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Symbolic Computation and New Exact Travelling Solutions for the (2+1)-Dimensional Zoomeron Equation

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Abstract

In this paper, we present Yan's sine-cosine method and Wazwaz's sine-cosine method to solve the (2+1)-dimensional Zoomeron equation. New exact travelling wave solutions are explicitly obtained with the aid of symbolic computation. The study confirms the power of the two schemes.

Keywords

Sine-Cosine Method, (2+1)-Dimensional Zoomeron Equation, Nonlinear Evolution Equations

1. Introduction

In recent years, nonlinear evolution equations (NLEES) are widely used to describe complex phenomena in various fields of sciences, such as physics, biology, chemistry, etc. Therefore, seeking exact solutions of nonlinear evolution equations (NLEES) plays an important role in mathematical physics. In the past decades, many effective methods have been presented, such as the inverse scattering method [1], Hirota bilinear method [2], the tanh-function method [3] [4], homogeneous balance method [5] [6], Jacobi elliptic function method [7] [8], the first-integral method [9] [10], the Exp-function method [11]-[13], the $\left(\frac{G'}{G}\right)$-expansion method [14]-[16] and so on.

Recently, Yan [17] directly obtained a simple transformation from the famous sine-Gordon equation. The simple transformation was used to get more solutions of a wide class of nonlinear wave equations [17]-[19]. The simple transformation which named sine-cosine method is based on the assumptions that the travelling wave solutions can be expressed by a trigonometric polynomial as follows:

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$$u(\xi)=\sum_{j=1}^{n}\cos^{j-1}\omega(\xi)\left[B_j\sin\omega(\xi)+A_j\cos\omega(\xi)\right]+A_0,\quad \frac{\mathrm{d}\omega}{\mathrm{d}\xi}=\sin\omega. \tag{1}$$

The degree of the polynomial can be determined by considering the homogeneous balance between the highest order derivative and nonlinear terms appearing in the given NLEE. The coefficients of the polynomial can be obtained by solving a set of algebraic equations resulted from the process of using the method. More recently, a new sine-cosine method was proposed by Wazwaz [20]. The new sine-cosine algorithm admits the use of the ansatzes

$$u(\xi)=\lambda\cos^{\beta}(\mu\xi),\quad |\xi|\le\frac{\pi}{2\mu}, \tag{2}$$

$$u(\xi)=\lambda\sin^{\beta}(\mu\xi),\quad |\xi|\le\frac{\pi}{2\mu}, \tag{3}$$

where λ, β and μ are parameters that will be determined later. By using Wazwaz's sine-cosine method, many nonlinear equations [20]-[28] have been successfully solved.

In the present paper, we will extend the two sine-cosine methods to the following (2+1)-dimensional Zoomeron equation:

$$\left(\frac{u_{xy}}{u}\right)_{tt}-\left(\frac{u_{xy}}{u}\right)_{xx}+2\left(u^2\right)_{xt}=0, \tag{4}$$

where $u(x,y,t)$ is the amplitude of the relevant wave mode; see [29]. To the best of our knowledge, there are a few articles about this equation. By applying the $\left(\frac{G'}{G}\right)$-expansion method, Abazari [30] obtained some periodic and soliton solutions to the Zoomeron equation. Recently, Alquran and Al-Khaled [31] studied the Zoomeron equation using the extended tanh, the exp-function and the $\operatorname{sech}^p-\tanh^p$ methods. In the subsequent section, we will illustrate the two sine-cosine methods in detail with the (2+1)-dimensional Zoomeron equation.

