Journal of Applied Mathematics and Physics, 2015, 3, 761-765
Published Online July 2015 in SciRes. http://www.scirp.org/journal/jamp
http://dx.doi.org/10.4236/jamp.2015.37092

Perturbation Solutions for Annular Flow of Small Gap

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Received 1 April 2015; accepted 23 June 2015; published 30 June 2015

Abstract

The perturbation method is used to solve the control equations of a three-dimensional annular flow inside a small gap. The nonlinear equations are separated into zeroth-order and first-order perturbation equations. The velocity and pressure distributions are solved successively by different numerical methods with the zeroth-order and first-order equation. Agreement in results is found with the present method and software ANSYS-CFX, which illustrates the applicability of perturbation method in solving complicated flow field inside small gaps.

Keywords

Perturbation, Annular Flow, Gap, Velocity, Pressure

1. Introduction

The annular flow inside small gaps between rotors and stators can be found in many fluid circumstances such as sliding bearings, radial dynamic pressure seals, submersible pumps and nuclear pumps. The study of dynamical effects related to gap fluid field has been one of the research hotspots of fluid mechanics. Fritz [1] investigated the dynamics model of an annular flow with large gap and simplified it to a two-dimensional incompressible fluid flow field. However, this model suffered from a disadvantage that it ignored the impact of the axial flow of the fluid. Hirs [2] proposed the theory of the turbulence overall flow which was applied to investigate the three-dimensional turbulent flow field of a seal ring by Childs [3]. Nelson [4] believed Moody wall friction coefficient equations [5] were more reasonable than Hirs wall friction coefficient equations in the study of the annular flow with gap. Antunes [6] studied the static and dynamic characteristics of an annular eccentric rotor with large gap based on the overall flow theory and Hirs wall friction coefficient equation. Moody wall friction coefficient equations are widely applied in the study of the dynamics of the seal ring subjected to the radial pressure. Sun [7] proposed the dynamic modeling of a 3D annular flow with large gap using the overall flow.

In this study, the perturbation method is used to solve three-dimensional control equations of an annular fluid flow inside a small gap that separates a rotating shaft and a fixed stator. The equations are expanded into zeroth-order and first-order perturbation equations of small eccentricity. The velocity and pressure distribution for the flow domain are solved successively with the zeroth-order and first-order equations by difference methods.

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How to cite this paper: Cao, X.J. and Wang, Y.F. (2015) Perturbation Solutions for Annular Flow of Small Gap. *Journal of Applied Mathematics and Physics*, **3**, 761-765. http://dx.doi.org/10.4236/jamp.2015.37092

2. Control Equations for Flow

Consider a small-sized control volume, *CV*, shown in **Figure 1**, across a fluctuating gap thickness, with the whole boundary *CS*. The continuity equation and the momentum equation of three-dimensional annular flow of a small gap can be written as:

$$\partial\left(\iiint_{CV}\rho dV\right)\Big/\partial t+\iint_{CS}\rho\mathbf{v}\cdot\boldsymbol{n}\mathrm{d}\varphi=0 \tag{1}$$

$$\partial\left(\iiint_{CV}\mathbf{v}\rho dV\right)\Big/\partial t+\iint_{CS}\mathbf{v}\rho\mathbf{v}\cdot\boldsymbol{n}\mathrm{d}\varphi=\sum_i\boldsymbol{F}_i \tag{2}$$

where ρ is fluid volume mass, $\mathbf{v}$ is the velocity of the fluid crossing *CS*, $\boldsymbol{n}$ is the local unit vector normal to the boundary. $\boldsymbol{F}_i$ is the external and volume force acting on the fluid in *CV*.

