Journal of Applied Mathematics and Physics, 2015, 3, 32-37
Published Online January 2015 in SciRes. http://www.scirp.org/journal/jamp
http://dx.doi.org/10.4236/jamp.2015.31005

Remarks on the Complexity of Signed *k*-Domination on Graphs

Chuan-Min Lee[1*], Cheng-Chien Lo[1], Rui-Xin Ye[2], Xun Xu[2], Xiao-Han Shi[2], Jia-Ying Li[2]

[1]Department of Computer and Communication Engineering, Ming Chuan University, The First American University in Asia, Taoyuan, Taiwan, Chinese Taipei
[2]Department of Electronic Information Engineering, Fuzhou University, Fuzhou, China
Email: [*]joneslee@mail.mcu.edu.tw

Received December 2014

Abstract

This paper is motivated by the concept of the *signed k-domination problem* and dedicated to the complexity of the problem on graphs. For any *fixed* nonnegative integer *k*, we show that the signed *k*-domination problem is NP-complete for doubly chordal graphs. For strongly chordal graphs and distance-hereditary graphs, we show that the signed *k*-domination problem can be solved in polynomial time. We also show that the problem is linear-time solvable for trees, interval graphs, and chordal comparability graphs.

Keywords

Graph Algorithm, Signed *k*-Domination, Strongly Chordal Graph, Tree, Fixed Parameter Tractable

1. Introduction

Let $G = (V, E)$ be a finite, undirected, simple graph. For any vertex $v \in V$, the open neighborhood of v in G is $N_G(v) = \{u \in V \mid (u,v) \in E\}$ and the closed neighborhood of v is $N_G[v] = N_G(v) \cup \{v\}$. The degree of a vertex v in G is $d_G(v) = |N_G(v)|$. We also use $V(G)$ and $E(G)$ to denote vertex set and edge set of G, respectively. If nothing else is stated, it is understood that $|V(G)| = n$ and $|E(G)| = m$. Let Y be a subset of real numbers. Let $f: V \to Y$ be a function which assigns to each $v \in V$ a value in Y. Let $f(S) = \sum_{u \in S} f(u)$ for any subset S of V and let $f(V)$ be the weight of f. In 2012, Wang [1] studied the notion of *signed k-domination* on graphs as follows. Let k be a fixed nonnegative integer and let $G = (V, E)$ be a graph. A *signed k-dominating function* of G is a function $f: V \to \{-1,1\}$ such that $f(N_G[v]) \geq k$ for every vertex $v \in V$. The *signed k-domination number* of G, denoted by $\gamma_{k,S}(G)$, is the minimum weight of a signed k-dominating function of G. The signed k-domination problem is to find a signed k-dominating function of G of minimum weight. Clearly, the signed k-domination problem is the signed domination problem if $k = 1$ [2]. Wang [1] presented several sharp lower bounds of these numbers for general graphs. In this paper, we study the signed k-domination problem for several well-known classes of graphs such as doubly chordal graphs, strongly chordal graphs, distance-hereditary graphs, trees, interval graphs, and chordal comparability graphs.

[*]Corresponding author.

How to cite this paper: Lee, C.-M., Lo, C.-C., Ye, R.-X., Xu, X., Shi, X.-H. and Li, J.-Y. (2015) Remarks on the Complexity of Signed *k*-Domination on Graphs. *Journal of Applied Mathematics and Physics*, **3**, 32-37.
http://dx.doi.org/10.4236/jamp.2015.31005

2. NP-completeness Results

Before presenting the NP-complete results, we restate the signed *k*-domination problem as decision problems as follows: Given a graph *G* = (*V*, *E*) and a nonnegative integer *k* and an integer λ, is $\gamma_{k,S}(G) \le \lambda$?

Theorem 1 [3] [4] For any integer *k* = 0 or 1, the signed *k*-domination problem on doubly chordal graphs and bipartite planar graphs is NP-complete

Theorem 2. For any fixed integer $k \ge 2$, the signed *k*-domination problem on doubly chordal graphs is NP-complete.

Proof. Clearly, the signed *k*-domination problem on doubly chordal graphs is in NP. By Theorem 1, the signed 0-domination and 1-domination problems on doubly chordal graphs are NP-complete. In the following, we show the NP-completeness of the signed *k*-domination problem on doubly chordal graphs by a polynomial-time reduction from the signed $(k-1)$-domination problem on doubly chordal graphs.

