

Framework of Penrose Transforms on D_p -Modules to the Electromagnetic Carpet of the Space-Time from the Moduli Stacks Perspective

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Abstract

Considering the different versions of the Penrose transform on D -modules and their applications to different levels of D_M -modules in coherent sheaves, we obtain a geometrical re-construction of the electro-dynamical carpet of the space-time, which is a direct consequence of the equivalence between the moduli spaces $\mathcal{M}_{G\text{-equivariant}}(G/H, \tilde{C}) \cong \mathcal{M}_{G\text{-equivariant}}(\text{CY}, \hat{C})$, that have been demonstrated in a before work. In this case, the equivalence is given by the Penrose transform on the quasi-coherent D_λ -modules given by the generalized Verma modules diagram established in the Recillas conjecture to the group $SO(1, n+1)$, and consigned in the D_p -modules on which have been obtained solutions in field theory of electromagnetic type.

Keywords

Penrose Transform, Moduli Stack, D_p -Modules, Verma Modules, Field Theory, Electro-Dynamical Stacks

1. Introduction

We consider the Penrose transformation, [1] through the correspondence

$$\begin{array}{ccc}
 & \mathbb{F} & \\
 \swarrow & & \searrow \\
 \mathbb{P} & & \mathbb{M}
 \end{array} \tag{1}$$

Let \mathcal{P} , be the Penrose transform [2] associated to the double fibration (1), used to represent the holomorphic solutions of the generalized wave equation [2], with parameter of helicity h [3,4]:

$$\square_h \phi = 0, \tag{2}$$

on some open subsets $U \subset \mathbb{M}$, in terms of cohomological classes of bundles of lines [2], on $U = \nu(\pi^{-1}(U)) \subset \mathbb{P}$, (where \mathbb{P} , is the super-projective space).

In the D -modules language the direct and inverse images of the double fibration comes given to D_M -modules that are D_p -modules through of the right functor equivalence:

$$R\Gamma(y; \mathcal{O}_p(k)) \cong R \operatorname{hom}_{\mathcal{D}_Y} (\Phi_F \mathcal{D}_p(-k), \mathcal{O}_Y)_y[-1], \quad (3)$$

where D -module transform of $\mathcal{D}_p(-k)$, is the D_M -module associated to the wave equation given in the equation (2), whose helicity is $h(k) = -(1+k/2)$.

Now in the context of the generalized D -modules (inside of the derived categories) we can use one version generalized of the Schmid-Radon transform, to cycles in the derived category and finally obtain for the Radon-Penrose Transform, the functor [5]

$$\Phi_S + \text{additional geometrical hypothesis}, \quad (4)$$

establishing an equivalence between the category $M(D_X$ -modules / flat connection), and the category $M(D_Y$ -modules/Singularities to along of the involutive manifold V /flat connection), [2,5,6].

Then our moduli space that we constructed is the categorization of equivalences:

$$\begin{aligned} \mathcal{M}_{d[S/Y]} = \{ & M(D_Y \text{-modules / Singularities to along} \\ & \text{of the involutive manifold } V \text{ / flat connection)} \rightleftharpoons \\ & H^*(U, \mathcal{O}) \}, \end{aligned} \quad (5)$$

considering the moduli space as base [2,6],

$$\mathcal{M}_{d[S/X]} = \{ M(D_X \text{-modules / flat connection)} \rightleftharpoons \ker(U, \square_{h(k)}) \}, \quad (6)$$

The additional geometrical hypothesis in the functor (4), comes established by the geometrical duality of Langlands [5], which says that the derived category of coherent sheaves on a moduli space $\mathcal{M}_{\text{Flat}}({}^L G, C)$, where C , is the complex given by [6]

$$C : \dots \xrightarrow{d^{j-1}} \mathcal{E}^{j-1} \xrightarrow{d^j} \mathcal{E}^j \xrightarrow{d^{j+1}} \mathcal{E}^{j+1} \xrightarrow{d^{j+2}} \mathcal{E}^{j+2} \xrightarrow{d^{j+3}} \dots, \quad (7)$$

which is equivalent to the derived category of D -modules on the moduli space of holomorphic vector G -bundles given by $B_G(C)$ [7], this shaped by the geometrical extension of a derived functor corresponding to one Langlands ramification with corresponding stack moduli given by ${}^L \text{Bun}_G^k$ [8].

Inside of this study, the transform on D -modules (see Kashiwara [9]), that arises is the Radon-Penrose transform for generalized flag manifolds in the framework of this quasi- G -equivariant D -modules (that are a class of the coherent D -modules useful in the manager of cycles to reconstruct and model the geometrical images [10,11] of the electrodynamics of the space-time through of the different photon fields that interact as strings, heterotic strings and D -branes [12]), is the generalized version that we want. Likewise, in particular their Radon integral operator defines and proves the necessary invariance of the geometrical images obtained by the D_p -modules transform, predicted by the Kashiwara theorem [12], and that guarantee the invariance in the solutions of the wave equation given in (2) in the frame of conformally [13] in the integral geometry context.

By the Recillas conjecture [14], and their application in the obtaining of Cěch complexes (complexes obtained in the development by derived sheaves (ver Kapustin [7])) in the tacking of strings through super-conformal spaces \mathbb{P}^S given in the corollary by [5], we have that the Penrose-Ward transform is done evident when the inverse images of the D_p -modules that are quasi-coherent D_λ -modules established by the diagram [14] (Verma modules diagram in the conjecture Recillas to the group $SO(1, n+1)$ [4,15]) result naturally inside of vector bundles language as image of the degeneration of cycles given inside the manifold signed in the equivalences of moduli spaces in the *theorem 4.1*, given in [16].

Likewise the duality between string theories (*string and D-branes*) stay established through of the intertwining operators of the Penrose transform and their isomorphism in all different string dualities, field/particle and the conformally and holonomy levels required in invariance of the space-time field theory.

2. Framework Penrose Transforms on D_p -Modules that are D_p -Modules

As we want the framework of the space-time through the different versions of the Penrose transforms and their

relation between them considering the corresponding intertwining operators (*integral transforms that are isomorphism between cohomological spaces of orbital spaces [17,18] of the space-time*), as for example, the twistor transform to the G -orbits of the complex holomorphic bundle of the complex Riemannian manifold.

