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Solving Intuitionistic Fuzzy Linear Programming Problem—II

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Abstract

Under non-random uncertainty, a new idea of finding a possibly optimal solution for linear programming problem is examined in this paper. It is an application of the intuitionistic fuzzy set concept within scope of the existing fuzzy optimization. Here, we solve a linear programming problem (LPP) in an intuitionistic fuzzy environment and compare the result with the solution obtained from other existing techniques. In the process, the result of associated fuzzy LPP is also considered for a better understanding.

Keywords

Fuzzy Linear Programming Problem (FLPP), Intuitionistic Fuzzy Set (IFS), Intuitionistic Fuzzy Number (IFN), Intuitionistic Fuzzy Linear Programming Problem (IFLPP)

1. Introduction

Linear programming is part of a very important area of applied mathematics called "optimization techniques". They may deal with hundreds of variables simultaneously, but they fail to handle imprecise data. Fuzzy linear programming was introduced to capture this imprecision in linear programming problem (LPP). Later, various modification methods have appeared from different interpretations. Intuitionistic fuzzy set (IFS) developed by Atanassov [1] [2] is one of them. In a fuzzy set, only the degree of acceptance of an element belongs to the set is considered. But in IFS a membership function, *i.e.*, degree of acceptance and a non-membership function, *i.e.*, degree of rejection are considered simultaneously so that the sum of both values for each element of the set is not exceeding one. Thus, intuitionistic fuzzy set [1] [2] [3] [4] has been found to be highly useful in dealing with imprecision in optimization techniques. Since this fuzzy set generalization can present the degrees of membership and non-membership of an element of the set with a degree of hesitancy, the knowledge and semantic representation becomes more meaningful and applicable.

The concept of maximizing decision under uncertainty was proposed by Bellman and Zadeh [5]. This concept of using fuzzy sets applied to the problems of mathematical programming by Tanaka and others. Zimmermann [6] presented a fuzzy approach to multi-objective linear programming problem. Fuzzy linear programming problem with fuzzy coefficients was formulated by Negoita [7] and are called robust programming. Dubois and Prade investigated optimization with linear fuzzy constraints [8]. Tanaka and Asai proposed a formulation of fuzzy linear programming with fuzzy constraints and suggested a method for its solution which is based on inequality relation between fuzzy numbers [9]. This ranking of fuzzy numbers plays a significant role in the study of optimization problems.

Recent years have witnessed growing interest in the study of decision-making problems with intuitionistic fuzzy sets/numbers [10]-[15]. Recently, we proposed a method to solve intuitionistic fuzzy linear programming problem (IFLPP) using a technique based on an earlier technique proposed by Zimmermann for solving fuzzy linear programming problems [15]. Authors in [16] presented an overview on IFS viz., some definitions, basic operations, some algebra, modal operators and also its normalization. Later, D. Dubey [17] proposed an approach based on value and ambiguity indices to solve LPPs with data as Triangular Intuitionistic Fuzzy Numbers (TIFN). Parvathi and Malathi [18] [19] worked on the intuitionistic fuzzy decisive set method, which is a combination of bisection method and phase one of the simplex method to obtain a feasible solution. In [20], the authors described a method to approximate a TIFN to a nearly approximated interval number. The average ranking index is also introduced here to find out order relations between two TIFNs. On ranking intuitionistic fuzzy numbers, some work had been reported in the literature. Mitchell [21] considered the problem of ranking a set of intuitionistic fuzzy numbers to define a fuzzy rank and a characteristic vagueness factor for each intuitionistic fuzzy number. Ranking using score function is introduced in [22]. Here, all the arithmetic operations of TIFN are based on (α,β)-cut method. Ranking of intuitionistic fuzzy number with expected interval is introduced in [23]. A. N. Gani and S. Abbas [24] worked on a new average method for finding an optimal solution for an intuitionistic fuzzy transportation problem. The main feature of this method is that it requires very simple arithmetical calculations and avoids large number of iterations. An accuracy function to defuzzify TIFN is also used here.

Angelov [25] [26] proposed optimization in an intuitionistic fuzzy environment. Hussain and Kumar [27] [28] [29] and Nagoor Gani and Abbas [24] proposed a method for solving intuitionistic fuzzy transportation problem. Ye [30] discussed expected value method for intuitionistic trapezoidal fuzzy multicriteria decision-making problems. Wan and Dong [31] used possibility degree method for interval-valued intuitionistic fuzzy numbers for decision making.

