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Reliability Analysis of a Redundant Cascade System by Using Markovian Approach

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Abstract

In the present work, it is assumed that the *n*-components are arranged in the hierarchial order. The *n*-cascade system surviving with loss of *m* components by *k* number of attacks is studied; the general equation for the reliability is obtained for the above said system; and the system reliability is computed numerically for 6-cascade system for 2-number of attacks.

Keywords

1. Introduction

Reliability of a system is the probability that a system will adequately perform its intended purpose for a given period of time under stated environmental conditions [1]. In some cases system failures occur due to certain type of stresses acting on them. Thus system composed of random strengths will have its strength as random variable and the stress applied on it will also be a random variable. A system fails whenever an applied stress exceeds strength of the system. Probability and reliability were explained by Shooman [2]. The applications of time dependent stress strength models are explained by Schatz R. *et al.* [3]. Reliability of a *n*-cascade system with stress attenuation was proposed by Pandit & Sriwastav [4]. Estimation of reliability was explained by William [5]. Raghavachar *et al.* [6] [7] studied the reliability of a cascade system with normal stress & strength distribution and survival function under stress attenuation in cascade reliability. Detailed explanation of mixture distributions is given in [8]. T. S. Uma Maheswari [9] explained reliability of cascade system with normal stress & exponential strength. T. S. Uma Maheswari [10] studied reliability comparison of an *n*-cascade system with the addition of an *n*-strength system. T. S. Uma Maheswari [11] explained reliability of single stress under *n*-strengths of life distribution. Rekha *et al.* [12] studied cascade system reliability with rayleigh distribution. In

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reliability theory, there are lots of real life situations where the concept of mixture distributions can be applied. For example, in life testing experiments, the systems will be failed due to different causes and the times to failure due to different reasons are likely to follow different distributions. Knowledge of these distributions is essential to eliminate cause of failures and thereby to improve the reliability. Maya, T. Nair [13] described the estimation of reliability based on finite mixture of pare to and beta distributions.

Stochastic process is a mathematical model that evolves over time in a probabilistic manner. A special kind of stochastic process is called Markov process, where the outcome of an experiment depends only on the outcome of the previous experiment, *i.e.*, the next state of the system depends only on the present state, not on preceding states. Cascade redundancy is the provision of alternative means or parallel paths in a system for accomplishing a given task such that all means must fail before causing a system failure. The reliability model is being studied here. The probabilities of component failure depend on the relative positions of the particular components along the hierarchy.

It is assumed that the n-components are arranged in the hierarchical order as $C_1, C_2, \cdots, C_n$. If C_i is the active component during an attack, let $p_i, p_{i,1}, \cdots, p_{i,j-1}$ and $p_{i,j}$ are the probabilities that C_i survives, C_i fails, but C_{i+1} survives etc., and $q_{i,n}$ be the probability that the components $C_i, C_{i+1}, \cdots, C_n$ fail in the same attack. Thus, we have, $p_i + p_{i,1} + \cdots + p_{i,n} + q_{i,n} = 1$ for $i = 1, 2, 3, \cdots, n$.

In the present, probability distribution of the number of attacks required for failure of a n-component hierarchical cascade system, as defined above, is investigated and the probability of the system surviving k attacks, sustained a loss of the first m components, is studied.

2. Notation and Explanations

$p_n(k)$: Probability that a n-component system fails in the kth attack.

$P_n(k)$: The corresponding probability distribution function $P_n(k) = \sum_{r=1}^{k} p_n(r)$.

$p_n(z)$: The corresponding probability generating function.

$R_{m,n}(k)$: The reliability of the n-component system surviving "k" attacks with a loss of "m" components.

$P_{m,n}(k)$: The probability of the n-component system surviving "k" attacks with a loss of "m" components.

$S_{i,j}$: The event that C_i survives "j" attacks.

$F_{i,j}$: The event that C_i fails at jth attacks.

$A_n(k)$: The event that the n-component system fails at the kth attack.

β: The serial number of attack at which C_{n-1} fails but C_n does not fail.

$A_n(k, \beta)$: The event that C_{n-1} fails at the βth attack and C_n fails at the kth attack. When $\beta < k$, C_n is attacked but survives the attacks number $\beta, \beta+1, \cdots, (k-1)$. When $\beta = k$, $A_n(k,k) = A_n(k) F_{n,k}$ *i.e.*, the component C_{n-1} fails in the kth attack and C_n also fails in the kth attack.

$B_{m,n}(k)$: The event that the system survives k attacks with a loss of the first m components.

π_n: The transition probability matrix for a n-component system. This is a $(n + 1)$th order matrix. It should be noted that the $(i + 1)$th row stands for the initial state s_i, $i = 0, 1, 2, \cdots, n$. Similarly, for the columns.