Since solutions of differential equations on manifolds and cohomological classes are part of the bases of this generalized Penrose transform, the theory of sheaf cohomology and D -modules was perfectly considered for their modern study. In [13], we used this theory to generalize and study the Penrose correspondence in (1). More recently in [15] has been studied the generalized Penrose transform between generalized flag manifolds over a complex algebraic group G , using M . The Kashiwara's correspondence (see [14]) between quasi- G -equivariant $-D_{G/H}$ -modules and some kind representation spaces which are (\mathfrak{g}, H) -modules (loosely, they are complex vector spaces endowed together with an action of the Lie algebra \mathfrak{g} , associated to G , and an action of H , which are compatible in some way) establishes the solid criteria to obtain equivalences between cohomological classes when H , is a closed algebraic subgroup of G .

Conjecture. 1. [14] We want a generalization of Penrose transform on the homogeneous space $SL(2, \mathbb{C})/U(1)$, or a Radon transform on equivariant- D -modules on generalized flag manifolds [13,15].

Let G , be a reductive algebraic group over \mathbb{C} , P , and Q , two parabolic subgroups containing the same Borel subgroup of G . Let $X = G/P$, $Y = G/Q$, and let S , be the unique closed G -Orbit in $X \times Y$, for the diagonal action. Then we can identify S , with $G/P \cap Q$. The natural correspondence

$$X \xleftarrow{\nu} S \xrightarrow{\pi} Y, \tag{8}$$

where ν , and π , are the restriction to S , of the projections of $X \times Y$, on X , and Y , induces an integral transform from X , to Y , which generalizes the classical Radon-Penrose transform. This subject has been investigated intensively both in the complex and real domains (to see Baston-Eastwood [1], D'Agnolo-Schapira [6], Marastoni [11], Gindikin [13]).

Our goal is to obtain one version of this transform in the framework of quasi- G -equivariant D -modules (see Kashiwara [7]), *i.e.* the functor

$$\mathcal{R}: M_G^b(\mathcal{M}_X) \longrightarrow M_G^b(\mathcal{M}_Y), \mathcal{R}(\mathcal{M}) = \underline{g}_* \underline{f}^{-1} \mathcal{M}, \tag{9}$$

called the Radon transform, which is predicted by the *conjecture 1*, given by [15] and completed by [11], where $M_G^b(\mathcal{M})$, denotes the derived category of quasi- G -equivariant D -modules with bounded cohomologies, and \underline{g}_* , and \underline{f}^{-1} , are the operations of direct image (integration) and inverse image (pull-back) for D -modules. More precisely, we consider a D_X -module of type $\mathcal{M} = \mathcal{DL} = \mathcal{D}_X \otimes_{\mathcal{O}_X} \mathcal{L}$, where \mathcal{L} , is an irreducible G -equivariant locally free \mathcal{O}_X -module. In this case is had that

$$H^p(\mathcal{R}(\mathcal{DL})) = 0, \forall p < 0,$$

The Grothendieck group of the category of quasi- G -equivariant D_X -modules of finite length is spanned by elements corresponding to the objects of the form \mathcal{DL} . This means that we can generate solutions in complex holomorphic manifolds as \mathcal{M} , through conformal solutions defined on bundle of lines, at least to each a of the derived category components of quasi- G -equivariant D -modules. From BGG-resolution and using some fact of the Dolbeault cohomology on \mathcal{L} , discussed by Cousin complexes and their sheaves we obtain a resolution of the quasi- G -equivariant D_S -module $\underline{v}^{-1}(\mathcal{DL})$, of the form:

$$0 \rightarrow \bigoplus_{k=1}^{r_n} D\mathcal{L}_{nk} \rightarrow \dots \rightarrow \bigoplus_{k=1}^{r_0} D\mathcal{L}_{0k} \rightarrow \underline{f}^{-1}(\mathcal{DL}) \rightarrow 0, \tag{10}$$

To obtain a generalization of the Kashiwara theorem in the context of the G -equivariant D -modules is necessary the geometric Zuckerman functor. This is the localization of the equivariant Zuckerman functor to the derived equivariant D -module categories on generalized flag varieties (this to apply in our study an appropriate D -module transform of the Penrose transform type). To this case, we define our categories of interest and construct the geometric Zuckerman functor from the basic D -module functors.

The category of (left) G -equivariant \mathcal{O}_X -module is denoted by $M_G(X)$. The following theorem is well known:

Theorem 2.1. If G , acts on X , freely, there is an equivalence of categories $M_G(X) \cong M(X/G)$.

For homogeneous spaces, we can make a stronger statement. Let $B^{\mathbb{C}}$, be a complex linear group, let $\text{Rep}(B^{\mathbb{C}})$, be the category of algebraic representations of $B^{\mathbb{C}}$.

Proof. [15]. ■

Theorem 2.2. If $X = G/B^c$, there is an equivalence of categories $M_G(X) \cong \text{Rep}(B^c)$.

Proof. [15]. ■

Let \mathcal{M}_G , a quasi- G -equivariant- $D_{G/H}$ -module belonging to the subcategory of $D_{G/H}$ -modules given by $M_{\text{good}}(\mathcal{D}_{G/H})$ (category of the G -equivariant- $D_{G/H}$ -modules).

The idea is to obtain equivalences between the different classes of $D_{G/H}$ -modules establishing the isomorphism between the categories $M(\mathcal{M}_G) = \text{Mod}_{\text{RS}(\nu)}(\mathcal{D}_{G/H})$, and $M_G = M_{\text{good}}(\mathcal{D}_{G/H})$, where $\text{RS}(\nu)$, must be the restricted Zuckerman functor to the subcategory of left G -equivariant- $D_{G/H}$ -modules to obtain the derived category that is product of a triangulated subcategory (as the given in [16]) for a factor category that determine us the Harish-Chandra functor between the categories. These equivalences in analogy to the established in (9) take the shape to the specific $D_{G/H}$ -modules

$$\mathcal{M}_{G/H}^G = \{M_G(\mathcal{D}_{G/H}) = \{\text{derived category of } D_{G/H} \text{ - modules - } G \text{ - equivariants}\} \rightleftharpoons M_G(\mathfrak{g}, H) = M(\mathcal{M}_G)\}, \quad (11)$$

considering the moduli space as base,

$$\mathcal{M}_{G/H}^{\text{equiv}G} = \{M(D_{G/H} \text{ - modules } G \text{ - equivariants} \rightleftharpoons \ker(U, Q_{\text{BRST}})\}, \quad (12)$$

If we consider $\mathcal{H}_G = M(B \backslash G/B) = M(\mathcal{D}_\lambda^{G/H})$, $\forall \lambda \in \mathfrak{h}_{\mathbb{Z}}^*$, then $\mathcal{H}_G = M(B \backslash G/B)$, from the group $G = G^c$. B -equivariant D -module on the flag manifold $X = G/B$, provide integral kernels and thus integral transforms, to know

$$H^0(X, \mathcal{L}_\lambda) \cong \ker(\tilde{U}, Q_{\text{BRST}}), \quad (13)$$

where $Q_{\text{BRST}} = \bar{\partial} + \Phi_1^i \theta_1 - \Phi_2^i \theta_1$, such that $Q_{\text{BRST}}^2 \phi = 0$, where $\Phi_1^i \square$ and $\square \Phi_2^i$, are Higgs fields on either side of the open string [7], which are implicitly these kernels?