In this paper, our aim is to propose a method to solve IFLPP when both the co-efficient matrix of the constraints and the cost co-efficients are represented as triangular intuitionistic fuzzy numbers and compare it with the case when both of them are triangular fuzzy numbers. Each problem is first converted into an equivalent crisp linear programming problem with the help of (α-β)-cut [20], which are then solved by standard optimization methods. A comparative study with other optimization techniques [15] [32] in fuzzy and intuitionistic fuzzy environment is undertaken and interesting results are presented.

This work seeks to study extensively the existing fuzzy [32] [33] and intuitionistic fuzzy [15] optimization techniques, and thereby develop an algorithm for finding solution of an intuitionistic fuzzy linear programming problem depending on the (α-β)-cut [20] and then compare it to a fuzzy environment depending only on the α-cut.

The paper is organized into seven sections. After a brief introductory section we present some basic concepts necessary for the development of a mechanism for solving intuitionistic fuzzy linear programming problems in Section 2. In Section 3, we discuss the mathematical formulation of our proposed technique to solve IFLPP when both the coefficient matrix of the constraints and cost coefficients are represented by triangular intuitionistic fuzzy numbers. In Section 4, we develop an algorithm and illustrate the same with some numerical examples. In Section 5, a comparative study between triangular intuitionistic fuzzy and triangular fuzzy environment is presented. Proposed result is also compared with other fuzzy [32] and intuitionistic fuzzy [15] optimization techniques. The present paper is concluded in section 6 which is followed by a list of references in the last section.

2. Preliminaries

Definition 1 [20]. Let $U=\{x_1,x_2,\cdots,x_n\}$ be a finite universal set. An intuitionistic fuzzy set (IFS) $\tilde{A}$ in a given universal set U is an object having the form

$$\tilde{A}=\left\{\left\langle x_j,\mu_{\tilde{A}}\left(x_j\right),\nu_{\tilde{A}}\left(x_j\right)\right\rangle : x_j\in U\right\}$$

where the functions $\mu_{\tilde{A}}:U\to[0,1]$ and $\nu_{\tilde{A}}:U\to[0,1]$ respectively define the degree of membership and the degree of non-membership of an element $x_j\in U$, such that they satisfy the following condition:

$$0\le\mu_{\tilde{A}}\left(x_j\right)+\nu_{\tilde{A}}\left(x_j\right)\le 1,\ \forall x_j\in U;$$

known as intuitionistic condition. The degree of acceptance $\mu_{\tilde{A}}(x)$ and of non-acceptance $\nu_{\tilde{A}}(x)$ can be arbitrary.

Definition 2 [18]. For all $\tilde{A}\in IFS(U)$, let $\pi_{\tilde{A}}\left(x_j\right)=1-\mu_{\tilde{A}}\left(x_j\right)-\nu_{\tilde{A}}\left(x_j\right)$, which is called the Atanassov's intuitionistic index of the element x_j in the set $\tilde{A}$ or the degree of uncertainty or the indeterministic part of x_j or a measure of hesitation. Obviously,

$$0 \le \pi_{\tilde{A}}(x) \le 1; \forall x_j \in U.$$

When $\pi_{\tilde{A}}(x) = 0$, $\forall x \in U$, *i.e.*, $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) = 1$, $\tilde{A}$ becomes a fuzzy set. Therefore, a fuzzy set is a special intuitionistic fuzzy set.

Definition 3 [16]. Let $\tilde{A}$ and $\tilde{B}$ be two Atanassov's IFSs defined on *U*. $\tilde{A} \subset \tilde{B}$ if and only if $\mu_{\tilde{A}}(x_j) \le \mu_{\tilde{B}}(x_j)$ and $\nu_{\tilde{A}}(x_j) \ge \nu_{\tilde{B}}(x_j)$; for all $x_j \in U$.

Definition 4 [16]. Let $\tilde{A}$ and $\tilde{B}$ be two Atanassov's IFSs defined on *U*. $\tilde{A} = \tilde{B}$ if and only if $\mu_{\tilde{A}}(x_j) = \mu_{\tilde{B}}(x_j)$ and $\nu_{\tilde{A}}(x_j) = \nu_{\tilde{B}}(x_j)$; for all $x_j \in U$.

Definition 5 [20]. An intuitionistic fuzzy set A of U is said to be normal if $\exists x_0 \in U$ such that $\mu_{\tilde{A}^j}(x_0) = 1$, (so $\nu_{\tilde{A}^j}(x_0) = 0$).

