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The New Infinite Sequence Complexion Solutions of a Kind of Nonlinear Evolutionary Equations

Lina Yi, Jundong Bao, Taogetusang*

College of Mathematical Science, Inner Mongolia Normal University, Inner Mongolia, China
Email: 573028703@qq.com, baojd@imnu.edu.cn, *tgts@imnu.edu.cn

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Abstract

The method combining the function transformation with the auxiliary equation is presented and the new infinite sequence complexion solutions of a class of nonlinear evolutionary equations are constructed. Step one, according to two function transformations, a class of nonlinear evolutionary equations is changed into two kinds of ordinary differential equations. Step two, using the first integral of the ordinary differential equations, two first order nonlinear ordinary differential equations are obtained. Step three, using function transformation, two first order nonlinear ordinary differential equations are changed to the ordinary differential equation that could be integrated. Step four, the new solutions, Bäcklund transformation and the nonlinear superposition formula of solutions of the ordinary differential equation that could be integrated are applied to construct the new infinite sequence complexion solutions of a class of nonlinear evolutionary equations. These solutions are consisting of two-soliton solutions, two-period solutions and solutions composed of soliton solutions and period solutions.

Keywords

Nonlinear Evolutionary Equation, Function Transformation, Nonlinear Superposition Formula of Solutions, Complexion New Solutions

1. Introduction

In Refs. [1]-[5], some methods were applied to research the following two nonlinear coupling systems, and a finite number of one soliton solutions were obtained.

*Corresponding author.

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$$\sigma_{xx} = -\sigma + \sigma^3 + l\sigma\rho^2, \tag{1}$$

$$\rho_{xx} = (h-l)\rho + m\rho^3 + l\rho\sigma^2 \tag{2}$$

where h, l and m are constants; ρ and σ are scalars.

$$\sigma_{tt} - c^2\sigma_{xx} = a\sigma - b\sigma^3 - b\sigma\rho^2, \tag{3}$$

$$\rho_{tt} - c^2\rho_{xx} = (a-4e)\rho - b\rho^3 - b\rho\sigma^2 \tag{4}$$

where a, b, c^2 and e are constants; ρ and σ are scalars.

In Refs. [6]-[9], the new two-soliton solutions of the following a class of nonlinear evolutionary equations are constructed.

$$u_{xt} = u + \frac{1}{6}\left(u^3 + 3uv^2\right)_{xx}, \quad v_{xt} = v + \frac{1}{6}\left(v^3 + 3u^2v\right)_{xx}. \tag{5}$$

$$u_{xt} = u + \frac{1}{6}\left(u^3 - 3uv^2\right)_{xx}, \quad v_{xt} = v - \frac{1}{6}\left(v^3 - 3u^2v\right)_{xx}. \tag{6}$$

$$u_{xt} = u + \frac{1}{6}\left(u^3\right)_{xx}, \quad v_{xt} = v + \frac{1}{2}\left(u^2v\right)_{xx}. \tag{7}$$

$$u_{xt} = u + \frac{1}{6}\left(u^3 + uv^2\right)_{xx}, \quad v_{xt} = v + \frac{1}{6}\left(v^3 + u^2v\right)_{xx}. \tag{8}$$

$$u_{xt} = u + \frac{1}{6}\left(u^3\right)_{xx}, \quad v_{xt} = v + \frac{1}{6}\left(u^2v\right)_{xx}. \tag{9}$$

In this paper, the method combining the function transformation with the auxiliary equation is applied to construct the new infinite sequence complexion solutions of a class of nonlinear evolutionary Equation (10) by the following steps.

Step one, in Part 2.1, according to function transformation (11), a class of nonlinear evolutionary Equation (10) is changed into two kinds of ordinary differential Equations (13), (14). And then with the help of the function transformation, ordinary differential Equations (13), (14) can be changed to ordinary differential Equations (19), (20). And the first integrals (21), (22) of the two ordinary differential equations are obtained. In Part 2.2, according to function transformation (12), a class of nonlinear evolutionary Equations (10) is changed into two kinds of ordinary differential Equations (25), (26). And then with the help of the function transformation, ordinary differential Equations (25), (26) can be changed to ordinary differential Equations (31), (32). And the first integrals (33), (34) of the two ordinary differential equations are obtained.

Step two, substituting the first integrals (21), (22) separately into the first equation of ordinary differential Equations (19), (20) yields two first order nonlinear ordinary differential Equations (23) and (24). Substituting the first integrals (33), (34) separately into the first equation of ordinary differential Equations (31), (32) yields two first order nonlinear ordinary differential Equations (35) and (36).

Step three, using function transformation, two first order nonlinear ordinary differential Equations (23) and (24) (or (23) and (24)) can be changed to the ordinary differential equation that could be integrated [10]-[13].

Step four, the new solutions, Bäcklund transformation and the nonlinear superposition formula of solutions of the ordinary differential equation that could be integrated [10]-[12] are applied to construct the new infinite sequence complexion solutions consisting of the Riemann θ function, the Jacobi elliptic function, the hyperbolic function, the trigonometric function and the rational function of a class of nonlinear evolutionary equations (10). These solutions are consisting of two-soliton solutions, two-period solutions and solutions composed of soliton solutions and period solutions.

$$\begin{cases} \delta_1 u_{xx} + \alpha_1 u_{tt} + \alpha_2 u_{xt} = \alpha_3 u + \alpha_4 u^3 + \alpha_5 v^2 u + \alpha_6\left(\gamma_1 u^3 + \alpha_7 uv^2\right)_{xx}, \\ \delta_2 v_{xx} + \beta_1 v_{tt} + \beta_2 v_{xt} = \beta_3 v + \beta_4 v^3 + \beta_5 u^2 v + \beta_6\left(\gamma_2 v^3 + \beta_7 vu^2\right)_{xx}. \end{cases} \tag{10}$$

Here $\alpha_j, \beta_i\,(i = j = 1, 2, \cdots, 7), \gamma_1, \gamma_2, \delta_1, \delta_2$ are constants.