The condition established in the kernel of equivalences inside the moduli space $\mathcal{M}_{G/H}^{\text{equiv}G}$, is analogous to the given by the isomorphism of the Penrose transform discussed in foot of page 2, in the section 2, in [5]. The idea is extend the harmonic condition to the functions to differential operators of derived sheaves that are in BRST-cohomology, such that the equivalences in (12) are defined by certain functors due to the duality between the BGG-resolution and meromorphic version of the Cousin complex¹ associated to \mathcal{L}_λ (bundle of lines associated to the flag manifold $X = G/H$, then H , is a Borel subgroup). These is the way of the classification of the differential operators with their moduli stacks. Now we will dedicate to obtain these moduli stacks to electromagnetism in field theory as representation of the space-time.

3. Electromagnetic Moduli Stacks to Space-Time

A concrete application of the moduli space $\mathcal{M}_{G/H}^G$, on $SU(2)$ -bundles and their complexification establishes that the moduli space of the relations between hyperbolic waves (*horocycles*) [18], and the Haar measure of the group action in $SU(2, 2)$, on \mathbb{M}^2 is the moduli space of the functors

$$\mathcal{M}_0 = \left\{ D_p(-k) \left| \begin{array}{l} \Gamma H_{SU(2)}^\bullet(\text{Hom}(\pi_1(X), U(2))) \\ = H^j(\Phi_F(D_p(-k))), \forall D_X, \\ D_Y \in M(D_p \text{ - modules}) \end{array} \right. \right\},$$

where the cohomological space $H^j(\Phi_F(D_p(-k)))$, is the space of the $SU(2)$ -equivariant functors related with the functors given in the relation (3) where these last come from the use of Penrose transform on D -modules [6]. But these functors are induced on orbits of $SU(2, 2)$, conserving their equivariance through of one integral transform that relates Harish-Chandra modules from the string level like D -branes (D -modules) with one intertwining operator of cohomological description as the given in geometrical Langlands correspondence established in [8]. Then by the Langlands ramifications the moduli stack correspond to the holomorphic ${}^L G$ -bundles on C , where C , is the module defined in the complexes succession (7). Here the group ${}^L G$, is de-

¹Cousin Dolbeault theory refers to the complex Dolbeault infinite dimension.

²In measurements of observables like curvature and torsion the corresponding Radon transform on spaces $F = \{L | L \subset \mathbb{C}^4\}$, is the John or classic Penrose transform, and their image points are used to measure the curvature through of light waves (*macroscopic case*) [19].

fined by the condition that the lattice of homomorphisms from $U(1)$, to a maximal torus of G , be isomorphic to the weight lattice of ${}^L G$.

Studies on representations to non-compactness Minkowski spaces such as that appear in electromagnetism to characterize the gauge fields in $SL(2, \mathbb{C})/U(1)$, and $SU(n)$, establish that when the gauge group is $SU(2)$, and G , is a non-compact maximal subgroup of $SO(3,1)$, the unique non-zero invariant solutions are Abelian Maxwell fields [20]. Then this fact produces $Q_{BRST}^2 = 0$, (zero branes stability) that is to say, the D -module of Q -branes are the torus [7] which is isomorphic to $U(1)$.³ What is the mosaic of the field theories of the space-time in this case?

The mosaic is very simple and only includes two theories of the electromagnetic nature shaping the frame of the study **Figure 1**.

In this case the two different electromagnetic theories, one in the commutative case (that includes the electromagnetic macroscopic fields) and other that includes the weakly electromagnetic type that conforms, joined with others considerations the gauge theory, can be the same if their semigroup operation is not invertible. To the 2-dimensional scheme of the conformation of the *one only theory* from two different theories (for example to the gauge $-U(1)$ -theories) the quotients of the homogeneous space can be visualized under the Q -branes square only (blue square in the **Figure 1**).

We consider the description of the grading (or classification) that appears physically in terms of the $U(1)_R$ (the re-normalizing of the group $U(1)$).

Considering the TQFT, and CFT, for branes and anti-branes wrapped on the entire space, the analysis from the gauge $-U(1)$ -theories is straightforward. The tachyon \mathcal{T} , is a degree zero operator. The term that we have that add to the boundary to describe a tachyon is the field descendant $\int_{\partial\mathcal{G}} [\mathcal{G}, \mathcal{T}]$, where \mathcal{G} , is the topologically twisted boundary supercharge (that is to say a term in the TFT OR TQFT). The operator \mathcal{G} , has $U(1)_R$, charge -1 , so $[\mathcal{G}, \mathcal{T}]$, has charge -1 . But a necessary condition to preserve supersymmetry is that boundary terms must be neutral under $U(1)_R$ (otherwise the $U(1)_R$, is broken which breaks the $\mathcal{N} = 2$, boundary supersymmetry⁴).

For other side, to lower dimensional sheaves, the relationship between the charge and grading is more subtle considering the dimension of the cohomological spaces to different vector bundles considered. For example, to the state corresponding to an element of $Ext^n(S, \mathcal{T})$, for two sheaves S , and \mathcal{T} , we need not have charge $U(1)_R$, equal to n , which could be the degree of the Ext , group will not match the charge of the state. Then the use of the twistor transform results very useful until certain point when there exist correspondence between sheaves that are corresponded to mutually supersymmetric branes. For example, the inverse twistor transform applied to the sheaf $\mathcal{O}(-k)$, corresponding to the k th - power of the tautological line bundle of super-projective space \mathbb{P}^3 [21], for example the bundle of lines $\mathcal{O}(1)^{\oplus 4}$, when is considered a twistor-string case with \mathbb{P}^3 .

