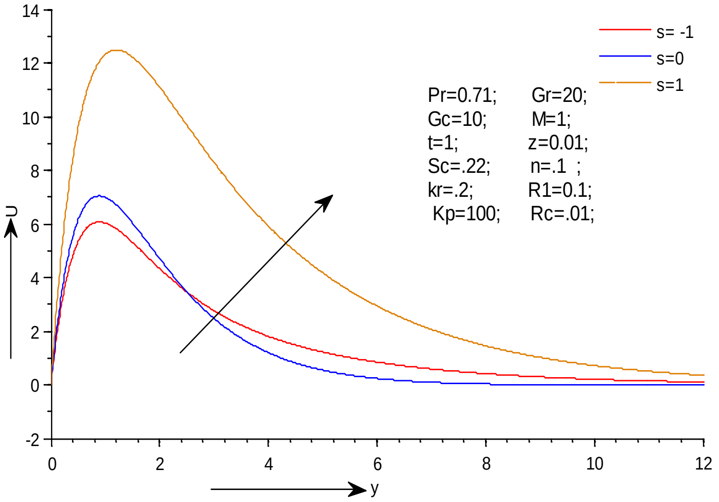

Figure 14. Effects of *S* on velocity.

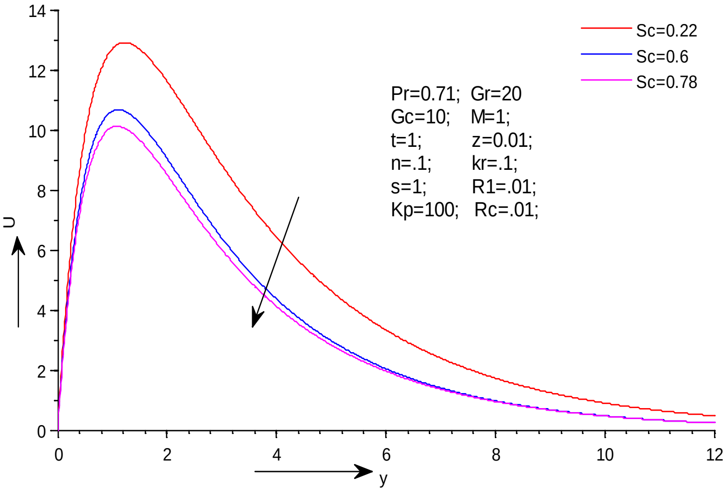

Figure 15. Effects of *Sc* on velocity.

Table 1. Effect of *Sc* on skin friction, Sherwood number, Nusselt number, z = 0.01, t = 1, n = 0.1.

Pr	*Gr*	*Gc*	*M*	*Sc*	*Kr*	*R1*	*Kp*	*Rc*	*S*	τ	Sh	Nu
0.71	20	10	1	**0.22**	0.1	0.1	100	0.1	1	1.7939	0.5513	0.3811
0.71	20	10	1	**0.66**	0.1	0.1	100	0.1	1	3.9012	1.0714	0.4061
0.71	20	10	1	**0.78**	0.1	0.1	100	0.1	1	5.5524	1.2913	0.4054

Table 2. Effect of *Pr* on skin friction, Sherwood number, Nusselt number, z = 0.01, t = 1, n = 0.1.

Pr	*Gr*	*Gc*	*M*	*Sc*	*Kr*	*R1*	*Kp*	*Rc*	*S*	τ	Sh	Nu
0.025	20	10	1	**0.22**	0.1	0.1	100	0.1	1	1.1192	0.2999	0.0194
0.71	20	10	1	**0.22**	0.1	0.1	100	0.1	1	9.8535	0.2999	0.3518
7	20	10	1	**0.22**	0.1	0.1	100	0.1	1	13.9820	0.2999	5.1590

Table 3. Effect of *Gr* on skin friction, Sherwood number, Nusselt number, z = 0.01, t = 1, n = 0.1.

Pr	*Gr*	*Gc*	*M*	*Sc*	*Kr*	*R1*	*Kp*	*Rc*	*S*	τ	Sh	Nu
0.71	**10**	10	1	0.22	0.1	0.1	100	0.1	1	6.5856	0.2999	0.3518
0.71	**15**	10	1	0.22	0.1	0.1	100	0.1	1	8.2196	0.2999	0.3518
0.71	**20**	10	1	0.22	0.1	0.1	100	0.1	1	9.8535	0.2999	0.3518

Table 4. Effect of *Gc* on skin friction, Sherwood number, Nusselt number, z = 0.01, t = 1, n = 0.1.