2. The Method Combining the Function Transformation with the Auxiliary Equation

Assume the solutions of a class of nonlinear evolutionary Equation (10) as

$$\begin{cases} u(x,t)=\frac{1}{2}\left[P_1(\xi_1)+Q_1(\eta_1)\right], \\ v(x,t)=\frac{1}{2}\left[P_1(\xi_1)-Q_1(\eta_1)\right]. \end{cases} \tag{11}$$

$$\begin{cases} u(x,t)=\frac{1}{2}\left[\exp\left[\int P_2(\xi_2)\mathrm{d}\xi_2\right]+\exp\left[\int Q_2(\eta_2)\mathrm{d}\eta_2\right]\right], \\ v(x,t)=\frac{1}{2}\left[\exp\left[\int P_2(\xi_2)\mathrm{d}\xi_2\right]-\exp\left[\int Q_2(\eta_2)\mathrm{d}\eta_2\right]\right], \end{cases} \tag{12}$$

where $\xi_j=\lambda_j x+\mu_j t,\eta_j=\nu_j x+\omega_j t,\lambda_j,\mu_j,\nu_j$ and ω_j are constants to be determined, and $\lambda_j\neq\nu_j$, $\mu_j\neq\omega_j\,(j=1,2)$.

2.1. The First Kind of Function Transformation with the Two Kinds of Ordinary Differential Equations

Substituting the first kind of function transformation (11) into a class of nonlinear evolutionary Equation (10) yields the following conclusions.

Case 1. When the coefficients of a class of nonlinear evolutionary equations satisfy the following conditions, the complexion solutions exist.

When $\alpha_i=\beta_i\,(i=1,2,\cdots,7),\delta_1=\delta_2,\beta_5=3\alpha_4,\beta_7=3\gamma_2$ a class of nonlinear evolutionary Equation (10) has the complexion solutions, and the solutions are determined by the following nonlinear ordinary differential equations.

$$\frac{\mathrm{d}^2P_1(\xi_1)}{\mathrm{d}\xi_1^2}=\frac{-3\beta_3P_1^4(\xi_1)-36\beta_6\lambda_1^2\gamma_1\left(P_1'(\xi_1)\right)^2+P_1^2(\xi_1)\left[-\alpha_5+6\left(\lambda_1^2\delta_2+\beta_1\mu_1^2+\lambda_1\mu_1\alpha_2\right)\left(P_1'(\xi_1)\right)^2\right]}{3\left[-3\beta_6\gamma_1\lambda_1^2+\left(\beta_1\mu_1^2+\lambda_1^2\delta_2+\lambda_1\mu_1\alpha_2\right)P_1^2(\xi_1)\right]P_1(\xi_1)}, \tag{13}$$

$$\frac{\mathrm{d}^2Q_1(\eta_1)}{\mathrm{d}\eta_1^2}=\frac{-3\beta_3Q_1^4(\eta_1)-36\beta_6\nu_1^2\gamma_1\left(Q_1'(\eta_1)\right)^2+Q_1^2(\eta_1)\left[-\alpha_5+6\left(\nu_1^2\delta_2+\beta_1\omega_1^2+\nu_1\omega_1\alpha_2\right)\left(Q_1'(\eta_1)\right)^2\right]}{3\left[-3\beta_6\gamma_1\nu_1^2+\left(\nu_1^2\delta_2+\nu_1\alpha_2\omega_1+\beta_1\omega_1^2\right)Q_1^2(\eta_1)\right]Q_1(\eta_1)}. \tag{14}$$

Case 2. The first integral of the second order nonlinear ordinary differential equations.

By the function transformation, the nonlinear ordinary differential Equations (13) and (14) are changed into the following two ordinary differential equations

$$\begin{cases} \frac{\mathrm{d}P_1}{\mathrm{d}\xi_1}=Y_1, \\ \frac{\mathrm{d}Y_1}{\mathrm{d}\xi_1}=\frac{-3\beta_3P_1^4-\alpha_5P_1^2+\left[6\left(\lambda_1^2\delta_2+\beta_1\mu_1^2+\lambda_1\mu_1\alpha_2\right)P_1^2-36\beta_6\lambda_1^2\gamma_1\right]Y_1^2}{3\left[-3\beta_6\gamma_1\lambda_1^2+\left(\beta_1\mu_1^2+\lambda_1^2\delta_2+\lambda_1\mu_1\alpha_2\right)P_1^2\right]P_1}, \end{cases} \tag{15}$$

$$\begin{cases} \frac{\mathrm{d}Q_1}{\mathrm{d}\eta_1}=X_1, \\ \frac{\mathrm{d}X_1}{\mathrm{d}\eta_1}=\frac{-3\beta_3Q_1^4-\alpha_5Q_1^2+\left[6\left(\nu_1^2\delta_2+\beta_1\omega_1^2+\nu_1\omega_1\alpha_2\right)Q_1^2-36\beta_6\nu_1^2\gamma_1\right]X_1^2}{3\left[-3\beta_6\gamma_1\nu_1^2+\left(\nu_1^2\delta_2+\nu_1\alpha_2\omega_1+\beta_1\omega_1^2\right)Q_1^2\right]Q_1}. \end{cases} \tag{16}$$