Definition 6 [20]. A subset (α, β)-cut of *U*, generated by IFS $\tilde{A}$, where $\alpha, \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \le 1$ is defined as

$$\tilde{A}_{\alpha,\beta} = \left\{ x_j \in U : \mu_{\tilde{A}}(x_j) \ge \alpha, \nu_{\tilde{A}}(x_j) \le \beta \right\}.$$

Thus, the (α, β)-cut of an intuitionistic fuzzy set to be denoted by $\tilde{A}_{(\alpha,\beta)}$, is defined as the crisp set of elements x which belong to $\tilde{A}$ at least to the degree α and which does not belong to $\tilde{A}$ at most to the degree β.

Definition 7 [20]. An intuitionistic fuzzy number (IFN) $\tilde{A}^j$ is

1) An intuitionistic fuzzy subset of the real line $\Re$;

2) Normal, *i.e.*, $\exists x_0 \in \Re$ such that $\mu_{\tilde{A}^j}(x_0) = 1$ (so $\nu_{\tilde{A}^j}(x_0) = 0$);

3) Convex for the membership function, *i.e.*,

$$\mu_{\tilde{A}^j}(\lambda x_1 + (1-\lambda) x_2) \ge \min\left\{ \mu_{\tilde{A}^j}(x_1), \mu_{\tilde{A}^j}(x_2) \right\}; \forall x_1, x_2 \in \Re, \lambda \in [0,1];$$

4) Concave for the non-membership function, *i.e.*,

$$\nu_{\tilde{A}^j}(\lambda x_1 + (1-\lambda) x_2) \le \max\left\{ \nu_{\tilde{A}^j}(x_1), \nu_{\tilde{A}^j}(x_2) \right\}; \forall x_1, x_2 \in \Re, \lambda \in [0,1].$$

Definition 8 [20]. A triangular intuitionistic fuzzy number (TIFN) $\tilde{A}^I = \langle a, l, r \rangle$ is a special IFS on the real number set $\Re$, whose membership function and non-membership functions are defined as follows:

$$\mu_{\tilde{A}^t}(x) = \begin{cases} \dfrac{x-a+l}{l} w_a; & a-l \le x < a \\ \dfrac{a+r-x}{r} w_a; & a \le x \le a+r \\ 0; & \text{otherwise,} \end{cases} \tag{1}$$

and

$$\nu_{\tilde{A}^t}(x) = \begin{cases} \dfrac{(a-x)+u_a(x-a+l)}{l}; & a-l \le x < a \\ \dfrac{(x-a)+u_a(a+r-x)}{r}; & a \le x \le a+r \\ 1; & \text{otherwise;} \end{cases} \tag{2}$$

where *l*, *r* are called spreads and *a* is called mean value. w_a and u_a represent the maximum degree of membership and minimum degree of non-membership respectively such that they satisfy the condition

$$0 \le w_a \le 1, 0 \le u_a \le 1 \text{ and } 0 \le w_a + u_a \le 1.$$

Definition 9 [18]. Any vector $x \in R^n$ which satisfies the constraints and non-negative restrictions is said to be an intuitionistic fuzzy feasible solution.

Let S be the set of all intuitionistic fuzzy feasible solutions. Any vector $x_0 \in S$ is said to be an intuitionistic fuzzy optimum solution if $Cx_0 \geq Cx \ \forall x \in S$ where $C = (c_1, c_2, \cdots, c_n)$ and $Cx = c_1x_1 + c_2x_2 + \cdots + c_nx_n$.

Definition 10 [20]. For $m_1 \leq m_2$ and $w_1 + w_2 \neq 0$, the value judgement index or acceptability index (AI) of $\tilde{A}^I_{\alpha,\beta} \leq \tilde{B}^I_{\alpha,\beta}$ is defined by

$$\mathrm{AI}\left(\tilde{A}^I_{\alpha,\beta} \leq \tilde{B}^I_{\alpha,\beta}\right) = \frac{m_2 - m_1}{2(w_1 + w_2)}.$$

3. Mathematical Formulation of IFLPP

Intuitionistic fuzzy optimization (IFO), a method of optimization under uncertainty, is put forward on the basis of intuitionistic fuzzy sets due to Atanassov [1]. It is an extension of fuzzy optimization in which the degrees of rejection of objective(s) and constraints are considered together with the degrees of their satisfaction.