Let $G=(V,E)$ be a doubly chordal graph with |*V*| = *n*. A *clique* is a subset of pairwise adjacent vertices in a graph. If a clique consists of *j* vertices, then it is called a *j-clique*. We construct a graph *H* from *G* by the following steps.

1) We construct a new vertex *u* and connect *u* to every vertex of *G*.
2) We construct $(k-1)$-cliques $K_1, K_2, \ldots, K_n$ and connect the vertex *u* to every vertex of K_i for $1 \le i \le n$. Note that $|K_i| = k-1$ for $1 \le i \le n$.

Clearly, the graph *H* is a doubly chordal graph [5]-[8] and can be constructed in polynomial time. In the following, we show that $\gamma_{k,S}(H) = \gamma_{k-1,S}(G) + n \cdot k - n + 1$.

Suppose that *g* is a *minimum* signed $(k-1)$-dominating function of *G*. Then, $g(V) = \gamma_{k-1,S}(G)$. Let $h: V(H) \to \{-1,1\}$ be a function of *H* defined by $h(v) = g(v)$ for every vertex $v \in V$ and $h(v) = 1$ for every vertex $v \in V(H) \setminus V$. It can be easily verified that *h* is a signed *k*-dominating function of *H*. We have

$$\gamma_{k,S}(H) \le h(V) + h(V(H) \setminus V) = \gamma_{k-1,S}(G) + h(u) + n \cdot (k-1) = \gamma_{k-1,S}(G) + n \cdot k - n + 1.$$

Conversely, let $i \in \{1, 2, \ldots, n\}$ and let *f* be a *minimum* signed *k*-dominating function of *H*. Since $K_i \cup \{u\}$ is a *k*-clique, $|N_H[v]| = k$ for every vertex $v \in K_i$ and thus $f(u) = f(v) = 1$. By the construction of *H*, the vertex *u* is adjacent to every vertex *v* of *G*. We know that $f(N_H[v]) = f(N_G[v] \cup \{u\}) = f(N_G[v]) + f(u) \ge k$. Then, $f(N_G[v]) \ge k-1$. Let $g: V \to \{-1,1\}$ be a function of *G* defined by $g(v) = f(v)$ for every vertex $v \in V$. The function *g* is a signed $(k-1)$-dominating function of *G*. We have $\gamma_{k-1,S}(G) \le g(V) = f(V(H)) - f(u) - n \cdot (k-1) = \gamma_{k,S}(H) - n \cdot k + n - 1$.

Therefore, $\gamma_{k-1,S}(G) + n \cdot k - n + 1 \le \gamma_{k,S}(H)$. Following the discussion above, we know that $\gamma_{k,S}(H) = \gamma_{k-1,S}(G) + n \cdot k - n + 1$. It implies that for any integer λ, $\gamma_{k-1,S}(G) \le \lambda$ if and only if $\gamma_{k,S}(H) \le \lambda + n \cdot k - n + 1$.

3. Polynomial-Time Solvable Results

In this section, we show that the signed *k*-domination problem is polynomial-time solvable for strongly chordal graphs and distance-hereditary graphs and linear-time solvable for trees, interval graphs, and chordal comparability graphs.

3.1. Strongly Chordal Graphs

Let $G=(V,E)$ be a graph. A *clique* is a subset of pairwise adjacent vertices of V. A vertex v is *simplicial* if and only if all vertices of $N_G[v]$ form a clique. The ordering $v_1, v_2, \ldots, v_n$ of the vertices of V is a *perfect elimination ordering* of *G* if for all $i \in \{1, 2, \ldots, n\}$, v_i is a simplicial vertex of the subgraph G_i of *G* induced by $\{v_i, v_{i+1}, \ldots, v_n\}$ [9]. Let $N_i[v]$ denote the closed neighborhood of v in G_i. A perfect elimination ordering is called a *strong elimination ordering* if it satisfies the following condition:

For $i < j < k$ if v_j and v_k belong to $N_i[v_i]$ in G_i, then $N_i[v_j] \subseteq N_i[v_k]$.