And by the following transformation (17) and (18), the two ordinary differential Equations (15) and (16) can be expressed by the form of (19) and (20).

$$\mathrm{d}\xi_1=3\left[-3\beta_6\gamma_1\lambda_1^2+\left(\beta_1\mu_1^2+\lambda_1^2\delta_2+\lambda_1\mu_1\alpha_2\right)P_1^2\right]\mathrm{d}\tau_1, \tag{17}$$

$$\mathrm{d}\eta_1 = 3\left[-3\beta_6\gamma_1\nu_1^2 + \left(\nu_1^2\delta_2 + \nu_1\alpha_2\omega_1 + \beta_1\omega_1^2\right)Q_1^2\right]\mathrm{d}\zeta_1. \tag{18}$$

$$\begin{cases} \dfrac{\mathrm{d}P_1}{\mathrm{d}\tau_1} = 3Y_1\left[-3\beta_6\gamma_1\lambda_1^2 + \left(\beta_1\mu_1^2 + \lambda_1^2\delta_2 + \lambda_1\mu_1\alpha_2\right)P_1^2\right], \\ \dfrac{\mathrm{d}Y_1}{\mathrm{d}\tau_1} = \dfrac{1}{P_1}\left[-3\beta_3P_1^4 - \alpha_5P_1^2 + \left[6\left(\lambda_1^2\delta_2 + \beta_1\mu_1^2 + \lambda_1\mu_1\alpha_2\right)P_1^2 - 36\beta_6\lambda_1^2\gamma_1\right]Y_1^2\right]. \end{cases} \tag{19}$$

$$\begin{cases} \dfrac{\mathrm{d}Q_1}{\mathrm{d}\zeta_1} = 3X_1\left[-3\beta_6\gamma_1\nu_1^2 + \left(\nu_1^2\delta_2 + \nu_1\alpha_2\omega_1 + \beta_1\omega_1^2\right)Q_1^2\right], \\ \dfrac{\mathrm{d}X_1}{\mathrm{d}\zeta_1} = \dfrac{1}{Q_1}\left[-3\beta_3Q_1^4 - \alpha_5Q_1^2 + \left[6\left(\nu_1^2\delta_2 + \beta_1\omega_1^2 + \nu_1\omega_1\alpha_2\right)Q_1^2 - 36\beta_6\nu_1^2\gamma_1\right]X_1^2\right]. \end{cases} \tag{20}$$

By calculating, the following first integral of the ordinary differential Equations (19) and (20) are obtained

$$Y_1^2 = \frac{6c_1P_1^8 + M_1P_1^6 + M_2P_1^4 - 2\alpha_5\beta_6\gamma_1\lambda_1^2P_1^2}{6\left(-3\beta_6\gamma_1\lambda_1^2 + \left[\delta_2\lambda_1^2 + \mu_1\left(\alpha_2\lambda_1 + \beta_1\mu_1\right)\right]P_1^2\right)^2}, \tag{21}$$

$$X_1^2 = \frac{6c_2Q_1^8 + N_1Q_1^6 + N_2Q_1^4 - 2\alpha_5\beta_6\gamma_1\nu_1^2Q_1^2}{6\left(-3\beta_6\gamma_1\nu_1^2 + \left[\delta_2\nu_1^2 + \omega_1\left(\alpha_2\nu_1 + \beta_1\omega_1\right)\right]Q_1^2\right)^2}. \tag{22}$$

Substituting the first integral (21) and (22) severally into the first equation of the ordinary differential Equations (19) and (20) yields the following two ordinary differential equations

$$\left(\frac{\mathrm{d}P_1}{\mathrm{d}\tau_1}\right)^2 = \frac{3}{2}\left(6c_1P_1^8 + M_1P_1^6 + M_2P_1^4 - 2\alpha_5\beta_6\gamma_1\lambda_1^2P_1^2\right), \tag{23}$$

$$\left(\frac{\mathrm{d}Q_1}{\mathrm{d}\zeta_1}\right)^2 = \frac{3}{2}\left(6c_2Q_1^8 + N_1Q_1^6 + N_2Q_1^4 - 2\alpha_5\beta_6\gamma_1\nu_1^2Q_1^2\right), \tag{24}$$

where $M_1 = 6\beta_3\delta_2\lambda_1^2 + 6\alpha_2\beta_3\lambda_1\mu_1 + 6\beta_1\beta_3\mu_1^2$, $M_2 = -9\beta_3\beta_6\gamma_1\lambda_1^2 + \alpha_5\delta_2\lambda_1^2 + \alpha_2\alpha_5\lambda_1\mu_1 + \alpha_5\beta_1\mu_1^2$, $N_1 = 6\beta_3\delta_2\nu_1^2 + 6\alpha_2\beta_3\nu_1\omega_1 + 6\beta_1\beta_3\omega_1^2$, $N_2 = -9\beta_3\beta_6\gamma_1\nu_1^2 + \alpha_5\delta_2\nu_1^2 + \alpha_2\alpha_5\nu_1\omega_1 + \alpha_5\beta_1\omega_1^2$, c_1 and c_2 are arbitrary constants.

2.2. The Second Kind of Function Transformation with the Two Kinds of Ordinary Differential Equations

Substituting the second kind of function transformation (12) into a class of nonlinear evolutionary Equation (10) yields the following conclusions.