There is no additional assumption about the nature of cost of decision variables and constraints. According to different considerations, distinct IFLPP could be obtained. We consider the case in which cost of decision variables and co-efficient matrix of constraints are represented as triangular intuitionistic fuzzy numbers and it is checked with a numerical example.

$$\max \tilde{Z} = \tilde{c}^I x = \sum_{k=1}^{n} \tilde{c}^I_k x_k$$

subject to

$$\sum_{k=1}^{n} \tilde{A}^I_{jk} x_k \leq \tilde{B}^I_j; 1 \leq j \leq m, x_k \geq 0; 1 \leq k \leq n \quad \text{where, } x = (x_1, x_2, \cdots, x_n)'.$$

Which is equivalent to,

$$\max \tilde{Z} = \sum_{k=1}^{n} (c_a, c_l, c_r)^I_k x_k$$

subject to

$$\sum_{k=1}^{n} (a_a, a_l, a_r)^I_{jk} x_k \leq (b_a, b_l, b_r)^I_j; \ j = 1, 2, \cdots, m.$$

Now, to solve the above IFLPP, first we find (α, β)-cut [20] of each TIFN as in the following:

$$\tilde{A}^I_{\alpha,\beta} = \begin{cases} A^I_\beta; & \text{if } \alpha < \dfrac{(1-\beta)w_a}{1-u_a} \\ A^I_\alpha; & \text{if } \alpha > \dfrac{(1-\beta)w_a}{1-u_a} \\ A^I_\beta \text{ or } A^I_\alpha; & \text{if } \alpha = \dfrac{(1-\beta)w_a}{1-u_a} \end{cases} \tag{3}$$

where $0 \leq \alpha \leq w_a, u_a \leq \beta \leq 1$ such that $0 \leq \alpha + \beta < 1$ and $0 \leq w_a + u_a < 1$.

Case 1: When

$$\alpha < \frac{(1-\beta) w_a}{1-u_a}, \tilde{A}^I_{\alpha,\beta} = A^I_\beta; \tag{4}$$

Now, according to the definition of TIFN, $\tilde{A}_\beta$ is a closed interval [20], denoted by $\tilde{A}_\beta = [A_L(\beta), A_R(\beta)]$, which can be calculated as,

$$A_L(\beta) = (a - l_a) + \frac{l_a(1-\beta)}{1-u_a}, \text{ and } A_R(\beta) = (a + r_a) - \frac{r_a(1-\beta)}{1-u_a}.$$

Then, the above IFLPP reduces to the following,

$$\max \tilde{Z} = \sum_{k=1}^{n} \left[(c_a - c_l) + \frac{c_l(1-\beta)}{1-u_a}, (c_a + c_r) - \frac{c_r(1-\beta)}{1-u_a} \right]_k x_k$$

subject to

$$\sum_{k=1}^{n} \left[(a_a - a_l) + \frac{a_l(1-\beta)}{1-u_a}, (a_a + a_r) - \frac{a_r(1-\beta)}{1-u_a} \right]_{jk} x_k$$
$$\le \left[(b_a - b_l) + \frac{b_l(1-\beta)}{1-u_a}, (b_a + b_r) - \frac{b_r(1-\beta)}{1-u_a} \right]_j;$$

i.e.,

$$\max \tilde{Z} = \left[(c_a - c_l) + \frac{c_l(1-\beta)}{1-u_a}, (c_a + c_r) - \frac{c_r(1-\beta)}{1-u_a} \right]_1 x_1$$
$$+ \left[(c_a - c_l) + \frac{c_l(1-\beta)}{1-u_a}, (c_a + c_r) - \frac{c_r(1-\beta)}{1-u_a} \right]_2 x_2$$
$$+ \cdots + \left[(c_a - c_l) + \frac{c_l(1-\beta)}{1-u_a}, (c_a + c_r) - \frac{c_r(1-\beta)}{1-u_a} \right]_n x_n$$

subject to

$$\left[(a_a - a_l) + \frac{a_l(1-\beta)}{1-u_a}, (a_a + a_r) - \frac{a_r(1-\beta)}{1-u_a} \right]_{11} x_1$$
$$+ \left[(a_a - a_l) + \frac{a_l(1-\beta)}{1-u_a}, (a_a + a_r) - \frac{a_r(1-\beta)}{1-u_a} \right]_{12} x_2$$
$$+ \cdots + \left[(a_a - a_l) + \frac{a_l(1-\beta)}{1-u_a}, (a_a + a_r) - \frac{a_r(1-\beta)}{1-u_a} \right]_{1n} x_n$$
$$\le \left[(b_a - b_l) + \frac{b_l(1-\beta)}{1-u_a}, (b_a + b_r) - \frac{b_r(1-\beta)}{1-u_a} \right]_1;$$