Nevertheless one important aspect that born of these relation that are explained by the twistor transform is the field consideration that gives origin to the shape of the geometry (at least in lower dimension) from the quantum states and their orbifolds visualized as the lines of their corresponding vector bundles shaping the physical stacks of the first topological electromagnetic theories to some classes of the space $SL(2, \mathbb{C})/U(1)$. But, furthermore using the "ramifications" to certain Langlands classes [8] we can to obtain the visualizing of the singularities of the field as sources of the proper fields, through of their extensions as rational curves. Remember

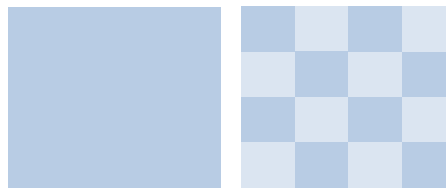


Figure 1. Re-normalizing of the Group in semi-groups of electrodynamical quotients from $SL(2, \mathbb{C})/U(1)$, when $Q_{BRST}^2 = 0$ [7].

³The co-weight lattice of G , is defined as the lattice of homomorphisms from $U(1)$, to a maximal torus τ , of G . The weight lattice of G , is the lattice of homomorphisms from τ , to $U(1)$.

⁴Twisting $\mathcal{N} = 2$, Super-Yang-Mills theory.

that we want discomposes singular regions of the space in regions of the conformal nature that can give the value of the singularity evaluated as zeros of polynomials in algebraic geometry and re-interpreted by the Penrose transform as varieties (varieties that are rational curves or algebraic curves). This is as to take a microscope and observe the flatness of the space in these singular regions (as the study frame given in the first square of the **Figure 1**) to these scales (for example to the QFT). To this last is very useful establish dualities between of set of equivalences from the moduli space $\mathcal{M}_{\text{Flat}}({}^L G, C)$, and the moduli space of the corresponding classes in (12) [5] (see **Figures 2 (a)** and **(b)**).

In the case of the complexes (7) the idea is discompose a complex with non-conformal properties in “factorable” pieces in complexes where flatness due to the holomorphicity exists and also conformability of \mathbb{M} , in these scales. A promissory result in that sense is the due to Kashiwara-Poicaré-Mebkhout on local cohomology where in the resolution proposed in the *Bulnes corollary* [14], we can investigate the cohomology⁵ of \mathbb{P}^3 , through the moduli space $\mathcal{M}(\mathbb{P}^3, 0) \cong \mathbb{P}^{3|4}$. To the electro-dynamical context, the moduli space is the corresponding to the super-projective $\mathcal{P}^{2|4} \cong \mathcal{L}^{4|6}$ (superminitwistor space).

What happen in the case when the space-time group is measured from electromagnetic fields in the TQFT, or TFT contexts as $SU(2) \otimes \mathbb{T}$?

4. Results

As has been mentioned before, to calibrate the space-time through the electromagnetic fields is necessary replace the Abelian group $U(1)$, in the K -invariant G -structure of the principal G -bundle of \mathbb{M} , given by $S_G(\mathbb{M})$, for the non-Abelian group $SU(2)$, since we want to realize an action through electromagnetic fields on a representation of the space-time given as $SU(2) \otimes \mathbb{T}$, where \mathbb{T} , is the torus $\mathbb{T} \cong U(1)$. Then $SO(4) \cong SU(2) \otimes \mathbb{T}$, where $SO(4)$, is a compact group of GL_4 .

For the isomorphism with $SO(4)$, a representation through of $U(1)$ -connections of the space-time will comes given for representations of the group $SU(2) \otimes \mathbb{T}$. An example of the application of these representations is the 2-dimensional hyperbolic space whose curvature can represents through the value of certain G -invariant integrals on lines of a bundle of lines on the principal G -bundle of $U(1)$, and whose that come from of the relation between co-cycles of the cohomology of G/L , with L , an open orbit and fields of G/K , (for example, the fields on horocycles as solutions of the wave equation on the hyperbolic plane when $G = SU(1,1)$, and $K = SO(2)$). The case that interest us is consider a G/L , as flag manifold and with $G = SL(2, C)$, and $L = \mathbb{T}$. Here G/K , is a 3-dimensional hyperbolic space.

The way to define the corresponding moduli stacks of electromagnetic nature is consider the following implications that are consequences of cohomology of derived sheaves of D_F -modules using the Cousin cohomology on $SU(2)$ -bundles [8] and after extend them to the complex case:

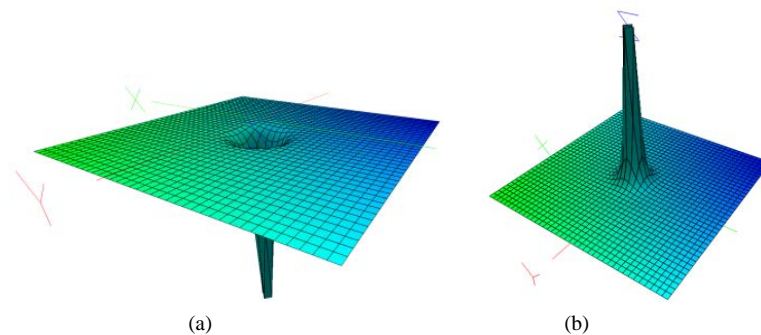


Figure 2. Mirror equivalences to sources and holes of the quantum electromagnetic fields. The first image corresponds to the hole or singularity of the $U(1)$ -connections. The two images correspond to the source of the gauge $-U(1)$ - field. Both geometrical objects are equivalent inside of the dualities of the moduli spaces $\mathcal{M}_{\text{Flat}}({}^L G, C)$, and Hitchin moduli space $\mathcal{M}_{\text{H}}(G, C)$.

⁵Remember that the twistor transform is an intertwining operator of cohomological description of isomorphisms from the different images of Penrose transforms of the same physical object (for example a hole can be a source and vice versa inside the bundle of lines context [5]).

$$\begin{aligned}
 & \text{Cohomology of } \widehat{\text{C}\hat{\text{e}}\text{ch}} \rightarrow \overline{\partial} - \text{Cohomology} \rightarrow \\
 & \text{Complexification of the Twistor model to Space - Time} \\
 & \rightarrow \text{hyper - functional of } \widehat{\text{C}\hat{\text{e}}\text{ch}},
 \end{aligned} \tag{14}$$

But this only can be realized directly through of different classes of the $U(1)$ –bundles of the space $SL(2, \mathbb{C})/U(1)$, that can be given only on those states inside the Lorentz norm.