Pr	*Gr*	*Gc*	*M*	*Sc*	*Kr*	*R1*	*Kp*	*Rc*	*S*	τ	Sh	Nu
0.71	10	**10**	1	0.60	0.1	0.1	100	0.1	1	6.5856	0.2999	0.3518
0.71	10	**15**	1	0.60	0.1	0.1	100	0.1	1	8.2444	0.2999	0.3518
0.71	10	**20**	1	0.60	0.1	0.1	100	0.1	1	9.9033	0.2999	0.3518

Table 5. Effect of *M* on skin friction, Sherwood number, Nusselt number, z = 0.01, t = 1, n = 0.1.

Pr	*Gr*	*Gc*	*M*	*Sc*	*Kr*	*R1*	*Kp*	*Rc*	*S*	τ	Sh	Nu
0.71	10	10	**1**	0.22	0.1	0.1	100	0.1	1	6.5856	0.2999	0.3518
0.71	10	10	**2**	0.22	0.1	0.1	100	0.1	1	2.9872	0.2999	0.3518
0.71	10	10	**5**	0.22	0.1	0.1	100	0.1	1	2.7055	0.2999	0.3518

Table 6. Effect of *Kr* on skin friction, Sherwood number, Nusselt number, z = 0.01, t = 1, n = 0.1.

Pr	*Gr*	*Gc*	*M*	*Sc*	*Kr*	*R1*	*Kp*	*Rc*	*S*	τ	Sh	Nu
0.71	10	10	1	0.22	0.1	0.1	100	0.1	1	6.5856	0.2999	0.3518
0.71	10	10	1	0.22	0.3	0.1	100	0.1	1	8.5864	0.3955	0.3634
0.71	10	10	1	0.22	0.5	0.1	100	0.1	1	10.0082	0.4662	0.3718

Table 7. Effect of *Kp* on skin friction, Sherwood number, Nusselt number, z = 0.01, t = 1, n = 0.1.

Pr	*Gr*	*Gc*	*M*	*Sc*	*Kr*	*R1*	*Kp*	*Rc*	*S*	τ	Sh	Nu
0.71	10	10	1	0.22	0.1	0.1	90	0.1	1	**6.5832**	0.2999	0.3518
0.71	10	10	1	0.22	0.1	0.1	95	0.1	1	**6.5844**	0.2999	0.3518
0.71	10	10	1	0.22	0.1	0.1	100	0.1	1	**6.5856**	0.2999	0.3518

Table 8. Effect of *S* on skin friction, Sherwood number, Nusselt number, z = 0.01, t = 1, n = 0.1.

Pr	***Gr***	***Gc***	***M***	***Sc***	***Kr***	***R1***	***Kp***	***Rc***	***S***	**τ**	**Sh**	**Nu**
0.71	10	10	1	0.22	0.1	0.1	100	0.1	**1**	6.5856	0.2999	0.3518
0.71	10	10	1	0.22	0.1	0.1	100	0.1	**3**	6.5808	0.2999	0.3568
0.71	10	10	1	0.22	0.1	0.1	100	0.1	**5**	6.5799	0.2999	0.3577

Table 9. Effect of *Rc* on skin friction, Sherwood number, Nusselt number, z = 0.01, t = 1, n = 0.1.

Pr	***Gr***	***Gc***	***M***	***Sc***	***Kr***	***R1***	***Kp***	***Rc***	***S***	**τ**	**Sh**	**Nu**
0.71	10	10	1	0.22	0.1	0.1	100	**0**	1	17.7144	0.2999	0.3518
0.71	10	10	1	0.22	0.1	0.1	100	**0.05**	1	3.2928	0.2999	0.3518
0.71	10	10	1	0.22	0.1	0.1	100	**0.2**	1	1.3171	0.2999	0.3518

Table 10. Effect of R_1 on skin friction, Sherwood number, Nusselt number, z = 0.01, t = 1, n = 0.1.