$$\left[(a_a - a_l) + \frac{a_l(1-\beta)}{1-u_a}, (a_a + a_r) - \frac{a_r(1-\beta)}{1-u_a} \right]_{21} x_1$$
$$+ \left[(a_a - a_l) + \frac{a_l(1-\beta)}{1-u_a}, (a_a + a_r) - \frac{a_r(1-\beta)}{1-u_a} \right]_{22} x_2$$
$$+ \cdots + \left[(a_a - a_l) + \frac{a_l(1-\beta)}{1-u_a}, (a_a + a_r) - \frac{a_r(1-\beta)}{1-u_a} \right]_{2n} x_n$$
$$\le \left[(b_a - b_l) + \frac{b_l(1-\beta)}{1-u_a}, (b_a + b_r) - \frac{b_r(1-\beta)}{1-u_a} \right]_2;$$

$$\vdots$$

$$\left[(a_a-a_l)+\frac{a_l(1-\beta)}{1-u_a},(a_a+a_r)-\frac{a_r(1-\beta)}{1-u_a}\right]_{m1}x_1$$

$$+\left[(a_a-a_l)+\frac{a_l(1-\beta)}{1-u_a},(a_a+a_r)-\frac{a_r(1-\beta)}{1-u_a}\right]_{m2}x_2$$

$$+\cdots+\left[(a_a-a_l)+\frac{a_l(1-\beta)}{1-u_a},(a_a+a_r)-\frac{a_r(1-\beta)}{1-u_a}\right]_{mn}x_n$$

$$\le\left[(b_a-b_l)+\frac{b_l(1-\beta)}{1-u_a},(b_a+b_r)-\frac{b_r(1-\beta)}{1-u_a}\right]_m .$$

Now, using the concept of comparison between interval numbers [20], we obtain if $m_1 \le m_2$ and $w_1+w_2 \ne 0$, the value judgement index or acceptability index (AI) of $\tilde{A}^I_{\alpha,\beta} \le \tilde{B}^I_{\alpha,\beta}$ is defined by

$$\mathrm{AI}\left(\tilde{A}^I_{\alpha,\beta} \le \tilde{B}^I_{\alpha,\beta}\right)=\frac{m_2-m_1}{2(w_1+w_2)}\ge 0 \text{ and } \max\tilde{Z}=\sum_{k=1}^n \tilde{c}^I_{\alpha,\beta}x_k$$

is constructed as,

$$\max\tilde{Z}=\sum_{k=1}^n \tilde{c}^I_{\alpha,\beta}x_k=\sum_{k=1}^n\frac{1}{2}\left[c_L(\beta)+c_R(\beta)\right]x_k .$$

Hence, the above IFLPP can be reformulated as:

$$\max\tilde{Z}=\frac{1}{2}\left[(c_a-c_l)+\frac{c_l(1-\beta)}{1-u_a}+(c_a+c_r)-\frac{c_r(1-\beta)}{1-u_a}\right]_1 x_1$$

$$+\frac{1}{2}\left[(c_a-c_l)+\frac{c_l(1-\beta)}{1-u_a}+(c_a+c_r)-\frac{c_r(1-\beta)}{1-u_a}\right]_2 x_2$$

$$+\cdots+\frac{1}{2}\left[(c_a-c_l)+\frac{c_l(1-\beta)}{1-u_a}+(c_a+c_r)-\frac{c_r(1-\beta)}{1-u_a}\right]_n x_n$$

i.e.,

$$\max\tilde{Z}=\left[c_a+\frac{c_r-c_l}{2}\frac{\beta-u_a}{1-u_a}\right]_1 x_1+\left[c_a+\frac{c_r-c_l}{2}\frac{\beta-u_a}{1-u_a}\right]_2 x_2$$

$$+\cdots+\left[c_a+\frac{c_r-c_l}{2}\frac{\beta-u_a}{1-u_a}\right]_n x_n$$

subject to

$$\left[b_a+\frac{b_r-b_l}{2}\frac{\beta-u_a}{1-u_a}\right]_1-\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{11}x_1$$

$$-\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{12}x_2-\cdots-\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{1n}x_n\ge 0$$

i.e.,

$$\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{11}x_1+\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{12}x_2$$

$$+\cdots+\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{1n}x_n\le\left[b_a+\frac{b_r-b_l}{2}\frac{\beta-u_a}{1-u_a}\right]_1 ;$$

$$\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{21}x_1+\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{22}x_2$$
$$+\cdots+\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{2n}x_n\le\left[b_a+\frac{b_r-b_l}{2}\frac{\beta-u_a}{1-u_a}\right]_2;$$
$$\vdots$$
$$\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{m1}x_1+\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{m2}x_2$$
$$+\cdots+\left[a_a+\frac{a_r-a_l}{2}\frac{\beta-u_a}{1-u_a}\right]_{mn}x_n\le\left[b_a+\frac{b_r-b_l}{2}\frac{\beta-u_a}{1-u_a}\right]_m.$$

Hence, solve the required equivalent crisp LPP using standard optimization methods.