Then by ingredients from the field theory we have that the Penrose transform gives the following identification:

$$H^2(\mathbb{P}, 2h - 2) = \{\text{hyperfunctions fields on } \mathbb{M}, \text{ of helicity } h\}, \tag{15}$$

But this space is too large to our purposes of field theory since it contains states of infinite norm like many radiative interactions that come from the background.

Then we consider the cohomological space

$$H^1(\mathbb{P}, 2h - 2) = \{\text{real analytic fields on } \mathbb{M}, \text{ of helicity } h\}, \tag{16}$$

which, using the *positive/negative* frequency decomposition due the standard polarization takes the form $H^1(\mathbb{P}^+, 2h - 2) \oplus H^1(\mathbb{P}^-, 2h - 2)$.

Then by the extension of functors given in the category [8], $M(\mathfrak{sl}_2, SU(2))$, we have

$$H^j(\mathbf{R}\Gamma_{U(1), SU(2)}^{\text{equivariant}}(\mathcal{D}(V))) = \mathbf{R}^P \Gamma_{U(1), SU(2)}(V), \tag{17}$$

$\forall j \in \mathbb{Z}^+$, and V , a Harish-Chandra module, where $\mathbf{R}^P \Gamma_{U(1), SU(2)}(V)$, is an equivariant image of the functor modulo $U(1)$, given in the moduli space \mathcal{M}_0 , and these are the functors $\mathbf{R}\Gamma(y; \mathcal{O}_{\mathbb{F}}(k))$, of the derived category $\mathcal{D}(V)$. Observe the D_p -modules as $D_{\mathbb{F}}$ -modules inside of the sheaf $\mathcal{O}_{\mathbb{F}}(k)$, that is to say, we want do correspond D_p -modules that come from a derived sheaf to certain class branes. These branes must be that come from of the electromagnetic fields to our research.

Theorem (F. Bulnes) 4.1. [2]. *The following isomorphism is valid*

$$\mathcal{M}_{\text{quasi-}G\text{-equivariant}}(G/H, \tilde{\mathcal{C}}) \cong \mathcal{M}_{\text{quasi-}G\text{-equivariant}}(CY, C^{\wedge*}),$$

being $C^{\wedge*}$, a Cousin Dolbeault complex

$$E^{n, \bullet} \otimes H^p R\mathcal{D}\mathcal{O}_{[X_k/X_{k+1}]}(\mathbb{M})[k] \cong E^{n, \bullet} \otimes H^p(R(\mathcal{D}\mathcal{O}_{X_l}(\lambda))).$$

Proof. [5,24]. ■

From the before theorem given in [5,8] we can give the application to a geometric electro-dynamical representation of the space-time considering conformally invariant two-dimensional theories⁶:

Theorem (F. Bulnes) 4.2. *Using the isomorphism between the moduli space $\mathcal{M}_{G\text{-equiv}}(CY, \hat{\mathcal{C}})$ and the moduli space to the stack $\mathcal{L}_{\lambda} \otimes \Omega^n$, we can obtain the moduli stacks $[c^4/z_2]$.*⁷

We want $\chi_G = \text{Hom}(T, c^{\times})$, [23] and obtain a result in real form spaces because we want use $U(1)$ –gauge theory, that is to say, “Abelianize” the $SU(2)$ –theory to find a decorated of the moduli stack where we can use the D –branes to tack the field carpet as a 2–image of the corresponding CFT-space (crystal field theory-space) obtained of the corresponding $U(1)$ –gerbe⁸ [7]:

via a homomorphism $\text{Hom}(Z(G), U(1))$, to know;

$$H^2(X, Z(G)) \xrightarrow{Z(G)-U(1)} H^2(X, U(1)), \tag{18}$$

Proof. Considering the twistor transform that appears in the composition $\mathcal{P} \circ \mathcal{T}$, [22] we have the Penrose

⁶Usually that stack is presented as $[X/c^{\times}]$, where to this presentation one associates a non-conformally invariant two-dimensional theory of the $U(1)$ –gauged sigma model. As stacks they are the same: $[c^4/z_2] = [X/c^{\times}]$.

⁷For example considering a decomposition given by $SO(4) = SU(2) \oplus SU(2)/z_2$.

⁸A gerbe is a construct in homological algebra and topology. A gerbe on a topological space X , is a stack G , of groupoids over X which is *locally non-empty* (each point in X , has an open neighbourhood U , over which the section category $G(U)$, of the gerbe is not empty) and *transitive* (for any two objects σ , and τ , of $G(U)$, for any open set U , there is an open covering $\{V_i\}$, of U , such that the restrictions of σ , and τ , to each V_i are connected by at least one morphism).

transform as a map congruent with $Z(G)$, modulo \mathbb{Z}_2 . The congruence of these maps defines the gerbe defined in (18) and shapes the composition of bundles [7,21,22]⁹

$$0 \rightarrow \mathbb{Z} \rightarrow \mathcal{E}_X \xrightarrow{\text{exp}} \mathcal{E}_X^* \rightarrow 1, \quad (19)$$

where \mathcal{E}_X^* , is the sheaf of the smooth invertible functions on X . Of this way the co-cycles obtained by the Penrose transform are Čech 1-co-cycles that come from the smooth principal $U(1)$ -bundle. The space \mathcal{E}_X , is a fine sheaf such that $H^2(X, \mathcal{E}_X) = 0$. Thus a principal $U(1)$ -bundle is a geometrical realization of one element of the cohomological space $H^1(X, \mathcal{E}_X^*)$. But we need establish the isomorphism between the moduli spaces $\mathcal{M}_{\text{Flat}}({}^L G, C)$, to D -modules and $(\mathcal{M}(G, C), \omega_K)$, of A -branes such and as is necessary to apply the geometrical Langlands correspondence between of the ${}^L \text{Bun}_G$ (bundles characterized on C) and the structure sheaves like the given in $\mathcal{O}(\mathbf{B})$ (with \mathbf{B} , a Hitchin base given by the cone $H^0(C, (\mathfrak{t} \times \omega_C) / \mathbb{W})$)¹⁰ to the *bosonic moduli space* that we want (remember $SU(2)$, is the electromagnetic group of weak interactions such as W^0, W^+, H). Remember that we want obtain a photonic carpet as a $U(1)_R$ -carpet of the space-time.