$R_{i,j}$: The (i, j)th element of π_n. Thus, $\pi_n = \begin{vmatrix} T_n & Q_n \\ O^l & 1 \end{vmatrix} = (p_{ij})$. The last row of this $(k+1)$th order matrix consists of zeros except for unity at the $(k+1)$th column.

$R_i(r)$: $p_i^{k+r-2} / \prod_{j \neq i=1}^{r} (p_i - p_j)$.

$p_{i,j}$: Probability that $C_i, C_{i+1}, \cdots, C_{j-1}$ fail and C_j survives during an attack; C_i is the first component facing this attack, components $C_{i-1}, C_{i-2}, \cdots, C_1$ having failed during earlier attacks, if any.

$q_{i,n}$: The probability that $C_i, C_{i+1}, \cdots, C_n$ fail at an attack when C_i is the first component facing this attack; components $C_{i-1}, C_{i-2}, \cdots, C_1$ having failed during earlier attacks, if any.

α_i: Serial number of attack at which C_i fails.

Obviously,

$$R_{ij} = \text{The } (i, j)\text{th element of } \pi_n = \begin{bmatrix} p_{i,j} & j < n+1 \\ q_{i,n} & j = n+1 \\ p_{i,i} = p_i & j = i \\ 0 & \text{otherwise} \end{bmatrix}$$

π_n = The trasition probability matrix for a n-component system

$$= \begin{bmatrix} p_1 & p_{1,2} & p_{1,3} & \cdots & p_{1,n} & q_{1,n} \\ 0 & p_2 & p_{2,3} & \cdots & p_{2,n} & q_{2,n} \\ 0 & 0 & p_3 & \cdots & p_{3,n} & q_{3,n} \\ - & - & - & - & - & q_n \\ - & - & - & - & - & - \\ 0 & 0 & 0 & & p_n & q_n \\ 0 & 0 & 0 & & 0 & 1 \end{bmatrix}$$

Let a system consist of three components C_1, C_2 and C_3. The system with the hierarchical ordering $C_1 C_2 C_3$ will have the transition probability matrix.

π_3 = The trasition probability matrix for a 3-component system

$$= \begin{bmatrix} p_1 & p_{1,2} & p_{1,3} & q_{1,3} \\ 0 & p_2 & p_{2,3} & q_{2,3} \\ 0 & 0 & p_3 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Reliability Evaluation

Reliability of a System for k Attacks

Here consider the probability that the system survives "k" attacks with a loss of the first "m" components. Let $R_{m,n}(k)$ is the reliability of the n-component system for "k" attacks with a loss of "m" components.

The Two Component System:

For $m = 0$, we get

$R_{0,2}(k)$ = Reliability of 2-component system for k attacks with loss of zero components = p_1^k

For $m = 1$, we have the corresponding event

$B_1(k)$ = The event that the system survives k attacks with loss of first component

$$= \bigcup_{\alpha=1}^{k} \left[S_{1,\alpha-1} \cap S_{(1,2),\alpha} \cap S_{2,k-\alpha} \right]$$

The corresponding probability

$R_{1,2}(k)$ = Reliability of 2 component system for k attacks with loss of one component

$$= p_{1,2} \left[p_1^k / (p_1 - p_2) + p_2^k / (p_2 - p_1) \right] = p_{1,2} \sum_{i=1}^{2} A_i(2)$$

here $A_i(2)$ means the event that ith component system fails at 2nd attack.

It is obvious that when $m = 2$

$P_{2,2}(k) = p_2(k)$ = Probability of 2-component system fails in the kth attack

$$= q_{1,2} p_1^{k-1} + p_{1,2} q_2 \left[p_1^k / (p_1 - p_2) + p_2^k / (p_2 - p_1) \right]$$

The Three Component System:

When $m = 0$, we get

$R_{0,3}(k)$ = Reliability of 3 component system for k attacks with loss of zero components = p_1^k

When $m = 1$, we have the corresponding event

$B_1(k)$ = The event that the system survives k attacks with loss of first component

$$= \bigcup_{\alpha=1}^{k} \left[S_{1,\alpha-1} \cap S_{(1,2),\alpha} \cap S_{3,k-\alpha} \right]$$

Hence

$R_{1,3}(k)$ = Reliability of 3 component system for k attacks with loss of one component

$$= p_{1,2}\left[p_1^k /(p_1 - p_2) + p_2^k /(p_2 - p_1) \right] = p_{1,2} \sum_{i=1}^{2} A_i(2),$$

here $A_i(2)$ means the event that ith component system fails at the 2nd attack.