Case 2: When

$$\alpha>\frac{(1-\beta)w_a}{1-u_a},\tilde{A}^I_{\alpha,\beta}=A^I_\alpha; \tag{5}$$

Now, according to the definition of TIFN, $\tilde{A}_\alpha$ is a closed interval [20], defined by

$$\tilde{A}_\alpha=\left[A_L(\alpha),A_R(\alpha)\right],$$

where $A_L(\alpha)=(a-l_a)+\frac{l_a\alpha}{w_a}$, and $A_R(\alpha)=(a+r_a)-\frac{r_a\alpha}{w_a}$.

The given problem reduces to the following:

$$\max\ \tilde{Z}=\sum_{k=1}^{n}\left[(c_a-c_l)+\frac{c_l\alpha}{w_a},(c_a+c_r)-\frac{c_r\alpha}{w_a}\right]_k x_k$$

subject to

$$\sum_{k=1}^{n}\left[(a_a-a_l)+\frac{a_l\alpha}{w_a},(a_a+a_r)-\frac{a_r\alpha}{w_a}\right]_{jk}x_k\le\left[(b_a-b_l)+\frac{b_l\alpha}{w_a},(b_a+b_r)-\frac{b_r\alpha}{w_a}\right]_j;$$

i.e.,

$$\max\tilde{Z}=\left[(c_a-c_l)+\frac{c_l\alpha}{w_a},(c_a+c_r)-\frac{c_r\alpha}{w_a}\right]_1x_1$$
$$+\left[(c_a-c_l)+\frac{c_l\alpha}{w_a},(c_a+c_r)-\frac{c_r\alpha}{w_a}\right]_2x_2$$
$$+\cdots+\left[(c_a-c_l)+\frac{c_l\alpha}{w_a},(c_a+c_r)-\frac{c_r\alpha}{w_a}\right]_nx_n$$

subject to

$$\left[(a_a-a_l)+\frac{a_l\alpha}{w_a},(a_a+a_r)-\frac{a_r\alpha}{w_a}\right]_{11}x_1$$
$$+\left[(a_a-a_l)+\frac{a_l\alpha}{w_a},(a_a+a_r)-\frac{a_r\alpha}{w_a}\right]_{12}x_2$$
$$+\cdots+\left[(a_a-a_l)+\frac{a_l\alpha}{w_a},(a_a+a_r)-\frac{a_r\alpha}{w_a}\right]_{1n}x_n$$
$$\le\left[(b_a-b_l)+\frac{b_l\alpha}{w_a},(b_a+b_r)-\frac{b_r\alpha}{w_a}\right]_1;$$

$$\left[\left(a_a-a_l\right)+\frac{a_l\alpha}{w_a},\left(a_a+a_r\right)-\frac{a_r\alpha}{w_a}\right]_{21}x_1$$
$$+\left[\left(a_a-a_l\right)+\frac{a_l\alpha}{w_a},\left(a_a+a_r\right)-\frac{a_r\alpha}{w_a}\right]_{22}x_2$$
$$+\cdots+\left[\left(a_a-a_l\right)+\frac{a_l\alpha}{w_a},\left(a_a+a_r\right)-\frac{a_r\alpha}{w_a}\right]_{2n}x_n$$
$$\le\left[\left(b_a-b_l\right)+\frac{b_l\alpha}{w_a},\left(b_a+b_r\right)-\frac{b_r\alpha}{w_a}\right]_2;$$
$$\vdots$$
$$\left[\left(a_a-a_l\right)+\frac{a_l\alpha}{w_a},\left(a_a+a_r\right)-\frac{a_r\alpha}{w_a}\right]_{m1}x_1$$
$$+\left[\left(a_a-a_l\right)+\frac{a_l\alpha}{w_a},\left(a_a+a_r\right)-\frac{a_r\alpha}{w_a}\right]_{m2}x_2$$
$$+\cdots+\left[\left(a_a-a_l\right)+\frac{a_l\alpha}{w_a},\left(a_a+a_r\right)-\frac{a_r\alpha}{w_a}\right]_{mn}x_n$$
$$\le\left[\left(b_a-b_l\right)+\frac{b_l\alpha}{w_a},\left(b_a+b_r\right)-\frac{b_r\alpha}{w_a}\right]_m.$$

Now, utilizing the concept of comparison between interval numbers [20], we obtain if $m_1 \le m_2$ and $w_1+w_2 \ne 0$, the value judgement index or acceptability index (AI) of $\tilde{A}^I_{\alpha,\beta} \le \tilde{B}^I_{\alpha,\beta}$ is defined by $\mathrm{AI}\left(\tilde{A}^I_{\alpha,\beta} \le \tilde{B}^I_{\alpha,\beta}\right)=\frac{m_2-m_1}{2\left(w_1+w_2\right)} \ge 0$.