Then to the Čech 1-co-cycles we need that come from holomorphic $U(1)$ -bundles. Then the corresponding sheaf of invertible holomorphic functions on X , will be \mathcal{O}_X^* .

Then a geometrical realization of a 0-gerbe is the cohomological group $H^2(X, \mathcal{O}_X^*)$.¹¹ But the 0-gerbe corresponds to a principal $U(1)$ -bundle (in this case a gerbe equal to S^1 -bundle with connection) where their topological realization is a cohomological space $H^2(X, \mathbb{Z})$. Then considering the twisted image of this cohomological space we obtain a characteristic class of gerbe in the space $H^2(X, Z(G))$ which is the pre-image of the Penrose transform

$$H^2(X, \mathcal{O}) \cong \{ \Phi \in \Gamma(U, \Omega^2) / \Phi = d\theta, \theta \in \Gamma(U, \Omega^1) \} \quad (20)$$

in the stack ${}^L \text{Bun}_G$. But we want extend the electromagnetic field solution to the stacks such that

$${}^L \text{Higgs}_G = \hat{T} {}^L \text{Bun}_G, \quad (21)$$

But our moduli space of the corresponding stack moduli must be the orbifolds in $\Omega^n \otimes \mathcal{L}$, since we want field extensions given by the Higgs field $\Phi \in \Gamma(\mathcal{L}^{\otimes \varrho})$. Different bundles implies fields that have different zero modes, which implies different anomalies which implies different physics measured from the electromagnetic gauges. All these different field physics obtained are ramifications of the same field following different patrons given by $\tilde{C} \rightarrow C \times \mathbf{B}$. For example on a non-compact worldsheet (see the **Figure 3**), the instantons that have charge $1/k$, corresponding to electrons with charge k , and massive particles can appear with periodicity established by the closed forms whose cohomological classes lands in lattice

$$H^2(\Sigma, 2\pi R\mathbb{Z}) \leq H_{dR}^2(\Sigma), \quad (22)$$

Then the Čech 1-cocycles are $H^1(X, S^1)$, who represents the kernel $Z(G)$, of the map (18) which is the space of the flat bundles, that is to say, the moduli space $\mathcal{M}_{\text{Flat}}({}^L G, C)$, who is image of the functor of finite range¹²

⁹In this point we have in the flat complex space that

$$\mathcal{O}[1]_{\text{in } \mathbb{C}^4} \rightarrow T \rightarrow T_a \xrightarrow{\nabla_a} \mathcal{O}_{\text{in } \mathbb{M}}[1] \cong \mathbb{C},$$

which is conformally invariant (since the vector bundles of \mathbb{C} , in \mathbb{M} , need of the twistor space deduced to the derived wave operators of the affine connections let be conformally invariants). Then considering the corresponding to the forms to curved case $(\nabla_a \nabla_b \phi)$, such that $(\nabla_a \nabla_b - \nabla_b \nabla_a)\phi$, is the curvature of T (essentially the Weyl curvature of \mathbb{M}) $\forall \phi$, a 0-form in $U(1)$ we find an analogous connection to 2-forms, proving the obtaining of elements of the space $\Omega^2(\mathbb{M})$. In this composition T_a , is a twistor image of bundle using the connection ∇_a . Then by the Penrose transform there is isomorphism between respective vector bundles. But from the congruence between (18) and their Penrose transform $\mathcal{P} \circ \mathcal{T}$, we have a stack modulo \mathbb{Z}_2 .

¹⁰ \mathbb{W} , is the Weyl group.

¹¹This cohomological space to all space-time \mathbb{M} , must be $H^2(\mathbb{M}, U(1))$, accord with the map (18).

¹² π_1 , is the homomorphism given by $\pi_1 = \pi_0(\text{Higgs})$. Also is explicitly given to this case (Higgs fields) by the map

$$\begin{array}{ccc} \mathcal{M}_{\text{H}}(G, C) & \xrightarrow{\phi} & X \\ \pi_0 \downarrow & & \nearrow \pi_1 \\ \mathcal{M}_{\text{Higgs}}(G, C) & & \end{array}$$

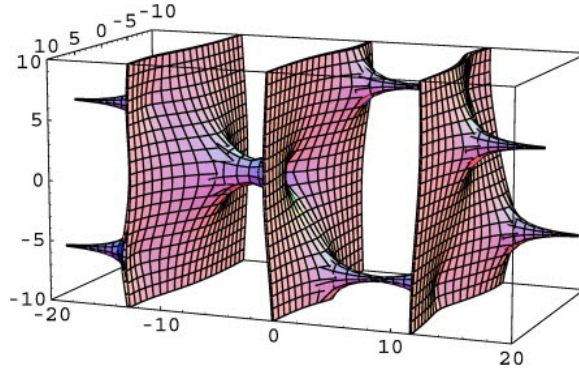


Figure 3. D -branes Moduli Stack $[X / \mathbb{C}^\times]$, calculated with the action of \mathbb{C}^\times , on Σ , lifting to an action $\tilde{C} \rightarrow C \times \Sigma$, where usually to the Riemannian surface Σ , we can consider the causal structure of \mathbb{M} . The physics are different and are corresponded to different “times” t , by a phase $t \mapsto \frac{|\tau|}{\tau} t$. The different physics can be viewed as sources (charges), holes (scattering or massive particles), warm holes (perturbative spaces), flat spaces (de Sitter regions, for example ∞ -neighborhoods in Lorentzian manifolds).

$$\Gamma H^2(\text{Hom}(Z(G), U(1))) \cong H^2(C, \pi_1({}^L G)), \tag{23}$$

But the MO-duality implies that the moduli spaces $\mathcal{M}_{\text{Flat}}({}^L G, C)$, and $\mathcal{M}_H(G, C)$, (with symplectic structure ω_k) are a mirror pair [25]. Then the flat \mathbf{B} -fields from $\mathcal{M}_H(G, C)$, take values in the space $H^2(C, Z(G)) = Z(G)$. But is well known that $Z(G)$ is naturally isomorphic to $\pi_1({}^L G)$. Newly by the arguments of the MO-duality we can map the class $w \in Z(G)$, defining the \mathbf{B} -field on $\mathcal{M}_H(G, C)$, to the corresponding element in $\pi_1({}^L G)$, labeling the connected component of $\mathcal{M}_H({}^L G, C)$, (and vice versa). Then to our case when $G = SU(2)$, then the moduli space $\mathcal{M}_H(G, C)$, is connected and has two possible flat \mathbf{B} -fields labeled by $Z(SU(2)) = \mathbb{Z}_2$. Then, for other side ${}^L G = SU(2) / \mathbb{Z}_2$, and therefore $\mathcal{M}_H({}^L G, C)$, has two components labeled by $\pi_1({}^L G) = \mathbb{Z}_2$.