When $m = 2$, the corresponding event

$B_2(k)$ = The event that the system survives k attacks with loss of first two component

$$= \bigcup_{\alpha=1}^{k}\bigcup_{\beta=\alpha}^{k} \left[S_{1,\alpha-1} \cap S_{(1,2),\alpha} \cap S_{3,\beta-\alpha} \cap S_{(2,3),\beta} \cap S_{3,k-\beta} \right] = \bigcup_{\alpha=1}^{k} \left[S_{1,\alpha-1} \cap S_{(1,3),\alpha} \cap S_{3,k-\alpha} \right]$$

Hence

$R_{2,3}(k)$ = Reliability of 3 component system for k attacks with loss of two components

$$= p_{1,3}\left[p_1^k /(p_1 - p_3) + p_3^k /(p_3 - p_1) + p_{1,2}p_{2,3}\sum_{i=1}^{3}\left[p_i^{k+1} / \prod_{j\neq i=1}^{3}(p_i - p_j) \right] \right]$$

It is obvious that

$p_{3,3}(k) = p_3(k)$ = Probability of 3-component system fails in the kth attack

$$= q_{1,3}p_1^k + p_{1,2}q_{2,3}\left[p_1^k /(p_1 - p_2) + p_2^k /(p_2 - p_1) \right]$$

$$+ p_{1,3}q_3\left[p_1^k /(p_1 - p_3) + p_3^k /(p_3 - p_1) \right] + p_{1,2}p_{2,3}q_3\sum_{i=1}^{3}\left[p_i^{k+1} / \prod_{j\neq i=1}^{3}(p_i - p_j) \right]$$

The Four Component System:

When $m = 0$, we get

$R_{0,4}(k)$ = Reliability of 4 component system for k attacks with loss of zero components = p_1^k

Similarly, for $m = 1$

$R_{1,4}(k)$ = Reliability of 4 component system for k attacks with loss of one component

$$= p_{1,2}\sum_{i=1}^{2} A_i(2)$$

For $m = 2$

$R_{2,4}(k)$ = Reliability of 4 component system for k attacks with loss of two components

$$= p_{1,3}\left[p_1^k /(p_1 - p_3) + p_3^k /(p_3 - p_1) + p_{1,2}p_{2,3}\sum_{i=1}^{3}\left[p_i^{k+1} / \prod_{j\neq i=1}^{3}(p_i - p_j) \right] \right]$$

For $m = 3$

$R_{3,4}(k)$ = Reliability of 4 component system for k attacks with loss of three components

$$= p_{1,4}\left[p_1^k /(p_1 - p_4) + p_4^k /(p_4 - p_1) + p_{1,2}p_{2,4}\sum_{i=1,2,4}\left[p_i^{k+1} / \prod_{j\neq i=1}^{4}(p_i - p_j) \right] \right.$$

$$\left. + p_{1,2}p_{2,3}p_{3,4}\sum_{i=1}^{4}\left[p_i^{k+1} / \prod_{j\neq i=1}^{4}(p_i - p_j) \right] \right]$$

For $m = 4$

$$p_{4,4}(k) = p_4(k) = \text{Probability of 4-component system fails in the } k\text{th attack}$$

$$= q_{1,4}p_1^k + p_{1,2}q_{2,4}\left[\frac{p_1^k}{(p_1 - p_2)} + \frac{p_2^k}{(p_2 - p_1)}\right] + p_{1,3}q_{3,4}\left[\frac{p_1^k}{(p_1 - p_3)} + \frac{p_3^k}{(p_3 - p_1)}\right]$$

$$+ p_{1,4}q_4\left[\frac{p_1^k}{(p_1 - p_4)} + \frac{p_4^k}{(p_4 - p_1)}\right] + p_{1,2}p_{2,3}q_{3,4}\sum_{i=1}^{4}\left[\frac{p_i^k}{\prod_{j\neq i=1}^{4}(p_i - p_j)}\right]$$

$$+ p_{1,2}p_{2,4}q_4 \sum_{i=1,2,4}\frac{p_i^k}{\prod_{j\neq i=1,2,4}(p_i - p_j)} + p_{1,2}p_{2,3}p_{3,4}q_4\sum_{i=1}^{4}\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^{4}(p_i - p_j)}\right]$$

The Five Component System:

For $m = 0$, we get

$$R_{0,5}(k) = \text{Reliability of 5 component system for } k \text{ attacks with loss of zero components} = p_1^k$$

For $m = 1$, the corresponding probability

$$R_{1,5}(k) = \text{Reliability of 5 component system for } k \text{ attacks with loss of one component}$$

$$= p_{1,2}\left[\frac{p_1^k}{(p_1 - p_2)} + \frac{p_2^k}{(p_2 - p_1)}\right] = p_{1,2}\sum_{i=1}^{2}A_i(2)$$