2. Yan's Sine-Cosine Method for the (2+1)-Dimensional Zoomeron Equation

In this section, we start out our study for Equation (4) by Yan's sine-cosine method. Firstly, making the following wave variable

$$u(x,y,t)=U(\xi),\quad \xi=\mu(x+cy-vt), \tag{5}$$

where c, and v and μ are constants to be determined later. Substitute Equation (5) into Equation (4) and integrating twice with respect to ξ, by setting the second constant of integration to zero, we obtain the following ODE:

$$c\mu^2\left(1-v^2\right)U''-2vU^3+RU=0, \tag{6}$$

where R is integration constant. According to Yan's sine-cosine method, we make an ansatz (1) for the solution of Equation (6). Balancing the terms U^3 and U'' in Equation (6) yields the leading order $n=1$ (from $3n=n+2$). Therefore, we can write the solution of Equation (6) in the form

$$U(\xi)=B_1\sin\omega+A_1\cos\omega+A_0,\quad \frac{\mathrm{d}\omega}{\mathrm{d}\xi}=\sin\omega, \tag{7}$$

and

$$\frac{\mathrm{d}\omega}{\mathrm{d}\xi}=\sin\omega, \tag{8}$$

where B_1, A_1 and A_0 are unknown constants.

Substituting (7) into (6), collecting the coefficients of $\sin^j\omega\cos^k\omega$ $(j=0,1;k=0,1,2,3)$ and set it to zero we obtain the following system of algebraic equations for: A_0, A_1, B_1, c, v, and μ:

$$
\begin{aligned}
&\text{constant}: -2vA_0^3 + RA_0 - 6vB_1^2A_0 = 0,\\
&\sin\omega: -2vB_1^3 + c\mu^2v^2B_1 - c\mu^2B_1 + RB_1 - 6vB_1A_0^2 = 0,\\
&\cos\omega: -6vB_1^2A_1 + RA_1 - 6vA_1A_0^2 - 2c\mu^2A_1 + 2c\mu^2v^2A_1 = 0,\\
&\sin\omega\cos\omega: -12vB_1A_1A_0 = 0,\\
&\cos^2\omega: 6vB_1^2A_0 - 6vA_1^2A_0 = 0,\\
&\sin\omega\cos^2\omega: -2c\mu^2v^2B_1 - 6vB_1A_1^2 + 2c\mu^2B_1 + 2vB_1^3 = 0,\\
&\cos^3\omega: 2c\mu^2A_1 + 6vB_1^2A_1 - 2vA_1^3 - 2c\mu^2v^2A_1 = 0.
\end{aligned}
$$

Solving the above system by Matlab gives

***Case* 1.**

$$A_0 = B_1 = 0,\quad c = \frac{-2RA_1^4}{\mu^2\left(R^2 - 4A_1^4\right)},\quad v = \frac{R}{2A_1^2}, \tag{9}$$

where R, μ and A_1 are arbitrary constants.

***Case* 2.**

$$A_0 = A_1 = 0,\quad c = \frac{RB_1^4}{\mu^2\left(R^2 - B_1^4\right)},\quad v = \frac{R}{B_1^2}, \tag{10}$$

where R, μ and B_1 are arbitrary constants.

***Case* 3.**

$$A_0 = 0,\quad B_1 = \pm iA_1,\quad c = \frac{-8RB_1^4}{\mu^2\left(R^2 - B_1^4\right)},\quad v = -\frac{R}{2B_1^2}, \tag{11}$$

where $i = \sqrt{-1}$, R, μ and A_1 are arbitrary constants.

Now, we consider Equation (8). By using the separation of variables method the solutions of Equation (8) are easily written in the following form

$$\sin\omega = \frac{2p\exp(\pm\xi)}{p^2\exp(\pm 2\xi)+1} = \operatorname{sech}(\xi), \tag{12}$$

or

$$\cos\omega = \frac{1 - p^2\exp(\pm 2\xi)}{p^2\exp(\pm\xi)+1} = \mp\tanh(\xi), \tag{13}$$

where $p = 1$ is the integration constant.

Finally, combining (5), (7), (12), (13) along with cases 1 - 3, we find the following three types of travelling wave solutions for Equation (4):

***Type* 1.**

$$u(x,y,t) = A_1\tanh\left(\mu\left(x - \frac{2RA_1^4}{\mu^2\left(R^2 - 4A_1^4\right)}y - \frac{R}{2A_1^2}t\right)\right), \tag{14}$$

where R, μ and A_1 are arbitrary constants.