Case 1. When the coefficients of a class of nonlinear evolutionary equations satisfy the following conditions, the complexion solutions exist.

When $\alpha_i = \beta_i\left(i = 1, 2, \cdots, 7\right), \delta_1 = \delta_2, \beta_5 = 3\alpha_4, \beta_7 = 3\gamma_2$ a class of nonlinear evolutionary Equation (10) has the complexion solutions, and the solutions are determined by the following nonlinear ordinary differential equations.

$$\frac{\mathrm{d}^2P_2\left(\xi_2\right)}{\mathrm{d}\xi_2^2} = -\frac{3\beta_3 + \left[27\beta_6\gamma_1\lambda_2^2P_2^2\left(\xi_2\right) + \alpha_5\right]\exp\left[2\int P_2\left(\xi_2\right)\mathrm{d}\xi_2\right] - \left(3\beta_1\mu_2^2 + 3\lambda_2\mu_2\alpha_2 + 3\lambda_2^2\delta_2\right)P_2^2\left(\xi_2\right)}{3\left(-\lambda_2^2\delta_2 + 3\beta_6\gamma_1\lambda_2^2\exp\left[2\int P_2\left(\xi_2\right)\mathrm{d}\xi_2\right] - \lambda_2\mu_2\alpha_2 - \beta_1\mu_2^2\right)}, \tag{25}$$

$$\frac{\mathrm{d}^2Q_2\left(\eta_2\right)}{\mathrm{d}\eta_2^2} = -\frac{3\beta_3 + \left[27\beta_6\gamma_1\nu_2^2Q_2^2\left(\eta_2\right) + \alpha_5\right]\exp\left[2\int Q_2\left(\eta_2\right)\mathrm{d}\eta_2\right] - \left(3\beta_1\omega_2^2 + 3\nu_2\omega_2\alpha_2 + 3\nu_2^2\delta_2\right)Q_2^2\left(\eta_2\right)}{3\left(-\nu_2^2\delta_2 + 3\beta_6\gamma_1\nu_2^2\exp\left[2\int Q_2\left(\eta_2\right)\mathrm{d}\eta_2\right] - \alpha_2\omega_2\nu_2 - \beta_1\omega_2^2\right)}. \tag{26}$$

The ordinary differential Equations (25) and (26) can be expressed by the following forms.

$$\begin{cases} \dfrac{\mathrm{d}Y_2}{\mathrm{d}\xi_2} = 2P_2Y_2, \\ \dfrac{\mathrm{d}P_2}{\mathrm{d}\xi_2} = -\dfrac{3\beta_3 + \alpha_5 Y_2 - \left(3\beta_1\mu_2^2 + 3\lambda_2\mu_2\alpha_2 + 3\lambda_2^2\delta_2 - 27\beta_6\gamma_1\lambda_2^2 Y_2\right)P_2^2}{3\left(-\lambda_2^2\delta_2 + 3\beta_6\gamma_1\lambda_2^2 Y_2 - \lambda_2\mu_2\alpha_2 - \beta_1\mu_2^2\right)}. \end{cases} \tag{27}$$

$$\begin{cases} \dfrac{\mathrm{d}X_2}{\mathrm{d}\eta_2} = 2Q_2X_2, \\ \dfrac{\mathrm{d}Q_2}{\mathrm{d}\eta_2} = -\dfrac{3\beta_3 + \alpha_5 X_2 - \left(3\beta_1\omega_2^2 + 3\nu_2\omega_2\alpha_2 + 3\nu_2^2\delta_2 - 27\beta_6\gamma_1\nu_2^2 X_2\right)Q_2^2}{3\left(-\nu_2^2\delta_2 + 3\beta_6\gamma_1\nu_2^2 X_2 - \alpha_2\omega_2\nu_2 - \beta_1\omega_2^2\right)}. \end{cases} \tag{28}$$

And by the following transformation (29) and (30), the two ordinary differential Equations (27), (28) can be expressed by the form of (31), (32).

$$\mathrm{d}\xi_2 = 3\left(-\lambda_2^2\delta_2 + 3\beta_6\gamma_1\lambda_2^2 Y_2 - \lambda_2\mu_2\alpha_2 - \beta_1\mu_2^2\right)\mathrm{d}\tau_2, \tag{29}$$

$$\mathrm{d}\eta_2 = 3\left(-\nu_2^2\delta_2 + 3\beta_6\gamma_1\nu_2^2 X_2 - \alpha_2\omega_2\nu_2 - \beta_1\omega_2^2\right)\mathrm{d}\zeta_2. \tag{30}$$

$$\begin{cases} \dfrac{\mathrm{d}Y_2}{\mathrm{d}\tau_2} = 6P_2Y_2\left(-\lambda_2^2\delta_2 + 3\beta_6\gamma_1\lambda_2^2 Y_2 - \lambda_2\mu_2\alpha_2 - \beta_1\mu_2^2\right), \\ \dfrac{\mathrm{d}P_2}{\mathrm{d}\tau_2} = -\left[3\beta_3 + \alpha_5 Y_2 - \left(3\beta_1\mu_2^2 + 3\lambda_2\mu_2\alpha_2 + 3\lambda_2^2\delta_2 - 27\beta_6\gamma_1\lambda_2^2 Y_2\right)P_2^2\right]. \end{cases} \tag{31}$$

$$\begin{cases} \dfrac{\mathrm{d}X_2}{\mathrm{d}\zeta_2} = 6Q_2X_2\left(-\nu_2^2\delta_2 + 3\beta_6\gamma_1\nu_2^2 X_2 - \alpha_2\omega_2\nu_2 - \beta_1\omega_2^2\right), \\ \dfrac{\mathrm{d}Q_2}{\mathrm{d}\zeta_2} = -\left[3\beta_3 + \alpha_5 X_2 - \left(3\beta_1\omega_2^2 + 3\nu_2\omega_2\alpha_2 + 3\nu_2^2\delta_2 - 27\beta_6\gamma_1\nu_2^2 X_2\right)Q_2^2\right]. \end{cases} \tag{32}$$

Case 2. The first integral of the second order nonlinear ordinary differential equations.