For $m = 2$,

$$R_{2,5}(k) = \text{Reliability of 5 component system for } k \text{ attacks with loss of two components}$$

$$= p_{1,3}\left[\frac{p_1^k}{(p_1 - p_3)} + \frac{p_3^k}{(p_3 - p_1)}\right] + p_{1,2}p_{2,3}\sum_{i=1}^{3}\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^{3}(p_i - p_j)}\right]$$

For $m = 3$,

$$R_{3,5}(k) = \text{Reliability of 5 component system for } k \text{ attacks with loss of three components}$$

$$= p_{1,4}\left[\frac{p_1^k}{(p_1 - p_4)} + \frac{p_4^k}{(p_4 - p_1)}\right] + p_{1,2}p_{2,4}\sum_{i=1,2,4}\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^{4}(p_i - p_j)}\right]$$

$$+ p_{1,2}p_{2,3}p_{3,4}\sum_{i=1}^{4}\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^{4}(p_i - p_j)}\right]$$

For $m = 4$,

$$R_{4,5}(k) = \text{Reliability of 5 component system for } k \text{ attacks with loss of four components}$$

$$= p_{1,5}\left[\frac{p_1^k}{(p_1 - p_4)} + \frac{p_4^k}{(p_4 - p_1)}\right] + p_{1,2}p_{2,5}\sum_{i=1,2,5}\left[\frac{p_i^{k+1}}{\prod_{i=1,2,5}(p_i - p_j)}\right]p_{1,3}p_{3,5}\sum_{i=1,3,5}\left[\frac{p_i^{k+1}}{\prod_{i=1,3,5}(p_i - p_j)}\right]$$

$$+ p_{1,2}p_{2,3}p_{3,4}p_{4,5}\sum_{i=1}^{5}\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^{5}(p_i - p_j)}\right]$$

For $m = 5$,

$p_{5,5}(k) = p_5(k) =$ Probability of 5-component system fails in the kth attack

$$= q_{1,5}p_1^k + p_{1,2}q_{2,5}\left[\frac{p_1^k}{(p_1-p_2)} + \frac{p_2^k}{(p_2-p_1)}\right] + p_{1,3}q_{3,5}\left[\frac{p_1^k}{(p_1-p_3)} + \frac{p_3^k}{(p_3-p_1)}\right]$$

$$+ p_{1,4}q_{4,5}\left[\frac{p_1^k}{(p_1-p_4)} + \frac{p_4^k}{(p_4-p_1)}\right] + p_{1,5}q_5\left[\frac{p_1^k}{(p_1-p_5)} + \frac{p_5^k}{(p_5-p_1)}\right]$$

$$+ p_{1,2}p_{2,3}q_{3,4}\sum_{i=1,2,3}\left[\frac{p_i^k}{\prod_{j\neq i=1}^3(p_i-p_j)}\right] + p_{1,3}p_{3,4}q_{4,5}\sum_{i=1,3,4}\left[\frac{p_i^k}{\prod_{j\neq i=1,3,4}(p_i-p_j)}\right]$$

$$+ p_{1,2}p_{2,5}q_5\sum_{i=1,2,5}\left[\frac{p_i^k}{\prod_{j\neq i=1,2,5}(p_i-p_j)}\right] + p_{1,3}p_{3,5}q_5\sum_{i=1,3,5}\left[\frac{p_i^k}{\prod_{j\neq i=1,3,5}(p_i-p_j)}\right]$$

$$+ p_{1,4}p_{4,5}q_5\sum_{i=1,4,5}\left[\frac{p_i^k}{\prod_{j\neq i=1,4,5}(p_i-p_j)}\right] + p_{1,2}p_{2,3}p_{3,4}p_{4,5}q_5\sum_{i=1}^5\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^5(p_i-p_j)}\right]$$

The Six Component System:

For $m = 0$, we get

$R_{0,6}(k) =$ Reliability of 6 component system for k attacks with loss of zero components $= p_1^k$

For $m = 1$, the corresponding probability

$R_{1,6}(k) =$ Reliability of 6 component system for k attacks with loss of one component

$$= p_{1,2}\left[\frac{p_1^k}{(p_1-p_2)} + \frac{p_2^k}{(p_2-p_1)}\right] = p_{1,2}\sum_{i=1}^2 A_i(2)$$

For $m = 2$,

$R_{2,6}(k) =$ Reliability of 6 component system for k attacks with loss of two components

$$= p_{1,3}\left[\frac{p_1^k}{(p_1-p_3)} + \frac{p_3^k}{(p_3-p_1)}\right] + p_{1,2}p_{2,3}\sum_{i=1}^3\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^3(p_i-p_j)}\right]$$