The control equations can be expressed based on overall flow theory and Moody's wall friction coefficient equations [5]:

$$\partial h/\partial t+r^{-1}\partial(hu_\theta)/\partial\theta+\partial(hu_z)/\partial z=0 \tag{3}$$

$$\begin{aligned}-h\rho^{-1}\partial p/\partial z&=u_z\sqrt{u_z^2+u_\theta^2}\,f_s\big/2+u_z\sqrt{u_z^2+(u_\theta-r\omega)^2}\,f_r\big/2\\&\quad+\partial(hu_z)/\partial t+r^{-1}\partial(hu_zu_\theta)/\partial\theta+\partial(hu_z^2)/\partial z\end{aligned} \tag{4}$$

$$\begin{aligned}-h(\rho r)^{-1}\partial p/\partial\theta&=u_\theta\sqrt{u_z^2+u_\theta^2}\,f_s\big/2+(u_\theta-r\omega)\sqrt{u_z^2+(u_\theta-r\omega)^2}\,f_r\big/2\\&\quad+\partial(hu_\theta)/\partial t+r^{-1}\partial(hu_\theta{}^2)/\partial\theta+\partial(hu_zu_\theta)/\partial z\end{aligned} \tag{5}$$

3. Perturbation Expression

For nonlinear ordinary and partial differential equations, perturbation methods can be used to quantify the change in solution with respect to unperturbed linear systems due to tiny disturbance to parameters. These methods have been used and developed in various fields with different backgrounds [8]-[10].

The eccentricity of the rotor shaft is defined as the small disturbance in this study. The thickness of the gap, the pressure and velocity of the annular flow are assumed to be:

$$h=h_0+\varepsilon h_1,\ p=p_0+\varepsilon p_1,u_z=u_{z0}+\varepsilon u_{z1},u_\theta=u_{\theta0}+\varepsilon u_{\theta1} \tag{6}$$

where ε is the small parameter, being small eccentricity of the shaft centroid in this study.

Equations (3) through (5) are separated in terms of order of ε, giving zeroth-order and first-order perturbation equations. The zeroth-order perturbation equations are as follows:

$$\partial(h_0u_{z0})/\partial z=0 \tag{7}$$

$$-h_0\rho^{-1}\partial p_0/\partial z=u_{z0}\sqrt{u_{z0}^2+u_{\theta0}^2}\,f_{s0}\big/2+u_{z0}\sqrt{u_{z0}^2+(u_{\theta0}-r\omega)^2}\,f_{r0}\big/2 \tag{8}$$

$$0=u_{\theta0}\sqrt{u_{z0}^2+u_{\theta0}^2}\,f_{s0}\big/2+(u_{\theta0}-r\omega)\sqrt{u_{z0}^2+(u_{\theta0}-r\omega)^2}\,f_{r0}\big/2+h_0u_{z0}\partial(u_{\theta0})/2 \tag{9}$$

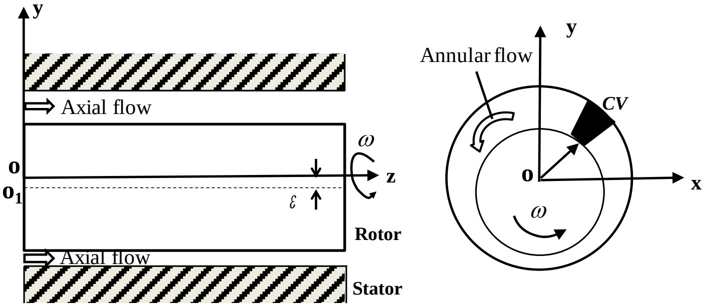


Figure 1. Geometry of the annular flow of gap.

The first-order perturbation equations are:

$$\partial h_1/\partial t + h_0 r^{-1}\partial u_{\theta 1}/\partial\theta + u_{\theta 0} r^{-1}\partial h_1/\partial\theta + h_0\partial u_{z1}/\partial z + u_{z0}\partial h_1/\partial z = 0 \tag{10}$$

$$\begin{aligned} -\rho^{-1}(h_1\partial p_0 + h_0\partial p_1)/\partial z = {} & r^{-1}\partial(h_1 u_{z0}u_{\theta 0} + h_0 u_{z1}u_{\theta 0} + h_0 u_{z0}u_{\theta 1})/\partial\theta + \partial(h_1 u_{z0}^2 + 2h_0 u_{z0}u_{z1})/\partial z \\ & + c_1 u_a u_{z0}(-c_2 e_s h_1/h_0^{\,2} - c_3\mu(h_1 u_a + h_0 u_c/u_a)/\rho h_0^{\,2} u_a^{\,2})/6(f_{s0}/c_1 - 1)^2 \\ & + c_1 u_b u_{z0}(-c_2 e_r h_1/h_0^{\,2} - c_3\mu(h_1 u_b + h_0 u_d/u_b)/\rho h_0^{\,2} u_a^{\,2})/6(f_{r0}/c_1 - 1)^2 \\ & + f_{s0}(u_{z1}u_a + u_{z0}u_c/u_a)/2 + f_{r0}(u_{z1}u_b + u_{z0}u_d/u_b)/2 + \partial(h_0 u_{z1} + h_1 u_{z0})/\partial t \end{aligned} \tag{11}$$