Pr	***Gr***	***Gc***	***M***	***Sc***	***Kr***	***R1***	***Kp***	***Rc***	***S***	**τ**	**Sh**	**Nu**
0.71	10	10	1	0.22	0.1	0.1	100	**0.1**	1	6.5856	0.2999	0.3518
0.71	10	10	1	0.22	0.1	0.3	100	**0.1**	1	6.5993	0.2999	0.3375
0.71	10	10	1	0.22	0.1	0.5	100	**0.1**	1	6.6130	0.2999	0.3232

Figure 2 exhibits the effect of chemical reaction (*Kr*) on concentration, it is noticed that the concentration of the fluid decreases as chemical reaction parameter increases. **Figure 3** shows the effect of Schmidt number Sc on concentration when the values of *Sc* increases, the concentration value decreases. From **Figure 4** we have noticed that temperature of the fluid increases as source or sink increases. **Figure 5** shows the temperature profile for the different values of *Sc*, from this figure we have noticed that increase in the value of Sc results in increase in the temperature profile. **Figure 6** depicts the effect of *Kr* on temperature, when the values of *Kr* increases the temperature value increases. **Figure 7** shows the effect of Prandtl number on temperature, when *Pr* value increases, the temperature decreases, similar type of results are noticed with Satyanarayana *et al.* [2]. **Figure 8** depicts the effect of *Gc* on velocity, the velocity of the fluid increases when *Gc* increases. **Figure 9** exhibits that the effect of *Gr* on velocity, from this figure we observed that, the velocity of the fluid increases when *Gr* increases. **Figure 10** shows the effect of *Kr* on velocity, the velocity of the fluid decreases in the increase of *Kr*. **Figure 11** illustrates velocity profiles for different values of *M*, from this figure we have observed that velocity of the fluid decreases when an increase in the values of *M*. **Figure 12** shows the velocity profile for different values of *Pr*. It is observed that increase in the value of *Pr* results in decrease in the velocity profile. **Figure 13** shows the velocity profile for different values of radiation absorption, from this figure it is noticed that an increase in the value of R_1 results a decrease in the velocity profile. **Figure 14** illustrates the effect of source/sink on velocity, from this figure it is noticed that velocity of the fluid increases for decreasing values of source/sink. **Figure 15** shows the effect of *Sc* on velocity, from this figure it is noticed that when *Sc* values increases the velocity of the fluid decreases. On the other hand, **Tables 1-4** show, the effect of *Sc*, *Pr*, *Gr* and *Gc* on the parameters skin friction, Sherwood number, Nusselt number. It can be observed that skin friction coefficient increased with the increase in *Sc*, *Pr*, *Gr*, *Gc*. It can be clearly observed that rate of heat transfer of the fluid increases for increase in *Sc* and it is not shown any effect in case of *Pr*, *Gr*, *Gc*. The Nusselt number increased as increase in the *Sc* and *Pr*, but it is constant in the case of *Gr* and *Gc*. Further, **Tables 5-10** show the effect of *M*, *Kr*, *Kp*, *S*, *Rc* and *R* on the parameters skin friction, Sherwood number, Nusselt number. It can be observed that skin friction coefficient increased with an increase in *Kr*, *Kp*, *Rc*, and R_1 whereas decreased in the increase of *M* and *S*. The rate of the heat transfer of the fluid increases with an increase in *Kr*, but it not shown any effect in case of *M*, *Kp*, *S*, *Rc* and R_1. The Nusselt number increased with an increase in *Kr*, *S* and it is decreased with an increase in R_1.

5. Conclusions

The present study is carried out to investigate the magneto convective flow of a non-Newtonian fluid through non-homogeneous porous medium past a vertical porous plate with variable suction. The dimensionless governing equations are solved by using the perturbation technique. The results for velocity and temperature are obtained and plotted graphically. The numerical results for skin friction and Nusselt number are computed in tables. The main conclusions of this study are as follows:

1. Velocity of the fluid increases with an increasing values of S, Gc, Gr. And it decreases in the case of Kr, Sc, Pr, M and R_1.

2. Temperature of the fluid increases with an increasing values of Kr, Sc and S, whereas decreased in the case of Pr.

3. Kr and Sc show negative impact on the concentration of the fluid.

4. Coeffecient of skin friction receives positive impact in case of Sc, Pr, Gr, Gc, Kr, Kp, Rc, while negative effect in the case of M and S. Sherwood number increases for increasing values of Sc and Kr. Coefficient of rate of heat transfer increases with an increase in Sc, Pr, Kr and S.