By calculating, the following first integral of the ordinary differential Equations (31), (32) are obtained.

$$P_2^2 = \frac{6c_3 - 2\alpha_5\beta_6\gamma_1\lambda_2^2 Y_2^3 + M_3 Y_2^2 + M_4 Y_2}{6\left[-3\beta_6\gamma_1\lambda_2^2 Y_2 + \delta_2\lambda_2^2 + \mu_2\left(\alpha_2\lambda_2 + \beta_1\mu_2\right)\right]^2 Y_2}, \tag{33}$$

$$Q_2^2 = \frac{6c_4 - 2\alpha_5\beta_6\gamma_1\nu_2^2 X_2^3 + N_3 X_2^2 + N_4 X_2}{6\left[-3\beta_6\gamma_1\nu_2^2 X_2 + \delta_2\nu_2^2 + \omega_2\left(\alpha_2\nu_2 + \beta_1\omega_2\right)\right]^2 X_2}. \tag{34}$$

Substituting the first integral (33) and (34) severally into the first equation of the ordinary differential Equations (31) and (32) yields the following two nonlinear ordinary differential equations

$$\left(\frac{\mathrm{d}Y_2}{\mathrm{d}\tau_2}\right)^2 = 6\left[-2\alpha_5\beta_6\gamma_1\lambda_2^2 Y_2^4 + M_3 Y_2^3 + M_4 Y_2^2 + 6c_3 Y_2\right], \tag{35}$$

$$\left(\frac{\mathrm{d}X_2}{\mathrm{d}\zeta_2}\right)^2 = 6\left[-2\alpha_5\beta_6\gamma_1\nu_2^2 X_2^4 + N_3 X_2^3 + N_4 X_2^2 + 6c_4 X_2\right], \tag{36}$$

where $M_3 = -9\beta_3\beta_6\gamma_1\lambda_2^2 + \alpha_5\left(\delta_2\lambda_2^2 + \alpha_2\lambda_2\mu_2 + \beta_1\mu_2^2\right)$, $M_4 = 6\beta_3\left[\delta_2\lambda_2^2 + \mu_2\left(\alpha_2\lambda_2 + \beta_1\mu_2\right)\right]$, $N_3 = -9\beta_3\beta_6\gamma_1\nu_2^2 + \alpha_5\left(\delta_2\nu_2^2 + \alpha_2\nu_2\omega_2 + \beta_1\omega_2^2\right)$, $N_4 = 6\beta_3\left[\delta_2\nu_2^2 + \omega_2\left(\alpha_2\nu_2 + \beta_1\omega_2\right)\right]$, c_3 and c_4 are arbitrary constants.

3. The New Infinite Sequence Complexion Solutions of a Class of Nonlinear Evolutionary Equations

In some cases, according to the ordinary differential Equations (23) and (24) (or (35) and (36)), the new infinite sequence complexion solutions of a class of nonlinear evolutionary Equation (10) are constructed.

3.1. The New Infinite Sequence Complexion Solutions Consisting of the Hyperbolic Function, the Trigonometric Function and the Rational Function

When $c_1 = c_2 = 0, \alpha_5\beta_6\gamma_1 \neq 0$, by the following superposition formula, the new infinite sequence complexion solutions consisting of the hyperbolic function, the trigonometric function and the rational function of a class of nonlinear evolutionary Equation (10) are obtained.

$$\begin{cases} u_{mn}(x,t) = \frac{1}{2}\left[(P_1)_m(\xi_1) + (Q_1)_n(\eta_1)\right] \quad (m = 1,2,\cdots), \\ v_{mn}(x,t) = \frac{1}{2}\left[(P_1)_m(\xi_1) - (Q_1)_n(\eta_1)\right] \quad (n = 1,2,\cdots), \\ \mathrm{d}\xi_1 = 3\left[-3\beta_6\gamma_1\lambda_1^2 + \left(\beta_1\mu_1^2 + \lambda_1^2\delta_2 + \lambda_1\mu_1\alpha_2\right)P_1^2\right]\mathrm{d}\tau_1, \\ \mathrm{d}\eta_1 = 3\left[-3\beta_6\gamma_1\nu_1^2 + \left(\nu_1^2\delta_2 + \nu_1\alpha_2\omega_1 + \beta_1\omega_1^2\right)Q_1^2\right]\mathrm{d}\zeta_1, \end{cases} \tag{37}$$

where $(P_1)_m(\xi_1)$ and $(Q_1)_n(\eta_1)$ are determined by the following superposition formula [10]-[13].