$$\begin{aligned} -h_0(\rho r)^{-1}\partial p_1/\partial\theta = {} & \partial(h_0 u_{\theta 1} + h_1 u_{\theta 0})/\partial t + r^{-1}\partial(h_1 u_{\theta 0}^2 + 2h_0 u_{\theta 1}u_{\theta 0})/\partial\theta + \partial(h_0 u_{z0}u_{\theta 1} + h_0 u_{z1}u_{\theta 0} + h_1 u_{z0}u_{\theta 0})/\partial z \\ & + c_1 u_a u_{\theta 0}(-c_2 e_s h_1/h_0^{\,2} - c_3\mu(h_1 u_a + h_0 u_c/u_a)/\rho h_0^{\,2} u_a^{\,2})/6(f_{s0}/c_1 - 1)^2 \\ & + c_1 u_b(u_{\theta 0} - r\omega)(-c_2 e_r h_1/h_0^{\,2} - c_3\mu(h_1 u_b + h_0 u_d/u_b)/\rho h_0^{\,2} u_b^{\,2})/6(f_{r0}/c_1 - 1)^2 \\ & + f_{s0}(u_{\theta 1}u_a + u_{\theta 0}u_c/u_a)/2 + f_{r0}(u_{\theta 1}u_b + (u_{\theta 0} - r\omega)u_d/u_b)/2 \end{aligned} \tag{12}$$

where

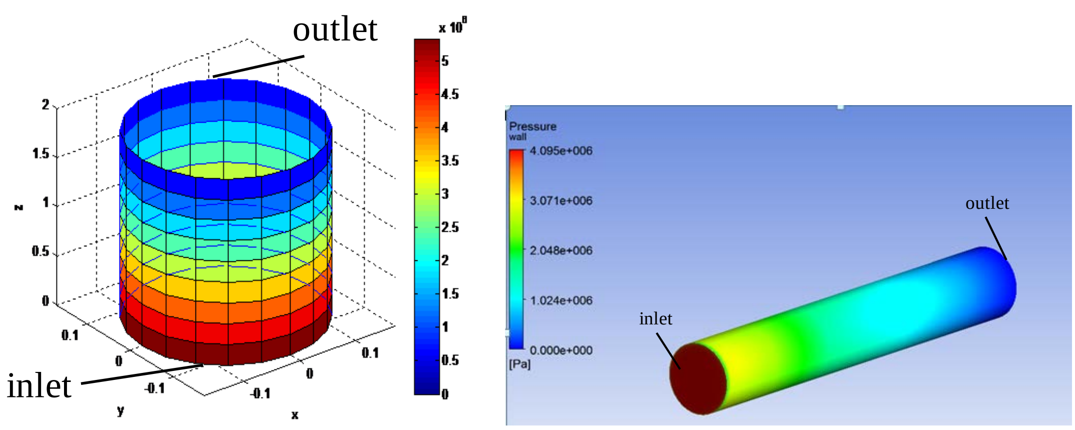
$$\begin{aligned} & u_a = \sqrt{(u_{z0}^2 + u_{\theta 0}^2)}, \quad u_b = \sqrt{(u_{z0}^2 + (u_{\theta 1} - r\omega)^2)} \\ & u_c = u_{z0}u_{z1} + u_{\theta 0}u_{\theta 1}, \quad u_d = u_{z0}u_{z1} + (u_{\theta 0} - r\omega)u_{\theta 1} \end{aligned} \tag{13}$$

The difference method is used to discretize the zeroth-order perturbation equations. The nonlinear partial differential equations are transformed into algebraic equations. The zeroth-order solution is substituted into the first-order perturbation equations to obtain the solution of the first order. The results of Equations (3) through (5) are the addition of zeroth-order perturbation solutions to Equations (7) through (9) and the first-order perturbation solutions to Equations (10) through (12).