For $m = 3$,

$R_{3,6}(k) =$ Reliability of 6 component system for k attacks with loss of three components

$$= p_{1,4}\left[\frac{p_1^k}{(p_1-p_4)} + \frac{p_4^k}{(p_4-p_1)}\right] + p_{1,2}p_{2,4}\sum_{i=1,2,4}\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^4(p_i-p_j)}\right]$$

$$+ p_{1,2}p_{2,3}p_{3,4}\sum_{i=1}^4\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^4(p_i-p_j)}\right]$$

For $m = 4$,

$R_{4,6}(n) =$ Reliability of 6 component system for k attacks with loss of four components

$$= p_{1,5}\left[\frac{p_1^k}{(p_1-p_4)} + \frac{p_4^k}{(p_4-p_1)}\right] + p_{1,2}p_{2,5}\sum_{i=1,2,5}\left[\frac{p_i^{k+1}}{\prod_{i=1,2,5}(p_i-p_j)}\right]$$

$$+ p_{1,3}p_{3,5}\sum_{i=1,3,5}\left[\frac{p_i^{k+1}}{\prod_{i=1,3,5}(p_i-p_j)}\right] + p_{1,2}p_{2,3}p_{3,4}p_{4,5}\sum_{i=1}^5\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^5(p_i-p_j)}\right]$$

For $m = 5$,

$R_{5,6}(k) = $ Reliability of 6 component system for k attacks with loss of five components

$$= p_{1,6}\left[\frac{p_1^k}{(p_1-p_4)} + \frac{p_4^k}{(p_4-p_1)}\right] + p_{1,2}p_{2,6}\sum_{i=1,2,6}\left[\frac{p_i^{k+1}}{\prod_{i=1,2,6}(p_i-p_j)}\right]$$

$$+ p_{1,3}p_{3,6}\sum_{i=1,3,5}\left[\frac{p_i^{k+1}}{\prod_{i=1,3,5}(p_i-p_j)}\right] + p_{1,2}p_{2,3}p_{3,4}p_{4,6}\sum_{i=1,2,3,4,6}\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1,2,3,4,6}(p_i-p_j)}\right]$$

$$+ p_{1,2}p_{2,3}p_{3,4}p_{4,5}p_{5,6}\sum_{i=1}^{6}\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^{6}(p_i-p_j)}\right]$$

For $m = 6$,

$p_{6,6}(k) = p_6(k) = $ Probability of 6-component system fails in the kth attack

$$= q_{1,6}p_1^k + p_{1,2}q_{2,6}\left[\frac{p_1^k}{(p_1-p_2)} + \frac{p_2^k}{(p_2-p_1)}\right] + p_{1,3}q_{3,6}\left[\frac{p_1^k}{(p_1-p_3)} + \frac{p_3^k}{(p_3-p_1)}\right]$$

$$+ p_{1,4}q_{4,6}\left[\frac{p_1^k}{(p_1-p_4)} + \frac{p_4^k}{(p_4-p_1)}\right] + p_{1,5}q_{5,6}\left[\frac{p_1^k}{(p_1-p_5)} + \frac{p_5^k}{(p_5-p_1)}\right]$$

$$+ p_{1,5}q_6\left[\frac{p_1^k}{(p_1-p_5)} + \frac{p_5^k}{(p_5-p_1)}\right] + p_{1,2}p_{2,3}q_{3,4}\sum_{i=1,2,3}\left[\frac{p_i^k}{\prod_{j\neq i=1,2,3}(p_i-p_j)}\right]$$

$$+ p_{1,3}p_{3,4}q_{4,5}\sum_{i=1,3,4}\left[\frac{p_i^k}{\prod_{j\neq i=1,3,4}(p_i-p_j)}\right] + p_{1,4}p_{4,5}q_{5,6}\sum_{i=1,4,5}\left[\frac{p_i^k}{\prod_{j\neq i=1,4,5}(p_i-p_j)}\right]$$

$$+ p_{1,2}p_{2,6}q_6\sum_{i=1,2,6}\left[\frac{p_i^k}{\prod_{j\neq i=1,2,6}(p_i-p_j)}\right] + p_{1,3}p_{3,6}q_6\sum_{i=1,3,6}\left[\frac{p_i^k}{\prod_{j\neq i=1,3,6}(p_i-p_j)}\right]$$

$$+ p_{1,4}p_{4,6}q_6\sum_{i=1,4,6}\left[\frac{p_i^k}{\prod_{j\neq i=1,4,6}(p_i-p_j)}\right] + p_{1,5}p_{5,6}q_6\sum_{i=1,5,6}\left[\frac{p_i^k}{\prod_{j\neq i=1,5,6}(p_i-p_j)}\right]$$