$$\begin{cases} (P_1)_m(\tau_1) = \pm\frac{\sqrt{-A_1 + (V_1)_m^2(\tau_1)}}{\sqrt{B_1 - 2\sqrt{C_1}(V_1)_m(\tau_1)}} \quad (m = 1,2,\cdots), \\ (V_1)_m(\tau_1) = \frac{\left(-M_0 \mp \sqrt{M_0^2 - 4L_0N_0}\right)R_1 + 2(L_0 - N_0R_1)(V_1)_{m-1}(\tau_1) + \left(M_0 \mp \sqrt{M_0^2 - 4L_0N_0}\right)(V_1)_{m-1}^2(\tau_1)}{2\left[L_0 + M_0(V_1)_{m-1}(\tau_1) + N_0(V_1)_{m-1}^2(\tau_1)\right]}, \\ (V_1)_0(\tau_1) = -\sqrt{-R_1}\tanh\left(\sqrt{-R_1}\tau_1\right) \quad \left(-R_1 = A_1 = -3\alpha_5\beta_6\gamma_1\lambda_1^2 < 0\right), \\ B_1 = \frac{3}{2}\left[-9\beta_3\beta_6\gamma_1\lambda_1^2 + \alpha_5\delta_2\lambda_1^2 + \alpha_2\alpha_5\lambda_1\mu_1 + \alpha_5\beta_1\mu_1^2\right], C_1 = \frac{3}{2}\left[6\beta_3\delta_2\lambda_1^2 + 6\alpha_2\beta_3\lambda_1\mu_1 + 6\beta_1\beta_3\mu_1^2\right]. \end{cases} \tag{38}$$

$$\begin{cases} (P_1)_m(\tau_1) = \pm\frac{\sqrt{-A_1 + (V_1)_m^2(\tau_1)}}{\sqrt{B_1 - 2\sqrt{C_1}(V_1)_m(\tau_1)}} \quad (m = 1,2,\cdots), \\ (V_1)_m(\tau_1) = \frac{\left(-M_0 \mp \sqrt{M_0^2 - 4L_0N_0}\right)R_1 + 2(L_0 - N_0R_1)(V_1)_{m-1}(\tau_1) + \left(M_0 \mp \sqrt{M_0^2 - 4L_0N_0}\right)(V_1)_{m-1}^2(\tau_1)}{2\left[L_0 + M_0(V_1)_{m-1}(\tau_1) + N_0(V_1)_{m-1}^2(\tau_1)\right]}, \\ (V_1)_0(\tau_1) = \sqrt{R_2}\tan\left(\sqrt{R_2}\tau_1\right) \quad \left(-R_1 = A_1 = -3\alpha_5\beta_6\gamma_1\lambda_1^2 > 0\right), \\ B_1 = \frac{3}{2}\left[-9\beta_3\beta_6\gamma_1\lambda_1^2 + \alpha_5\delta_2\lambda_1^2 + \alpha_2\alpha_5\lambda_1\mu_1 + \alpha_5\beta_1\mu_1^2\right], C_1 = \frac{3}{2}\left[6\beta_3\delta_2\lambda_1^2 + 6\alpha_2\beta_3\lambda_1\mu_1 + 6\beta_1\beta_3\mu_1^2\right]. \end{cases} \tag{39}$$

$$\begin{cases} (P_1)_m(\tau_1) = \pm\frac{\sqrt{-A_1 + (V_1)_m^2(\tau_1)}}{\sqrt{B_1 - 2\sqrt{C_1}(V_1)_m(\tau_1)}} \quad (m = 1,2,\cdots), \\ (V_1)_m(\tau_1) = \frac{2L_0(V_1)_{m-1}(\tau_1) + \left(M_0 \mp \sqrt{M_0^2 - 4L_0N_0}\right)(V_1)_{m-1}^2(\tau_1)}{2\left[L_0 + M_0(V_1)_{m-1}(\tau_1) + N_0(V_1)_{m-1}^2(\tau_1)\right]} \quad (m = 1,2,\cdots), \\ (V_1)_0(\tau_1) = -\frac{1}{\tau_1} \quad \left(-R_1 = A_1 = -3\alpha_5\beta_6\gamma_1\lambda_1^2 = 0\right), \\ B_1 = \frac{3}{2}\left[-9\beta_3\beta_6\gamma_1\lambda_1^2 + \alpha_5\delta_2\lambda_1^2 + \alpha_2\alpha_5\lambda_1\mu_1 + \alpha_5\beta_1\mu_1^2\right], C_1 = \frac{3}{2}\left[6\beta_3\delta_2\lambda_1^2 + 6\alpha_2\beta_3\lambda_1\mu_1 + 6\beta_1\beta_3\mu_1^2\right]. \end{cases} \tag{40}$$

$$\begin{cases} (Q_1)_n(\zeta_1) = \pm\dfrac{\sqrt{-A_2+(V_2)_n^2(\zeta_1)}}{\sqrt{B_2-2\sqrt{C_2}(V_2)_n(\zeta_1)}} \quad (n=1,2,\cdots), \\ (V_2)_n(\zeta_1) = \dfrac{\left(-M_0 \mp \sqrt{M_0^2-4L_0N_0}\right)R_2 + 2(L_0-N_0R_2)(V_2)_{n-1}(\zeta_1) + \left(M_0 \mp \sqrt{M_0^2-4L_0N_0}\right)(V_2)_{n-1}^2(\zeta_1)}{2\left[L_0+M_0(V_2)_{n-1}(\zeta_1)+N_0(V_2)_{n-1}^2(\zeta_1)\right]}, \\ (V_2)_0(\zeta_1) = -\sqrt{-R_2}\tanh\left(\sqrt{-R_2}\zeta_1\right) \quad \left(-R_2 = A_2 = -3\alpha_5\beta_6\gamma_1\nu_1^2 < 0\right), \\ B_2 = \dfrac{3}{2}\left[-9\beta_3\beta_6\gamma_1\nu_1^2+\alpha_5\delta_2\nu_1^2+\alpha_2\alpha_5\nu_1\omega_1+\alpha_5\beta_1\omega_1^2\right], C_2 = \dfrac{3}{2}\left[6\beta_3\delta_2\nu_1^2+6\alpha_2\beta_3\nu_1\omega_1+6\beta_1\beta_3\omega_1^2\right]. \end{cases} \tag{41}$$