***Type* 2.**

$$u(x,y,t) = B_1\operatorname{sech}\left(\mu\left(x + \frac{RB_1^4}{\mu^2\left(R^2 - B_1^4\right)}y - \frac{R}{B_1^2}t\right)\right), \tag{15}$$

where R, μ and B_1 are arbitrary constants.

***Type* 3.**

$$u(x,y,t)=A_1\left(\tanh\left(\mu\left(x-\frac{8RA_1^4}{\mu^2\left(R^2-A_1^4\right)}y+\frac{R}{2A_1^2}t\right)\right)\pm i\,\mathrm{sech}\left(\mu\left(x-\frac{8RA_1^4}{\mu^2\left(R^2-A_1^4\right)}y+\frac{R}{2A_1^2}t\right)\right)\right), \tag{16}$$

where $i=\sqrt{-1}$, R, μ and A_1 are arbitrary constants.

Then if we take $A_1=\mp\frac{1}{2}\sqrt{\frac{-2R}{w}}$ and $\mu=\frac{1}{2}\sqrt{\frac{2R}{c\left(w^2-1\right)}}$ in the new form of (14), it is easy that our results can reduce to Abazari's [30] result (21a). When setting $B_1=\mp\frac{\sqrt{R}}{\sqrt{w}}$, our solution (15) will be same as Alquran's [31] result (15). It is worth to note that our solution (16) is not derived in [30] [31].

3. Wazwaz's Sine-Cosine Method for the (2+1)-Dimensional Zoomeron Equation

Now, we use Wazwaz's sine-cosine method to handle Equation (4). Substituting (2) into (6) gives

$$-c\left(1-v^2\right)\mu^2\beta^2\lambda\cos^{\beta}(\mu\xi)+c\left(1-v^2\right)\mu^2\lambda\beta(\beta-1)\cos^{\beta-2}(\mu\xi)-2v\lambda^3\cos^{3\beta}(\mu\xi)+R\lambda\cos^{\beta}(\mu\xi)=0. \tag{17}$$

The equation is satisfied only if the following system of algebraic equations hold

$$\begin{aligned}&\beta-2=3\beta,\\&c\left(1-v^2\right)\mu^2\beta(\beta-1)=2v\lambda^2,\\&R\lambda=c\left(1-v^2\right)\mu^2\beta^2\lambda.\end{aligned} \tag{18}$$

Solving the system (18) leads to the following sets of solutions:

$$\begin{aligned}&\beta=-1,\\&\lambda=\pm\sqrt{\frac{R}{v}},\\&c=\frac{R}{\left(1-v^2\right)\mu^2}\end{aligned} \tag{19}$$

where R, μ and v are any arbitrary constant. Therefore, the solution of Equation (4) is

$$u(x,y,t)=\pm\sqrt{\frac{R}{v}}\sec\left(\mu\left(x+\frac{R}{\left(1-v^2\right)\mu^2}y-vt\right)\right). \tag{20}$$

Now, if we use the ansatze (3) instead of (2), then we get the same system (18) and therefore, the solution is

$$u(x,y,t)=\pm\sqrt{\frac{R}{v}}\csc\left(\mu\left(x+\frac{R}{\left(1-v^2\right)\mu^2}y-vt\right)\right). \tag{21}$$

To the best of our knowledge, solutions (20) and (21) have not been reported in the literature.

4. Conclusion

The two sine-cosine methods have been successfully applied here to seek exact solutions of the (2+1)-dimensional Zoomeron equation. As a result, a series of new exact solutions are obtained and some solutions given in [30] [31] are only our special cases. The solution procedure is very simple, and the obtained solution is very concise. It is shown that the sine-cosine method provides a very effective and powerful mathematical tool for solving nonlinear equations in mathematical physics.

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