$$+ p_{1,2}p_{2,3}p_{3,4}q_{4,5}\sum_{i=1}^{5}\left[\frac{p_i^k}{\prod_{j\neq i=1}^{5}(p_i-p_j)}\right] + p_{1,2}p_{2,3}p_{3,4}q_{5,6}\sum_{i=1}^{6}\left[\frac{p_i^k}{\prod_{j\neq i=1}^{6}(p_i-p_j)}\right]$$

$$+ p_{1,2}p_{2,3}p_{3,4}q_{5,6}q_6\sum_{i=1}^{6}\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^{6}(p_i-p_j)}\right]$$

The general equation for probability of n-component system fails in the kth attack

$p_{n,n}(k) = p_n(k) = $ Probability of n-component system fails in the kth attack

$$= q_{1,n}p_1^k + \sum_{r=2}^{n}p_{1,r}q_{r,n}\left[\frac{p_1^k}{(p_1-p_r)} + \frac{p_r^k}{(p_r-p_1)}\right] + \sum_{r=2,s=r+1}^{n}p_{1,r}p_{r,s}q_{s,n}\sum_{i=1,r,s}\left[\frac{p_i^k}{\prod_{j\neq i=1,r,s}(p_i-p_j)}\right]$$

$$+ \sum_{r=2,}^{n}p_{1,r}p_{r,n}q_{n,n}\sum_{i=1,r,n}\left[\frac{p_i^k}{\prod_{j\neq i=1,r,n}(p_i-p_j)}\right] + \cdots + p_{1,r}p_{r,r+1}p_{r+1,r+2}\cdots p_{n-1,n}q_{n,n}\sum_{i=1}^{n}\left[\frac{p_i^{k+1}}{\prod_{j\neq i=1}^{n}(p_i-p_j)}\right]$$

The general equation for reliability of n-component system for kth attack

$$R_{n,n}(k) = 1 - p_n(k)$$

$$= 1 - \left[q_{1,n} p_1^k + \sum_{r=2}^{n} p_{1,r} q_{r,n} \left[\frac{p_1^k}{(p_1 - p_r)} + \frac{p_r^k}{(p_r - p_1)} \right] \right.$$

$$+ \sum_{r=2,s=r+1}^{n} p_{1,r} p_{r,s} q_{s,n} \sum_{i=1,r,s} \left[\frac{p_i^k}{\prod_{j \neq i=1,r,s}(p_i - p_j)} \right] + \sum_{r=2,}^{n} p_{1,r} p_{r,n} q_{n,n} \sum_{i=1,r,n} \left[\frac{p_i^k}{\prod_{j \neq i=1,r,n}(p_i - p_j)} \right]$$

$$+ \cdots + p_{1,r} p_{r,r+1} p_{r+1,r+2} \cdots p_{n-1,n} q_{n,n} \sum_{i=1}^{n} \left[\frac{p_i^{k+1}}{\prod_{j \neq i=1}^{n}(p_i - p_j)} \right] \right]$$

4. Example

Let us consider a transition probability matrix of order 7 for 6-component system

$$\pi_6 = \begin{bmatrix} 0.40 & 023 & 0.18 & 0.10 & 0.05 & 0.03 & 0.01 \\ 0 & 0.35 & 0.20 & 0.18 & 0.12 & 0.10 & 0.05 \\ 0 & 0 & 0.30 & 0.22 & 0.20 & 0.15 & 0.13 \\ 0 & 0 & 0 & 0.33 & 0.24 & 0.22 & 0.21 \\ 0 & 0 & 0 & 0 & 0.45 & 0.30 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 0.55 & 0.45 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.40 & 023 & 0.18 & 0.10 & 0.05 & 0.03 \\ 0 & 0.35 & 0.20 & 0.18 & 0.12 & 0.10 \\ 0 & 0 & 0.30 & 0.22 & 0.20 & 0.15 \\ 0 & 0 & 0 & 0.33 & 0.24 & 0.22 \\ 0 & 0 & 0 & 0 & 0.45 & 0.30 \\ 0 & 0 & 0 & 0 & 0 & 0.55 \end{bmatrix} \begin{bmatrix} 0.01 \\ 0.05 \\ 0.13 \\ 0.21 \\ 0.25 \\ 0.45 \end{bmatrix} \begin{bmatrix} 0.086 \\ 0.156 \\ 0.203 \\ 0.228 \\ 0.258 \\ 0.248 \end{bmatrix} \begin{bmatrix} 0.150 \\ 0.192 \\ 0.199 \\ 0.192 \\ 0.190 \\ 0.136 \end{bmatrix} \begin{bmatrix} 0.173 \\ 0.178 \\ 0.160 \\ 0.139 \\ 0.124 \\ 0.075 \end{bmatrix} \begin{bmatrix} 0.1613 \\ 0.1417 \\ 0.1146 \\ 0.0919 \\ 0.0756 \\ 0.0411 \end{bmatrix} \begin{bmatrix} 0.1319 \\ 0.1022 \\ 0.0759 \\ 0.0575 \\ 0.0446 \\ 0.0226 \end{bmatrix} \begin{bmatrix} 0.0986 \\ 0.0689 \\ 0.0477 \\ 0.0347 \\ 0.0258 \\ 0.0125 \end{bmatrix}$$