$$\begin{cases} (Q_1)_n(\zeta_1) = \pm\dfrac{\sqrt{-A_2+(V_2)_n^2(\zeta_1)}}{\sqrt{B_2-2\sqrt{C_2}(V_2)_n(\zeta_1)}} \quad (n=1,2,\cdots), \\ (V_2)_n(\zeta_1) = \dfrac{\left(-M_0 \mp \sqrt{M_0^2-4L_0N_0}\right)R_2 + 2(L_0-N_0R_2)(V_2)_{n-1}(\zeta_1) + \left(M_0 \mp \sqrt{M_0^2-4L_0N_0}\right)(V_2)_{n-1}^2(\zeta_1)}{2\left[L_0+M_0(V_2)_{n-1}(\zeta_1)+N_0(V_2)_{n-1}^2(\zeta_1)\right]}, \\ (V_2)_0(\zeta_1) = \sqrt{R_2}\tan\left(\sqrt{R_2}\zeta_1\right) \quad \left(-R_2 = A_2 = -3\alpha_5\beta_6\gamma_1\nu_1^2 > 0\right), \\ B_2 = \dfrac{3}{2}\left[-9\beta_3\beta_6\gamma_1\nu_1^2+\alpha_5\delta_2\nu_1^2+\alpha_2\alpha_5\nu_1\omega_1+\alpha_5\beta_1\omega_1^2\right], C_2 = \dfrac{3}{2}\left[6\beta_3\delta_2\nu_1^2+6\alpha_2\beta_3\nu_1\omega_1+6\beta_1\beta_3\omega_1^2\right]. \end{cases} \tag{42}$$

$$\begin{cases} (Q_1)_n(\zeta_1) = \pm\dfrac{\sqrt{-A_2+(V_2)_n^2(\zeta_1)}}{\sqrt{B_2-2\sqrt{C_2}(V_2)_n(\zeta_1)}} \quad (n=1,2,\cdots), \\ (V_2)_n(\zeta_1) = \dfrac{2L_0(V_2)_{n-1}(\zeta_1) + \left(M_0 \mp \sqrt{M_0^2-4L_0N_0}\right)(V_2)_{n-1}^2(\zeta_1)}{2\left[L_0+M_0(V_2)_{n-1}(\zeta_1)+N_0(V_2)_{n-1}^2(\zeta_1)\right]}, \\ (V_2)_0(\zeta_1) = -\dfrac{1}{\zeta_1} \quad \left(-R_2 = A_2 = -3\alpha_5\beta_6\gamma_1\nu_1^2 = 0\right), \\ B_2 = \dfrac{3}{2}\left[-9\beta_3\beta_6\gamma_1\nu_1^2+\alpha_5\delta_2\nu_1^2+\alpha_2\alpha_5\nu_1\omega_1+\alpha_5\beta_1\omega_1^2\right], C_2 = \dfrac{3}{2}\left[6\beta_3\delta_2\nu_1^2+6\alpha_2\beta_3\nu_1\omega_1+6\beta_1\beta_3\omega_1^2\right]. \end{cases} \tag{43}$$

In (38)-(43), L_0, M_0 and N_0 are arbitrary constants not all equal to zero.

Case 1. The hyperbolic function type new infinite sequence complexion two-soliton solutions of a class of nonlinear evolutionary Equation (10).

Substituting the solution determined by the superposition formula (38), (41) into (37) yields the hyperbolic function type new infinite sequence complexion two-soliton solutions.

Case 2. The new infinite sequence complexion solutions consisting of the hyperbolic function and the trigonometric function of a class of nonlinear evolutionary Equation (10).

Substituting the solution determined by the superposition formula (38), (42) into (37) yields the new infinite sequence complexion solutions composed of soliton solutions and the period solutions.

Case 3. The new infinite sequence complexion solutions consisting of the hyperbolic function and the rational function of a class of nonlinear evolutionary Equation (10).

Substituting the solution determined by the superposition formula (38) and (43) into (37) yields the new infinite sequence complexion solutions consisting of the hyperbolic function and the rational function.

Case 4. The trigonometric function type new infinite sequence two-period solutions of a class of nonlinear evolutionary Equation (10).

Substituting the solution determined by the superposition formula (39), (42) into (37) yields the trigonometric function type new infinite sequence two-period solutions.

Case 5. The new infinite sequence complexion solutions consisting of the trigonometric function and the rational function of a class of nonlinear evolutionary Equation (10).

Substituting the solution determined by the superposition formula (39), (43) into (37) yields the new infinite sequence complexion solutions consisting of the trigonometric function and the rational function.

Case 6. The rational function type new infinite sequence solutions of a class of nonlinear evolutionary Equation (10).

Substituting the solution determined by the superposition formula (40) and (43) into (37) yields the rational function type new infinite sequence solutions.

When $c_3 = c_4 = 0$, with the help of the ordinary differential Equations (35) and (36), the new infinite sequence complexion two-soliton and two-periods solutions consisting of the hyperbolic function, the trigonometric function and the rational function of a class of nonlinear evolutionary Equation (10) can also be obtained (not given here).