Farer [10] showed that a graph is *strongly chordal* if and only if it has a strong elimination ordering. Currently, the fastest algorithm to recognize a strongly chordal graph and give a strong elimination ordering takes $O(m \log n)$ [11] or $O(n^2)$ time [12]. Strongly chordal graphs include many interesting classes of graphs such as trees, block graphs, interval graphs, and directed path graphs [13]. In the paper [3], Lee and Chang introduced the concept of *L-domination*. The definition of *L-domination* is as follows.

Let ℓ, d, I_1, F_r be fixed integer such that $\ell, d > 0$ and $F_r = I_1 + \ell \cdot d$. Let Y be the set $\{I_1, I_1 + d, I_1 + 2d, \ldots, I_1 + (\ell - 1) \cdot d\}$. Suppose that $G = (V, E)$ is a graph. Let L be a labeling function which assigns to each $v \in V$ a label $L(v) = (t(v), k(v))$, where $t(v) \in Y \cup \{F_r\}$ and $k(v)$ is a fixed integer. An *L-dominating function* of a graph $G = (V, E)$ is a function $f : V \to Y$ satisfying the following two conditions:

1) If $t(v) \neq F_r$, then $f(v) = t(v)$.
2) $f(N_G[v]) \geq k(v)$ for every vertex $v \in V$.

The *L-domination number* of *G*, denoted by $\gamma_L(G)$, is the minimum weight of an *L*-dominating function of *G*. The *L-domination domination problem* is to find an *L*-dominating function of *G* of minimum weight. Lee and Chang obtained the following result.

Theorem 4 [3] For any strongly chordal graph *G*, the *L*-domination problem can be solved in $O(n + m)$ time if a strong elimination ordering of *G* is given.

We show a connection between and the signed *k*-domination problem and a special case of the *L*-domination problem in Theorem 3.

Theorem 5. Suppose that $\ell = 2$, $d = 1$, $I_1 = -1$, $F_r = I_1 + \ell \cdot d$, and $Y = \{I_1, I_1 + d, \ldots, I_1 + (\ell - 1) \cdot d\}$. Let *k* be a nonnegative integer and let $G = (V, E)$ be a graph in which each $v \in V$ is associated with a label $L(v) = (F_r, k)$. Then, a minimum *L*-dominating function of *G* is equivalent to a minimum signed *k*-dominating function of *G*.

Proof. Clearly, $Y = \{-1, 1\}$. We assume that f is a minimum *L*-dominating function of *G* and each $v \in V$ is associated with a label $L(v) = (F_r, k)$. Then, $f(N_G[v]) \geq k$ and f is a signed *k*-dominating function of *G*. We have $\gamma_{k,S}(G) \leq \gamma_L(G)$. Conversely, we assume that g is a minimum signed *k*-dominating set of *G*. Then, $g(N_G[v]) \geq k$ for every vertex $v \in V$. It can be easily verified that *g* is an *L*-dominating function of *G*. We have $\gamma_L(G) \leq \gamma_{k,S}(G)$. Following the discussion above, we know that $\gamma_{k,S}(G) \leq \gamma_L(G)$ and $\gamma_L(G) \leq \gamma_{k,S}(G)$. Hence, $\gamma_L(G) = \gamma_{k,S}(G)$ and the theorem holds.

Theorem 6. For any nonnegative integer *k*, the signed *k*-domination problem on a strongly chordal graph *G* can be solved in $O(n + m)$ time if a strong elimination ordering of *G* is given.

Proof. The theorem follows from Theorems 4 and 5.

Theorem 7. For any nonnegative integer *k*, the signed *k*-domination problem is linear-time solvable for trees.

Proof. Trees are both chordal and strongly chordal [13]. Let *G* be a tree. A perfect elimination ordering $v_1, v_2, \ldots, v_n$ of the vertices in *G* can be obtained in linear time [14]. Since *G* is a tree, v_i has at most one neighbor in G_i for any $i \in \{1, 2, \ldots, n\}$. Otherwise, $N_i[v_i]$ forms a clique with at least three vertices and it contradicts the assumption that *G* is a tree. Therefore, the ordering $v_1, v_2, \ldots, v_n$ is also a strong elimination ordering of *G*. Following Theorem 6, we know that the signed *k*-domination problem is linear-time solvable for trees.

Theorem 8. For any nonnegative integer *k*, the signed *k*-domination problem is linear-time solvable for interval graphs.