The objective can now be restated as,

$$\max \tilde{Z}=\sum_{k=1}^{n}\tilde{c}^I_{\alpha,\beta}x_k=\sum_{k=1}^{n}\frac{1}{2}\left[c_L\left(\alpha\right)+c_R\left(\alpha\right)\right]x_k.$$

Hence, the above FLPP is reformulated to:

$$\max\ \tilde{Z}=\left[c_a+\frac{c_r-c_l}{2}\frac{w_a-\alpha}{w_a}\right]_1x_1+\left[c_a+\frac{c_r-c_l}{2}\frac{w_a-\alpha}{w_a}\right]_2x_2$$
$$+\cdots+\left[c_a+\frac{c_r-c_l}{2}\frac{w_a-\alpha}{w_a}\right]_n x_n$$

subject to

$$\left[b_a+\frac{b_r-b_l}{2}\frac{w_a-\alpha}{w_a}\right]_1-\left[a_a+\frac{a_r-a_l}{2}\frac{w_a-\alpha}{w_a}\right]_{11}x_1$$
$$-\left[a_a+\frac{a_r-a_l}{2}\frac{w_a-\alpha}{w_a}\right]_{12}x_2-\cdots-\left[a_a+\frac{a_r-a_l}{2}\frac{w_a-\alpha}{w_a}\right]_{1n}x_n\ge 0$$

i.e.,

$$\left[a_a+\frac{a_r-a_l}{2}\frac{w_a-\alpha}{w_a}\right]_{11}x_1+\left[a_a+\frac{a_r-a_l}{2}\frac{w_a-\alpha}{w_a}\right]_{12}x_2$$
$$+\cdots+\left[a_a+\frac{a_r-a_l}{2}\frac{w_a-\alpha}{w_a}\right]_{1n}x_n\le\left[b_a+\frac{b_r-b_l}{2}\frac{w_a-\alpha}{w_a}\right]_1$$

$$\left[a_a + \frac{a_r - a_l}{2}\frac{w_a - \alpha}{w_a}\right]_{21} x_1 + \left[a_a + \frac{a_r - a_l}{2}\frac{w_a - \alpha}{w_a}\right]_{22} x_2$$
$$+\cdots+\left[a_a + \frac{a_r - a_l}{2}\frac{w_a - \alpha}{w_a}\right]_{2n} x_n \le \left[b_a + \frac{b_r - b_l}{2}\frac{w_a - \alpha}{w_a}\right]_2$$
$$\vdots$$
$$\left[a_a + \frac{a_r - a_l}{2}\frac{w_a - \alpha}{w_a}\right]_{m1} x_1 + \left[a_a + \frac{a_r - a_l}{2}\frac{w_a - \alpha}{w_a}\right]_{m2} x_2$$
$$+\cdots+\left[a_a + \frac{a_r - a_l}{2}\frac{w_a - \alpha}{w_a}\right]_{mn} x_n \le \left[b_a + \frac{b_r - b_l}{2}\frac{w_a - \alpha}{w_a}\right]_m .$$

Hence, we solve the required equivalent crisp LPP in standard optimization methods.

It appears as a simple fuzzy LPP which depends on α and w_a only. β is not explicitly used in its formulation.

Case 3: When

$$\alpha = \frac{(1-\beta)w_a}{1-u_a}, \tilde{A}^I_{\alpha,\beta} = A^I_\alpha \text{ or } A^I_\beta; \tag{6}$$

We can choose anyone of the above two formulations.

4. Algorithm

Input: An Intuitionistic fuzzy LPP in mathematical form.

Output: Converging solution and corresponding decision.

Step 1: Calculate separately the α-cut and β-cut of each TIFN as follows:

Let $\tilde{A}^I = \langle a, l_a, r_a; w_a, u_a \rangle$ then

$$\tilde{A}_\alpha = \left[A_L(\alpha), A_R(\alpha)\right],$$

where

$$A_L(\alpha) = (a - l_a) + \frac{l_a \alpha}{w_a} \text{ and } A_R(\alpha) = (a + r_a) - \frac{r_a \alpha}{w_a}.$$

similarly, $\tilde{A}_\beta = \left[A_L(\beta), A_R(\beta)\right]$, which can be calculated as

$$A_L(\beta) = (a - l_a) + \frac{l_a(1-\beta)}{1-u_a}, \text{ and } A_R(\beta) = (a + r_a) - \frac{r_a(1-\beta)}{1-u_a}.$$