3.2. The New Infinite Sequence Complexion Solutions Consisting of the Riemann Function, the Jacobi Elliptic Function, the Hyperbolic Function, the Trigonometric Function and the Rational Function

With the help of the relative conclusions of the second kind of elliptic equation [11]-[13], by analyzing, the infinite sequence solutions of the ordinary differential Equations (35) and (36) are obtained. Then substituting these solutions into function transformation (44) yields the new infinite sequence complexion solutions consisting of the Riemann θ function, the Jacobi elliptic function, the hyperbolic function, the trigonometric function and the rational function of a class of nonlinear evolutionary Equation (10) in some kinds of cases. These solutions are consisting of two-soliton solutions, two-period solutions and solutions composed of soliton solutions and period solutions (not given concretely here).

$$\begin{cases} u_{mn}(x,t) = \frac{1}{2}\left[\exp\left[\int (P_2)_m(\xi_2)\,\mathrm{d}\xi_2\right] + \exp\left[\int (Q_2)_n(\eta_2)\,\mathrm{d}\eta_2\right]\right] \quad (m,n=1,2,\cdots), \\ v_{mn}(x,t) = \frac{1}{2}\left[\exp\left[\int (P_2)_m(\xi_2)\,\mathrm{d}\xi_2\right] - \exp\left[\int (Q_2)_n(\eta_2)\,\mathrm{d}\eta_2\right]\right] \quad (m,n=1,2,\cdots), \\ \mathrm{d}\xi_2 = 3\left(-\lambda_2^2\delta_2 + 3\beta_6\gamma_1\lambda_2^2 Y_2 - \lambda_2\mu_2\alpha_2 - \beta_1\mu_2^2\right)\mathrm{d}\tau_2, \\ \mathrm{d}\eta_2 = 3\left(-\nu_2^2\delta_2 + 3\beta_6\gamma_1\nu_2^2 X_2 - \alpha_2\omega_2\nu_2 - \beta_1\omega_2^2\right)\mathrm{d}\zeta_2. \end{cases} \tag{44}$$

Case 1. When $\gamma_1 = 0, \alpha_5\beta_6 \neq 0, c_3c_4 \neq 0$, the new infinite sequence complexion solutions consisting of the Riemann θ function, the Jacobi elliptic function, the hyperbolic function, the trigonometric function and the rational function of a class of nonlinear evolutionary Equation (10) are obtained.

Case 2. When $\beta_6 = 0, \alpha_5\gamma_1 \neq 0, c_3c_4 \neq 0$, the new infinite sequence complexion solutions consisting of the Riemann θ function, the Jacobi elliptic function, the hyperbolic function, the trigonometric function and the rational function of a class of nonlinear evolutionary Equation (10) are obtained.

Case 3. When $\alpha_5 = 0, \beta_6\gamma_1 \neq 0, c_3c_4 \neq 0$, the new infinite sequence complexion solutions consisting of the Riemann θ function, the Jacobi elliptic function, the hyperbolic function, the trigonometric function and the rational function of a class of nonlinear evolutionary Equation (10) are obtained.

4. Conclusions

In Refs. [1]-[5], a finite number of one function type one soliton new solutions consisting of the Jacobi elliptic function, the hyperbolic function, the trigonometric function and the rational function of the nonlinear evolutionary Equations (1)-(4) were constructed. In Refs. [6]-[9], a finite number of complexion function type two-soliton new solutions consisting of the Jacobi elliptic function, the hyperbolic function, the trigonometric function and the rational function of the nonlinear evolutionary Equation (5) were constructed.

In this paper, the method combining the function transformation with the auxiliary equation is presented, and the problem of solving the solutions of a class of nonlinear evolutionary Equation (10) concluding the nonlinear evolutionary Equations (1)-(9) is considered; the new infinite sequence complexion solutions consisting of the Riemann θ function, the Jacobi elliptic function, the hyperbolic function, the trigonometric function and the rational function are obtained. These solutions are consisting of two-soliton solutions, two-period solutions and solutions composed of soliton solutions and period solutions.

When the coefficients of nonlinear evolutionary Equations (1) are $\alpha_3 = \beta_3 = h - l = -1$, $\alpha_4 = \beta_4 = m = 1$, then they satisfy the condition $\alpha_i = \beta_i \left(i = 1, 2, \cdots, 7\right), \delta_1 = \delta_2, \beta_5 = 3\alpha_4, \beta_7 = 3\gamma_2$ that the new infinite sequence complexion solutions exist. So, according to the method given in the Part 3, the new infinite sequence complexion two-soliton and two-periods and so on solutions of nonlinear evolutionary Equation (1) are constructed.

When the coefficients of a class of nonlinear evolutionary Equation (10) are $\delta_1 = \delta_2 = \alpha_1 = \beta_1 = \alpha_4 = \alpha_5 = \beta_4 = \beta_5 = 0$, $\alpha_2 = \beta_2 = \alpha_3 = \beta_3 = \gamma_1 = \gamma_2 = 1, \alpha_6 = \beta_6 = \frac{1}{6}, \alpha_7 = \beta_7 = 3$, it satisfies the condition that the new infinite sequence complexion solutions exist. Then according to the conclusions given in Part 3 in the paper, the new infinite sequence complexion two-soliton solutions, two-period solutions and solutions composed of soliton solutions and period solutions of nonlinear evolutionary Equation (5) are constructed.

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