Proof. An interval graph *G* is the intersection graph of a set of intervals on a line. That is, each interval corresponds to a vertex of *G* and two vertices are adjacent if and only if the corresponding intervals intersect. The set of intervals constitutes an *interval model* of the graph. Booth and Lueker [15] gave the first linear-time algorithm for recognizing interval graphs and constructing interval models for the interval graphs.

Let *I* be an interval model of an interval graph *G*. Each interval in the interval model has a right endpoint and a left endpoint. Without loss of generality, we may assume that all endpoints of the intervals in *I* are pairwise distinct, since, when they are not, it is easy to make this true without altering the represented graph. Let $l(v)$ and $r(v)$ denote the left and right endpoints of the interval corresponding to *v*. We order the vertices of *G* by the increasing order of right endpoints of the intervals in *I*, and let the ordering be $v_1, v_2, \ldots, v_n$. For any $i, j \in \{1, 2, \ldots, n\}$ with $i < j$, we know that $r(v_i) < r(v_j)$ and $l(v_j) < r(v_i)$ if v_i is adjacent to v_j in *G*. Therefore, the vertices of $N_i[v_i]$ form a clique and v_i is a simplicial vertex of G_i. The ordering $v_1, v_2, \ldots, v_n$ is a perfect elimination ordering and can be obtained in linear time.

For $i < j < k$, we assume v_j and v_k belong to $N_i[v_i]$ in G_i. Since $v_1, v_2, \ldots, v_n$ is a perfect elimination ordering, v_j is adjacent to v_k and $r(v_j) < r(v_k)$ and $l(v_k) < r(v_i) < r(v_j)$. Then, every v_p in $N_i[v_j]$ is adjacent to v_k. We have $N_i[v_j] \subseteq N_i[v_k]$. The ordering $v_1, v_2, \ldots, v_n$ is also a strong elimination ordering of *G*. By Theorem 6, we know that the signed *k*-domination problem is linear-time solvable for interval graphs.

Theorem 9. For any nonnegative integer *k*, the signed *k*-domination problem is linear-time solvable for chordal comparability graphs.

Proof. Let $G = (V, E)$ be a graph. A vertex v in G is a *simple vertex* if for any two neighbors x and y of v, either the closed neighborhood of y is a subset of the closed neighborhood of x or the closed neighborhood of x is a subset of the closed neighborhood of y. An ordering $v_1,v_2,\ldots,v_n$ is a *simple elimination ordering* if for each $1 \le t \le n$, the vertex v_i is a simple vertex of the subgraph G_i induced by the vertices $v_i,v_2,\ldots,v_n$.

A simple elimination ordering of a chordal comparability graph can be obtained in linear time [16]. Sawada and Spinrad [17] presented a linear-time algorithm to transform a simple elimination ordering of a strongly chordal graph to a strong elimination ordering. Therefore, the theorem is true.

3.2. Distance-Hereditary Graphs

The *distance* between two vertices u and v of a graph G is the number of edges of a shortest path from u to v. If any two distinct vertices have the same distance in every connected induced subgraph containing them, then G is a *distance-hereditary* graph. In 1997, Chang, Hsieh, and Chen [18] showed that distance-hereditary graphs can be defined recursively.

Theorem 10 [18] Distance-hereditary graphs can be defined as follows.

1) A graph consisting of only one vertex is distance-hereditary, and the twin set is the vertex itself.
2) If G_1 and G_2 are disjoint distance-hereditary graphs with the twin sets $TS(G_1)$ and $TS(G_2)$, respectively, then the graph $G = G_1 \cup G_2$ is a distance-hereditary graph and the twin set of G is $TS(G_1) \cup TS(G_2)$. G is said to be obtained from G_1 and G_2 by a *false twin operation.*
3) If G_1 and G_2 are disjoint distance-hereditary graphs with the twin sets $TS(G_1)$ and $TS(G_2)$, respectively, then the graph G obtained by connecting every vertex of $TS(G_1)$ to all vertices of $TS(G_2)$ is a distance-hereditary graph and the twin set of G is $TS(G_1) \cup TS(G_2)$. G is said to be obtained from G_1 and G_2 by a *true twin operation.*
4) If G_1 and G_2 are disjoint distance-hereditary graphs with the twin sets $TS(G_1)$ and $TS(G_2)$, respectively, then the graph G obtained by connecting every vertex of $TS(G_1)$ to all vertices of $TS(G_2)$ is a distance-hereditary graph and the twin set of G is $TS(G_1)$. G is said to be obtained from G_1 and G_2 by a *pendant vertex operation.*