$$= \begin{bmatrix} 0.40 & 023 & 0.18 & 0.10 & 0.05 \\ 0 & 0.35 & 0.20 & 0.18 & 0.12 \\ 0 & 0 & 0.30 & 0.22 & 0.20 \\ 0 & 0 & 0 & 0.33 & 0.24 \\ 0 & 0 & 0 & 0 & 0.45 \end{bmatrix} \begin{bmatrix} 0.03 \\ 0.10 \\ 0.15 \\ 0.22 \\ 0.30 \end{bmatrix} \begin{bmatrix} 0.1015 \\ 0.147 \\ 0.1634 \\ 0.1566 \\ 0.14 \end{bmatrix} \begin{bmatrix} 0.1264 \\ 0.1291 \\ 0.1115 \\ 0.0853 \\ 0.056 \end{bmatrix} \begin{bmatrix} 0.1116 \\ 0.0896 \\ 0.0645 \\ 0.0416 \\ 0.0224 \end{bmatrix} \begin{bmatrix} 0.0829 \\ 0.0522 \\ 0.0329 \\ 0.0191 \\ 0.009 \end{bmatrix} \begin{bmatrix} 0.0535 \\ 0.0300 \\ 0.0159 \\ 0.0085 \\ 0.0036 \end{bmatrix} \begin{bmatrix} 0.0322 \\ 0.0156 \\ 0.0073 \\ 0.0036 \\ 0.0014 \end{bmatrix}$$

$$= \begin{bmatrix} 0.40 & 023 & 0.18 & 0.10 \\ 0 & 0.35 & 0.20 & 0.18 \\ 0 & 0 & 0.30 & 0.22 \\ 0 & 0 & 0 & 0.33 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.12 \\ 0.20 \\ 0.24 \end{bmatrix} \begin{bmatrix} 0.1076 \\ 0.1252 \\ 0.1128 \\ 0.0792 \end{bmatrix} \begin{bmatrix} 0.1001 \\ 0.0806 \\ 0.0513 \\ 0.0261 \end{bmatrix} \begin{bmatrix} 0.0704 \\ 0.0432 \\ 0.0211 \\ 0.0086 \end{bmatrix} \begin{bmatrix} 0.0428 \\ 0.0209 \\ 0.0082 \\ 0.0028 \end{bmatrix} \begin{bmatrix} 0.0237 \\ 0.0095 \\ 0.0031 \\ 0.0009 \end{bmatrix} \begin{bmatrix} 0.0123 \\ 0.0041 \\ 0.0011 \\ 0.0003 \end{bmatrix}$$

$$= \begin{bmatrix} 0.40 & 023 & 0.18 \\ 0 & 0.35 & 0.20 \\ 0 & 0 & 0.30 \end{bmatrix} \begin{bmatrix} 0.10 \\ 0.18 \\ 0.22 \end{bmatrix} \begin{bmatrix} 0.121 \\ 0.107 \\ 0.066 \end{bmatrix} \begin{bmatrix} 0.0849 \\ 0.0507 \\ 0.0198 \end{bmatrix} \begin{bmatrix} 0.0492 \\ 0.217 \\ 0.0059 \end{bmatrix} \begin{bmatrix} 0.0257 \\ 0.0088 \\ 0.0018 \end{bmatrix} \begin{bmatrix} 0.0126 \\ 0.0034 \\ 0.0005 \end{bmatrix} \begin{bmatrix} 0.0126 \\ 0.0034 \\ 0.0005 \end{bmatrix} \begin{bmatrix} 0.059 \\ 0.0013 \\ 0.0002 \end{bmatrix}$$

$$= \begin{bmatrix} 0.40 & 023 \\ 0 & 0.35 \end{bmatrix} \begin{bmatrix} 0.18 \\ 0.2 \end{bmatrix} \begin{bmatrix} 0.118 \\ 0.07 \end{bmatrix} \begin{bmatrix} 0.0633 \\ 0.0245 \end{bmatrix} \begin{bmatrix} 0.0309 \\ 0.0086 \end{bmatrix} \begin{bmatrix} 0.0144 \\ 0.0030 \end{bmatrix} \begin{bmatrix} 0.0064 \\ 0.0010 \end{bmatrix} \begin{bmatrix} 0.0028 \\ 0.0004 \end{bmatrix}$$

The first element in the ith column matrix represents the probability of failure of the system at the end of ith attack $i.e.$, $p_k(i)$ for $k = 2, 3, 4$ and 5, and $i = 1, 2, 3, 4$ and 5.