Finally the sheaf of the open-string states (states that come from the D -branes) is the sheaf of holomorphic functions on $\mathcal{M}_H(G, C)$, in the complex structure of the symplectic complex structure $\omega_I = \omega_J + i\omega_K$ ¹³. Then $\mathcal{M}_H(G, C)$, can be identified with $\mathcal{M}_{\text{Higgs}}(G, C)$, which the bosonic moduli space that we want. ■

Example 4.1. We consider the Abelian Hodge theory approach, and let $\text{Loc}_C({}^L G)$, with C , the curve of their corresponding ramification with the same geometry to construct the pair or Langlands data (M, δ) , where $M = \text{Bun}_{\text{Higgs}}$, and

$$\delta : M \rightarrow M \otimes \Omega_{\text{Higgs}/B}^1, \tag{24}$$

This connection is a meromorphic relative flat connection acting along of fibers of the Hitchin map:

$$h : \text{Bun}_{\text{Higgs}} \rightarrow B, \tag{25}$$

Furthermore by construction the bundle with connection (M, δ) , is a Hecke eigen D -module with eigenvalue (E, ∇) , (as the found in [8]) but with respect to an Abelianized version

¹³Since has been used the Hitchin base (to involve a the torus \mathfrak{t} , and relate the symplectic form ω_k with ω_c . Then the isomorphism given by the Penrose transform must be (because we want include the Hitchin base to singularities of the field seen these as sources or holes) $H^2(M, \Omega^n \otimes \mathcal{L}) \cong \ker(U, \bar{\partial} + Q)$, where $\bar{\partial} + Q$, is the new connection considering the Langlands parameter to regularize the connection given by the global Langlands category of sheaves $\text{Loc}_C({}^L G)$, [8] to elements $Q \in \mathcal{D}_{\text{BRST}}$, in the Penrose transform problem when we want obtain the functor (4).

$$U(1), U(2), SO(2). {}^L\Phi_{Abelian}^i : D^b(\text{Bun}_{\text{Higgs}}, \mathcal{O}) \rightarrow D^b(\text{Bun}_{\text{Higgs}} \times C, \mathcal{O}), \quad (26)$$

of Hecke functors. These are defined again for $i = 1, 2, 3, \dots, n-1$, as integral transforms with respect to the trivial local system on the abelianized Hecke correspondences $\mathcal{H}_{G,\lambda}$, with $\text{Bun}_{\text{Higgs}}$, and $\text{Bun}_{\text{Higgs}} \times C$, in the classic scheme of the double fibration of the integral geometry (Penrose transforms!) on micro-local objects of these bundles. Their stack moduli is ${}^L\text{Higgs}$.

Corollary (F. Bulnes) 4.1. *An 2-dimensional A-brane model of the electromagnetic carpet is given by the moduli space $\mathcal{M}_{\text{Higgs}}(G, C)$, with 2d-dimension with $d=1$, and charge e , such that*

$$\omega_l = \frac{1}{e^2} \int_C \text{Tr}(\delta A \wedge \delta A - \delta \phi \wedge \delta \phi). \quad (27)$$

Proof. Applying [26], and considering the example 4.1, with the moduli space $\mathcal{M}_{\text{Higgs}}(G, C)$, with the hypothesis given for our corollary, the image under the Penrose transform on double fibration in $\text{Bun}_{\text{Higgs}} \times C$, that is to say, for example, the cohomological space $H^1(\mathbb{CP}^1, 2h-2)$, (this formulation is through the integral geometry) can establish a 2-dimensional image given as fibration (not vector bundle (mini-twistor surface)) in $\mathbb{CP}^1 \times \mathbb{CP}^1$ [5]¹⁴. Then the 2-dimensional image of the electromagnetic carpet is the generated by the rotations and sources such as in the **Figure 4**. ■

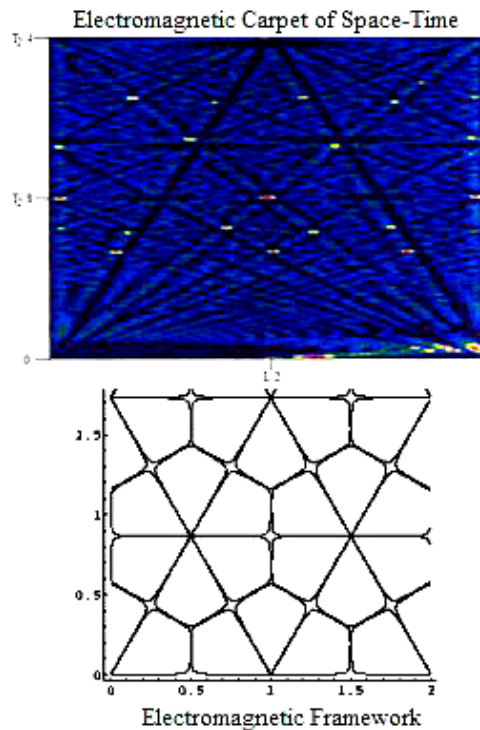


Figure 4. Electromagnetic carpet of space-time obtained by quantum electromagnetic waves using the coefficients $|\psi|^2$, from the energy of the waves to quantum level [27]. The model is obtained simulating the waves in the corresponding $SL(2, \mathbb{C})/U(1)$, model, using a two-points function on the torus. Of this way, is obtained the framework using this orbital space predicted by the gauge theory. In the carpet (blue surface) the nodes are states of energy ϕ , and lines (or routes) are the paths framework. The nodes are the singularities or sources given in the **Figure 2**.

¹⁴ \mathbb{CP}^1 , is the Riemann sphere S^2 .