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Appendix

$$m_1 = \frac{S_c + \sqrt{S_c^2 + 4S_c\kappa_r}}{2} \qquad m_3 = \frac{S_c + \sqrt{S_c^2 + 4S_c\left(\kappa_r + \frac{n}{4}\right)}}{2}$$

$$A_1 = \frac{S_c m_1}{m_1^{\,2} - m_1 s_c - S_c\left(\kappa_r + \frac{n}{4}\right)} \qquad A_2 = \frac{R_1 P_r}{m_1^{\,2} - m_1 P_r + SP_r},\quad \begin{aligned} A_3 &= R_1 P_r\left(1 - A_1\right)\\ A_4 &= \left(R_1 A_1 + m_1 A_2\right)P_r\\ A_5 &= m_5 P_r\left(1 - A_2\right)\end{aligned}$$

$$m_5 = \frac{P_r + \sqrt{P_r^2 - 4SP_r}}{2} \qquad m_7 = \frac{P_r + \sqrt{P_r^2 - 4P_r\left(S - \frac{n}{4}\right)}}{2} \qquad m_9 = \frac{1 + \sqrt{1 + 4\left(M^2 + \frac{1}{\kappa_p}\right)}}{2}$$

$$A_6 = \frac{A_3}{m_3^{\,2} - m_3 P_r + P_r\left(S - \frac{n}{4}\right)} \qquad A_7 = \frac{A_4}{m_1^{\,2} - m_1 P_r + P_r\left(S - \frac{n}{4}\right)} \qquad A_8 = \frac{A_5}{m_5^2 - m_5 P_r + P_r\left(S - \frac{n}{4}\right)}$$

$$\begin{aligned} A_9 &= -\left(G_c + A_2 G_r\right),\\ A_{10} &= -G_r\left(1 - A_2\right),\end{aligned} \qquad A_{11} = \frac{A_9}{m_1^{\,2} - m_1 - \left(M^2 + \frac{1}{\kappa_p}\right)} \qquad A_{12} = \frac{A_{10}}{m_5^2 - m_5 - \left(M^2 + \frac{1}{\kappa_p}\right)}$$

$$\begin{aligned} A_{13} &= -\left(A_{11} + A_{12}\right),\\ m_{11} &= m_9,\end{aligned} \qquad A_{14} = \frac{A_{13} m_9^3}{m_9^2 - m_9 - \left(M^2 + \frac{1}{\kappa_p}\right)} \qquad A_{15} = \frac{A_{11} m_1^3}{m_1^2 - m_1 - \left(M^2 + \frac{1}{\kappa_p}\right)}$$

$$A_{16} = \frac{A_{12} m_5^2}{m_5^2 - m_5 - \left(M^2 + \frac{1}{\kappa_p}\right)} \qquad \begin{aligned} A_{17} &= -\left(A_{14} + A_{15} + A_{16}\right),\\ A_{18} &= m_9 A_{13} - \frac{A_{13}}{\kappa_p},\end{aligned} \qquad A_{19} = -Gr\left(1 - A_6 - A_7 - A_8\right),$$

$$A_{20} = m_5 A_{12} - A_8 G_r - \frac{A_{12}}{\kappa_p} \qquad A_{21} = -\left(A_6 G_r + G_c\left(1 - A_1\right)\right), \qquad A_{22} = m_1 A_{11} - A_7 G_r - A_1 G_c - \frac{A_{11}}{\kappa_p},$$

$$A_{23} = \frac{A_{18}}{m_9^2 - m_9 - \left(M^2 + \frac{1}{\kappa_p} + \frac{n}{4}\right)}, \qquad A_{24} = \frac{A_{19}}{m_7^2 - m_7 - \left(M^2 + \frac{1}{\kappa_p} + \frac{n}{4}\right)},$$

$$A_{25} = \frac{A_{20}}{m_5^2 - m_5 - \left(M^2 + \frac{1}{\kappa_p} + \frac{n}{4}\right)}, \qquad m_{13} = \frac{1 + \sqrt{1 + 4\left(M^2 + \frac{1}{\kappa_p} + \frac{n}{4}\right)}}{2},$$