Let us find the reliability of above system in 2 number of attacks

$$p_{6,6}(2) = p_6(2) = \text{Probability of 6-component system fails in the 2th attack}$$

$$= q_{1,6} p_1^2 + p_{1,2} q_{2,6} \left[\frac{p_1^2}{(p_1 - p_2)} + \frac{p_2^2}{(p_2 - p_1)} \right] + p_{1,3} q_{3,6} \left[\frac{p_1^2}{(p_1 - p_3)} + \frac{p_3^2}{(p_3 - p_1)} \right]$$

$$+ p_{1,4} q_{4,6} \left[\frac{p_1^2}{(p_1 - p_4)} + \frac{p_4^2}{(p_4 - p_1)} \right] + p_{1,5} q_{5,6} \left[\frac{p_1^2}{(p_1 - p_5)} + \frac{p_5^2}{(p_5 - p_1)} \right]$$

$$+ p_{1,5} q_6 \left[\frac{p_1^2}{(p_1 - p_5)} + \frac{p_5^2}{(p_5 - p_1)} \right] + p_{1,2} p_{2,3} q_{3,6} \sum_{i=1,2,3} \left[\frac{p_i^k}{\prod_{j \neq i=1,2,3}(p_i - p_j)} \right]$$

$$+ p_{1,3} p_{3,4} q_{4,6} \sum_{i=1,3,4} \left[\frac{p_i^2}{\prod_{j \neq i=1,3,4}(p_i - p_j)} \right] + p_{1,4} p_{4,5} q_{5,6} \sum_{i=1,4,5} \left[\frac{p_i^2}{\prod_{j \neq i=1,4,5}(p_i - p_j)} \right]$$

$$+ p_{1,2} p_{2,6} q_6 \sum_{i=1,2,6} \left[\frac{p_i^2}{\prod_{j \neq i=1,2,6}(p_i - p_j)} \right] + p_{1,3} p_{3,6} q_6 \sum_{i=1,3,6} \left[\frac{p_i^2}{\prod_{j \neq i=1,3,6}(p_i - p_j)} \right]$$

$$+ p_{1,4} p_{4,6} q_6 \sum_{i=1,4,6} \left[\frac{p_i^2}{\prod_{j \neq i=1,4,6}(p_i - p_j)} \right] + p_{1,5} p_{5,6} q_6 \sum_{i=1,5,6} \left[\frac{p_i^2}{\prod_{j \neq i=1,5,6}(p_i - p_j)} \right]$$

$$+ p_{1,2} p_{2,3} p_{3,4} q_{4,6} \sum_{i=1}^{5} \left[\frac{p_i^2}{\prod_{j \neq i=1}^{5}(p_i - p_j)} \right] + p_{1,2} p_{2,3} p_{3,4} q_{5,6} \sum_{i=1}^{6} \left[\frac{p_i^2}{\prod_{j \neq i=1}^{6}(p_i - p_j)} \right]$$

$$+ p_{1,2} p_{2,3} p_{3,4} q_{5,6} q_6 \sum_{i=1}^{6} \left[\frac{p_i^3}{\prod_{j \neq i=1}^{6}(p_i - p_j)} \right]$$

$$= 0.0016 + 0.0086 + 0.01638 + 0.01533 + 0.0106 + 0.0115 + 0.0059 + 0.00832 + 0.0060$$

$$+ 0.01035 + 0.01215 + 0.01434 + 0.00990 + 0.000000212 + 0.0000001 + 0.0000002$$

$$p_6(2) = 0.1310$$

$$\therefore \text{Reliability of 6-cascade system in 2 number of attacks} = 1 - p_6(2) = 0.8689$$

5. Conclusion

The present work deals with the cascade reliability model represented as Markovian model. Reliability of stress strength model is derived with Markovian Approach. In this paper, the reliability of n-cascade system for k attacks with loss of m components has been derived for $n > 4$ and the general formula for reliability of n cascade system for k number of attacks has been derived. Using above general equation reliability has been calculated numerically for 6-cascade system for 2-number of attacks.

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