4. Results of the Flow Field

Numerical simulations using software ANSYS-CFX [11] are carried out to verify the perturbation solution obtained from the present method. An example is illustrated with boundary conditions of a 5 m/s velocity at inlet and a zero pressure at outlet. The results of pressure and circumferential velocity obtained through the perturbation equations are compared with those obtained through ANSYS-CFX, as illustrated in **Figure 2** and **Figure 3**.

The results displayed in **Figure 2(a)** are the solution to the zeroth-order perturbation equations. It is noticed that the pressure distribution is axially symmetric. The closer of a position to the inlet is, the greater its pressure will be, which is qualitatively consistent with the results through ANSYS-CFX shown in **Figure 2(b)**. Similarly, the circumferential velocity obtained through the zeroth-order perturbation equations in **Figure 3** shows the same trend with those calculated by ANSYS-CFX. The profile of velocity is also centrosymmetric, and the farther to the inlet is, the greater the circumferential velocity will be. The solutions to the first-order perturbation,



(a) (b)

Figure 2. Comparison of pressure distribution. (a) Perturbation solution; (b) ANSYS-CFX.

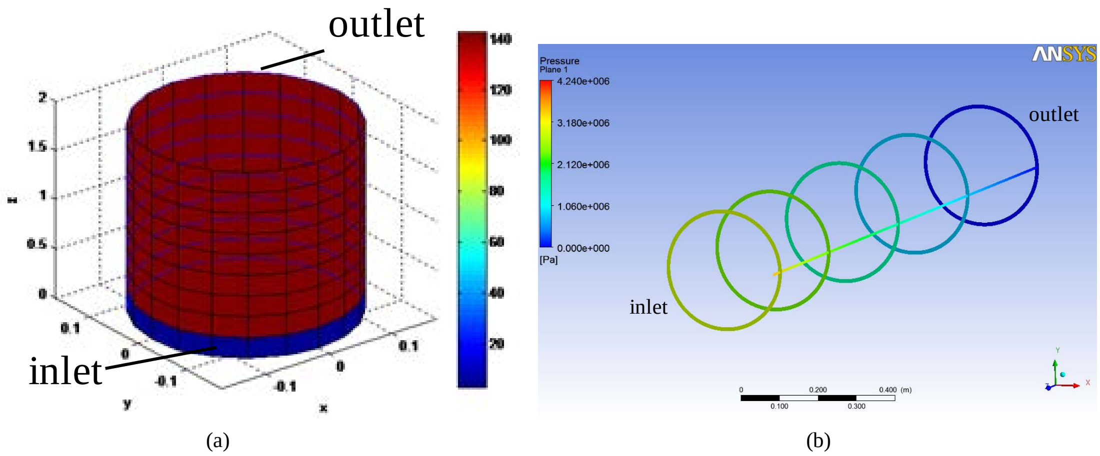


Figure 3. Comparison of circumferential velocity distribution. (a) Perturbation method; (b) ANSYS-CFX.

equations also agree well with the results through ANSYS-CFX qualitatively which will not be discussed any further.

5. Conclusion

The solution to a three-dimensional annular flow inside a small gap between rotor and stator is obtained using zeroth-order and first-order perturbed control equations. The perturbed solutions are compared with the numerical results through ANSYS-CFX and they are found to agree qualitatively. Consequently, it is applicable to solve three-dimensional nonlinear control equations of the small-gap annular fluid when the eccentricity of axis is small enough compared with the average clearance between the rotor and the stator.

Acknowledgements

This paper is sponsored by the National Basic Research Program of China (Grant 2015CB057300), the Key Specific Projects of Liaoning Scientific Innovation (201303002), Liaoning Provincial Science and Technology Programs(2014010499-301), Program of Cultivated Key Project of Dalian University of Technology, and the Free Exploration Project of State Key Laboratory of Structural Analysis for Industrial Equipment (S14204).

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