$$A_{26} = \frac{A_{21}}{m_3^2 - m_3 - \left(M^2 + \frac{1}{\kappa_p} + \frac{n}{4}\right)}, \qquad A_{27} = \frac{A_{22}}{m_1^2 - m_1 - \left(M^2 + \frac{1}{\kappa_p} + \frac{n}{4}\right)},$$

$$A_{28} = -\left(A_{23} + A_{24} + A_{25} + A_{26} + A_{27}\right), \qquad A_{29} = \frac{n}{4} m_{13}^2 A_{28} + m_{13}^3 A_{28}$$

$$A_{30}=m_{11}A_{17}-\frac{A_{17}}{K_p}\quad A_{31}=m_9A_{14}-\frac{A_{14}}{\kappa_p}+A_{13}m_9^3+\frac{n}{4}m_9^2A_{23}+m_9^3A_{23},$$

$$A_{32}=\frac{n}{4}m_7^2A_{24}+m_7^3A_{27}\quad A_{33}=m_5A_{16}-\frac{A_{16}}{\kappa_p}+A_{12}m_5^3+\frac{n}{4}m_5^2A_{25}+m_5^3A_{25}$$

$$A_{34}=\frac{n}{4}m_3^2A_{26}+m_3^3A_{26},\quad m_{15}=m_{13,}\qquad A_{35}=\frac{n}{4}m_1^2A_{27}+m_1^3A_{27}+m_1A_{15}+m_1^3A_{11}-\frac{A_{15}}{K_p},$$

$$A_{36}=\frac{A_{29}}{m_{13}^2-m_{13}-\left(M^2+\frac{1}{\kappa_p}+\frac{n}{4}\right)},\qquad A_{37}=\frac{A_{30}}{m_{11}^2-m_{11}-\left(M^2+\frac{1}{K_p}+\frac{n}{4}\right)},$$

$$A_{38}=\frac{A_{31}}{m_9^2-m_9-\left(M^2+\frac{1}{\kappa_p}+\frac{n}{4}\right)},\qquad A_{39}=\frac{A_{32}}{m_7^2-m_7-\left(M^2+\frac{1}{\kappa_p}+\frac{n}{4}\right)},$$

$$A_{40}=\frac{A_{33}}{m_5^2-m_5-\left(M^2+\frac{1}{\kappa_p}+\frac{n}{4}\right)},\qquad A_{41}=\frac{A_{34}}{m_3^2-m_3-\left(M^2+\frac{1}{\kappa_p}+\frac{n}{4}\right)},$$

$$A_{42}=\frac{A_{35}}{m_1^2-m_1-\left(M^2+\frac{1}{\kappa_p}+\frac{n}{4}\right)},\qquad A_{43}=-\left(A_{36}+A_{37}+A_{38}+A_{39}+A_{40}+A_{41}+A_{42}\right),$$

$$A_{44}=R_cA_{17},\qquad A_{45}=A_{13}+R_cA_{14},$$

$$A_{46}=A_{12}+R_cA_{16},\quad A_{47}=A_{11}+R_cA_{15},\qquad A_{49}=A_{28}+R_cA_{36},\quad A_{50}=R_cA_{37},$$

$$A_{48}=R_cA_{43},\quad A_{51}=A_{23}+R_cA_{38},\qquad A_{52}=A_{24}+R_cA_{39},$$

$$A_{53}=A_{25}+R_cA_{40},\qquad A_{54}=A_{26}+R_cA_{41},\quad A_{55}=A_{27}+R_cA_{42},$$

Nomenclature

C0: Species concentration
C: Non-dimensional species concentration
D: Molecular diffusivity
Gc: Grashof number for mass transfer
Gr: Grashof number for heat transfer
g: Acceleration due to gravity
K0: Permeability of the medium
Kp: Permeability/porosity parameter
k: Thermal diffusivity
M: Magnetic parameter
N: Nusselt number
Pr: Prandtl number ε: a small positive constant
S: Heat source parameter
Sc: Schmidt number
Sh: Sherwood number

R_1: Radiation absorption.
Kr: Chemical reaction
M: Magnetic parameter
B_0: Magnetic field of uniform strength
σ: Electrical conductivity
ρ: Density of the fluid
t: Time
β: Volumetric coefficient of expansion for heat transfer
β^*: Volumetric coefficient of expansion with species concentration
R_c: Elastic parameter.
υ: Kinematic coefficient of viscosity.
v_0: Constant suction velocity.