Following Theorem 10, a distance-hereditary graph G can be represented as a binary ordered decomposition tree and the decomposition tree can be obtained in linear-time [18]. In this decomposition tree, each leaf is a single vertex graph, and each internal node represents one of the three operations: pendant vertex operation (labeled by P), true twin operation (labeled by T), and false twin operation (labeled by F). Therefore, the decomposition tree is called a PTF-tree.

Definition 1. Suppose that $G = (V, E)$ is a distance-hereditary graph. Let $TS(G)$ be the twin set of G. Let a and b be integers such that $0 \le a, b \le |V|$ and $-|V| \le t \le |V|$. A (t, a, b)-*function* $f : V \to \{-1, 1\}$ of G is a function satisfying the following three conditions.

1) $a + b = |TS(G)|$.
2) The function f assigns the value 1 to a vertices in $TS(G)$ and the value -1 to b vertices in $TS(G)$.
3) For a vertex $v \in V$, $f(N_G[v]) + t \ge k$ if $v \in TS(G)$; Otherwise, $f(N_G[v]) \ge k$.

We define $\gamma(G,t,a,b) = \max\{f(V(G)) \mid f \text{ is a } (t,a,b)\text{-function of } G\}$. If there does not exist a (t, a, b)-function of G, then $\gamma(G,t,a,b) = \infty$. It is clear that

$$\gamma_{k,S}(G) = \min\{\gamma(G,0,a,b) \mid 0 \le a,b \le |TS(G)|\}.$$

We give the following lemmas to compute $\gamma(G,t,a,b)$ for a distance-hereditary graph G. The correctness of Lemmas 2 - 5 can be proved by the arguments similar to those for proving Lemmas 1 - 4 in Section III. B of the paper [4].

Lemma 2. Suppose that $G = (V, E)$ is a graph of only one vertex v. Then,

$$\gamma(G,t,a,b) = \begin{cases} 1 & \text{if } a = 1,\ b = 0,\ t \ge k-1; \\ 0 & \text{if } a = 0,\ b = 1,\ t \ge k+1; \\ \infty & \text{otherwise.} \end{cases}$$

Lemma 3. Suppose that $G = (V, E)$ is formed from two disjoint distance-hereditary graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ by a false twin operation. Then,

$$\gamma(G,t,a,b)=\min\{\gamma(G_1,t,a_1,b_1)+\gamma(G_2,t,a_2,b_2)\},$$

where $a_1+a_2=a$ and $b_1+b_2=b$.

Lemma 4. Suppose that $G=(V,E)$ is formed from two disjoint distance-hereditary graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ by a true twin operation. Then,

$$\gamma(G,t,a,b)=\min\{\gamma(G_1,t+a_2-b_2,a_1,b_1)+\gamma(G_2,t+a_1-b_1,a_2,b_2)\},$$

where $a_1+a_2=a$ and $b_1+b_2=b$.

Lemma 5. Suppose that $G=(V,E)$ is formed from two disjoint distance-hereditary graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ by a pendant vertex operation. Then,

$$\gamma(G,t,a,b)=\max\{\gamma(G_1,t+a_2-b_2,a,b)+\gamma(G_2,a-b,a_2,b_2)\},$$

where $a_2+b_2=|TS(G_2)|$.

Theorem 11. For any nonnegative integer *k*, the signed *k*-domination problem can be solved in polynomial time for distance-hereditary graphs.

Proof. Following Lemmas 2 - 5 and the recursive definition of distance-hereditary graphs in Theorem 10, we can design a dynamic programming algorithm to compute the signed *k*-domination number of a distance-hereditary graph *G* in polynomial time. Moreover, it is not difficult to see that a minimum signed *k*-dominating function of a distance-hereditary graph *G* can be obtained in polynomial time, too.

Acknowledgements

This research was partially supported under Research Grants: NSC-102-2221-E-130-004 and MOST-103-2221-E-130-009 in Taiwan.