For other way, the gauge field A , can be naturally coupled to sources which are transformed to unitary representations from $U(1)_R$, of $G = SU(2)$. These are electrical sources, since they create a charge like a Coulumb charge of the form

$$A_\theta^a \approx T^a \frac{\rho^2}{r}, \tag{28}$$

where T^a , $a=1$, and $\dim G$, are generators of G , in the representation R , from $U(1)_R$. Then if we consider to the space-time $\mathbb{M} = \mathbb{R}^3 \times \mathbb{R}$, with r , the distance from the origin in \mathbb{R}^3 , and we assume that the world-line of the source is given for $r=0$, [7], then we can establish that magnetic sources are labeled by irreducible representations of different groups as $U(1), U(2), SO(2)$.

We can establish using a path integral as the given by

$$Z = \sum_\nu \int \mathcal{D}A \exp\{-I(A)\} \tag{29}$$

the traces inside the symplectic structure given in (27), and draw the electro-physical carpet of the space-time with all constitutive sources “charges”. Here ν , is the principal G -bundle on M , where M , is the compact component of \mathbb{M} . For example, to vector bundle given by $\nu = \mathcal{O}(1)^{\otimes 4}$, which is basically the functor image $\Gamma(\mathcal{L}^{\otimes 4})$, (that appeared in the demonstration of the theorem 4.2) necessary to represent the space-time in their string $4d$ - model. Note that in QFT one sums over all isomorphism classes of ν .

It’s necessary the BRST-cohomology and the equivalence between QFT, with a re-interpretation in graph quantum theory using the string theory with a photon of the class of the boson field as calculated by $\Phi \in \Gamma(\mathcal{L}^{\otimes 4})$. Joining the paths of this *boson floor* we obtain the corresponding Q -fields to every instanton in path integral as given by [7]

$$Z = \sum_\nu \int \mathcal{D}A \mathcal{D}A \mathcal{D}A \exp\{\sum_j^3 i \mathfrak{S}_{Q_j}\} \tag{30}$$

having the transformation

$$Q_a \rightarrow \exp\{-i\Phi/2\}Q_a, Q^a \rightarrow \exp\{-i\Phi/2\}/Q^a, \exp\{i\Phi\} = |\tau|/\tau, \tag{31}$$

Then the three actions contemplated in (30) are consigned in $(Q_a + Q^a + \exp\{i\Phi\})$ [7].

From the *corollary* 4.1, and the before discussion with the frame (31) we have the following electromagnetic carpet and the corresponding their framework.

5. Conclusions

Develop a electromagnetic theory of space-time is equivalent to extend the electromagnetic forces to the gauge context using the concepts as supersymmetry from the conformality and holonomicity of the electromagnetic fields always present from the bosonic symmetry generators given from the dualities of the moduli spaces $\mathcal{M}_{\text{Flat}}({}^L G, C)$, and $\mathcal{M}_H(G, C)$. using their corresponding physical stacks that comes from their connections with ramifications [8].

Using the hypothesis of that all singularities of the space-time \mathbb{M} , are sources in different moduli stacks (mirror theory) of a Calabi-Yau manifold of the space-time \mathbb{M} , corresponding to the superprojective flat spaces $\mathbb{P}^{n/m}$, [5,14], we have that in their super-mini-twistor spaces the flatness is traduced in superconformality to these spaces, and their line integrals give the values of the singularities. The singularities co-shape varieties of different degrees of genus.

The functor given in (17) say us that the equivalences necessities to comply the dualities established in the theorem 4.2, must be considering the Harish-Chandra modules that are quasi- G -equivariants $-D_{G/H}$ - modules such and as must do it on moduli space \mathcal{M}_0 , in the level of the functors obtained by the Penrose transform to the problem of ramification signed in the theorem 4.2, on *geometrical Langlands correspondences* and their corresponding derived category (the Hecke category) given in [8]. Inside of the set of dualities established on moduli stacks, the *MO*-dualities (*Montonen-Olive duality*) are of special attention due to their application in twisted theory of the different classes that appear when is used the twistor transform inside the Penrose transform, this in frame of the field problem (for example the correspondences between A -branes and D -branes [7] or the behavior of the field in different orbits (see **Figure 5**) to after extend it to all space-time, and of this

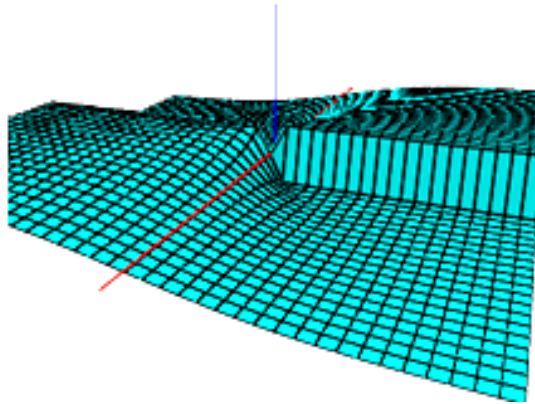


Figure 5. Hyperbolic flat model of the space-time with one singularity of space (could be one diagram in **Figure 4 or 6**) This model is product of two orbital spaces [17,28], and conform the surface of photon back-reaction in presence of singularity, this visualized as a super-massive particle. This simulation is shaped for complex hyperbolic surface $z(x, y) = (1/1.9) \tanh(2x/y)$ [14, 27]. The massive particle corresponds to the sources of photons and these are their nodes in the **Figure 4**, corresponds to the energy states average given by the path integrals that connect them. Every node of energy that appears in the electromagnetic framework is the singularity of the hyperbolic space used by the twistor transform [21], and their neared space to the link-wave in Feynman diagrams in **Figure 6**, is the surface that is showed.

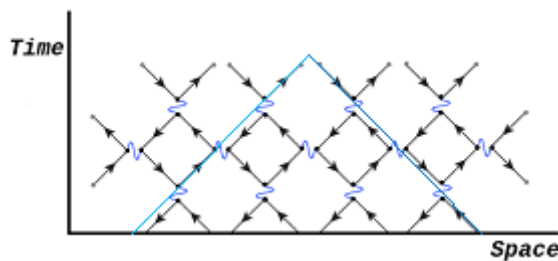


Figure 6. From the way established in the **Figure 4**, is obtained the framework using this orbital space predicted by the gauge theory. In the carpet (blue surface) the nodes are states of energy ϕ , and lines (or ruts) are the paths framework.

manner to obtain the electromagnetic field carpet that we want. The *Theorem 4.2*, is the first intent through the elements and methods of the integral geometry to obtain different geometrical models, considering different electromagnetic interactions or adding they until to arrive to the curved case, which will can be included considering the bundle $\mathbb{M} \rightarrow S^4$.

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