References

[1] Wang, C. (2012) The Signed *k*-Domination Numbers in Graphs. *Ars Combinatoria*, **106**, 205-211.

[2] Hattingh, J.H., Henning, M.A. and Slater, P.J. (1995) The Algorithmic Complexity of Signed Domination in Graphs. *Australasian Journal of Combinatorics*, **12**, 101-112.

[3] Lee, C.-M. and Chang, M.-S. (2008) Variations of Y-Dominating Functions on Graphs. *Discrete Mathematics*, **308**, 4185-4204. http://dx.doi.org/10.1016/j.disc.2007.08.080

[4] Lee, C.-M. (2014) Remarks on the Complexity of Non-negative Signed Domination. *Journal of Networks*, **9**, 2051-2058. http://dx.doi.org/10.4304/jnw.9.8.2051-2058

[5] Brandstädt, A., Dragan, F.F., Chepoi, V.D. and Voloshin, V. (1998) Dually Chordal Graphs. *SIAM Journal on Discrete Mathematics*, **11**, 437-455. http://dx.doi.org/10.1137/S0895480193253415

[6] Cai, L., Chen, J., Downey, R.G. and Fellows, M.R. (1997) Advice Classes of Parameterized Tractability. *Annals of Pure and Applied Logic*, **84**, 119-138. http://dx.doi.org/10.1016/S0168-0072(95)00020-8

[7] Downey, R.G. and Fellows, M.R. (1999) Parameterized Complexity. Monographs in Computer Science, Springer-Verlag.

[8] Moscarini, M. (1993) Doubly Chordal Graphs, Steiner Trees, and Connected Domination. *Networks*, **23**, 59-69. http://dx.doi.org/10.1002/net.3230230108

[9] Rose, D.J. (1970) Triangulated Graphs and the Elimination Process. *Journal of Mathematical Analysis and Applications*, **32**, 597-609. http://dx.doi.org/10.1016/0022-247X(70)90282-9

[10] Farber, M. (1983) Characterizations of Strongly Chordal Graphs. *Discrete Mathematics*, **43**, 173-189. http://dx.doi.org/10.1016/0012-365X(83)90154-1

[11] Paige, R. and Tarjan, R.E. (1987) Three Partition Refinement Algorithms. *SIAM Journal on Computing*, **16**, 973-989. http://dx.doi.org/10.1137/0216062

[12] Spinrad, J.P. (1993) Doubly Lexical Ordering of Dense 0-1 Matrices. *Information Processing Letters*, **45**, 229-235. http://dx.doi.org/10.1016/0020-0190(93)90209-R

[13] Brandstädt, A., Le, V.B. and Spinrad, J.P. (1999) Graph Classes: A Survey. SIAM Monographs on Discrete Mathematics and Applications, Philadelphia 1999. http://dx.doi.org/10.1137/1.9780898719796

[14] Tarjan, R.E. and Yannakakis, M. (1984) Simple Linear Algorithms to Test Chordality of Graphs, Test Acyclicity of Hypergraphs, and Selectively Reduce Acyclic Hypergraphs. *SIAM Joural on Computing*, **13**, 566-579. http://dx.doi.org/10.1137/0213035

[15] Booth, S. and Lueker, S. (1976) Testing for the Consecutive Ones Property, Interval Graphs, Graph Planarity Using PQ-Trees Algorithms. *Journal of Computer and System Sciences*, **13**, 335-379. http://dx.doi.org/10.1016/S0022-0000(76)80045-1

[16] Borie, R.B. and Spinrad, J.P. (1999) Construction of a Simple Elimination Scheme for a Chordal Comparability Graph in Linear Time. *Discrete Applied Mathematics*, **91**, 287-292. http://dx.doi.org/10.1016/S0166-218X(98)00137-1

[17] Sawada, J. and Spinrad, J.P. (2003) From a Simple Elimination Ordering to a Strong Elimination Ordering in Linear Time. *Information Processing Letters*, **86**, 299-302. http://dx.doi.org/10.1016/S0020-0190(03)00228-X

[18] Chang, M.-S., Hsieh, S.-Y. and Chen, G.-H. (1997) Dynamic Programming on Distance-Hereditary Graphs. *Proceedings of the* 8*th International Symposium o Algorithms and Computation*, Lecture Notes in Computer Science, Vol. 1350, 344-353.