Step 2: Now depending on the above calculations, we find (α-β)-cut of each TIFN *i.e.*, $\tilde{A}^I_{\alpha,\beta}$ as follows:

$$\tilde{A}^I_{\alpha,\beta} = \begin{cases} A^I_\beta; & \text{if } \alpha < \dfrac{(1-\beta)w_a}{1-u_a} \\ A^I_\alpha; & \text{if } \alpha > \dfrac{(1-\beta)w_a}{1-u_a} \\ A^I_\beta \text{ or } A^I_\alpha; & \text{if } \alpha = \dfrac{(1-\beta)w_a}{1-u_a} \end{cases} \tag{7}$$

Step 3: Accordingly, we have the formulation:

$$\max \tilde{Z} = \sum_{k=1}^{n} \tilde{c}^{I}_{k_{\alpha,\beta}} x_k$$

subject to $\sum_{k=1}^{n} \tilde{A}^{I}_{\alpha,\beta} x_k \le \tilde{B}^{I}_{\alpha,\beta}$; $1 \le j \le m$.

Step 4: For the constraints, utilizing the concept of comparison between interval numbers [20], we obtain if $m_1 \le m_2$ and $w_1 + w_2 \ne 0$, the value judgement index or acceptability index (AI) of $\tilde{A}^{I}_{\alpha,\beta} \le \tilde{B}^{I}_{\alpha,\beta}$ is defined by

$$\mathrm{AI}\left(\tilde{A}^{I}_{\alpha,\beta} \le \tilde{B}^{I}_{\alpha,\beta}\right) = \frac{m_2 - m_1}{2\left(w_1 + w_2\right)} \ge 0.$$

Step 5: For the objective function $\max \tilde{Z} = \sum_{k=1}^{n} \tilde{c}^{I}_{\alpha,\beta} x_k$ is constructed as $\max \tilde{Z} = \sum_{k=1}^{n} \tilde{c}^{I}_{\alpha,\beta} x_k = \sum_{k=1}^{n} \frac{1}{2}\left[c_L\left(\alpha \text{ or } \beta\right) + c_R\left(\alpha \text{ or } \beta\right)\right] x_k$.

Step 6: Solve the ordinary Linear programming problem using simplex technique.

To illustrate the same let us consider the problem as in the following.

Example 1: Let us consider an IFLPP as in the following:

$$\begin{aligned} \max\ & f\left(x_1, x_2\right) = (2,1,2)^I x_1 + (3,1,1)^I x_2 \\ \text{s.t.}\ & (1,1,2)^I x_1 + (2,1,3)^I x_2 \le (4,2,1)^I \\ & (3,2,3)^I x_1 + (1,1,2)^I x_2 \le (6,1,1)^I \end{aligned} \tag{8}$$

Here, $\bar{c}_1 = (2,1,2)$. let $\alpha = 0.9, \beta = 0.09$, $\alpha \in [0, 0.925), \beta \in [0.07, 1]$, We first calculate (α,β)-cut of $\bar{c}_1$.

$$\begin{aligned} \hat{c}_\alpha &= \left[c_L(\alpha), c_R(\alpha)\right], \\ \hat{c}_L(0.9) &= 1 + \frac{1 \times 0.9}{0.925} = 1.9729, \\ \hat{c}_R(0.9) &= 4 - \frac{2 \times 0.9}{0.925} = 2.0541, \\ \tilde{c}_\alpha &= \hat{c}_{0.9} = [1.9729, 2.0541]. \end{aligned} \tag{9}$$

$$\begin{aligned} \hat{c}_\beta &= \left[c_L(\beta), c_R(\beta)\right], \\ \hat{c}_L(0.09) &= 1 + \frac{(1 - 0.09) \times 1}{1 - 0.07} = 1.9784, \\ \hat{c}_R(0.09) &= 4 - \frac{(1 - 0.09) \times 2}{1 - 0.07} = 2.0431, \end{aligned} \tag{10}$$

Since, $\alpha < \frac{1-\beta}{1-u_a}\omega_a$, $\tilde{c}^{I}_{0.9,0.09} = \hat{c}_{0.09} = [1.9784, 2.0431]$.

For $\bar{c}_2 = (3,1,1)$, (α,β)-cut is $[2.978, 3.022]$.

Similarly,

For $(1,1,2)$, (α,β)-cut is $[0.978, 1.044]$;

For $(2,1,3)$, (α,β)-cut is $[1.978, 2.066]$;

For $(4,2,1)$, (α,β)-cut is $[3.956, 4.022]$;

For $(3,2,3)$, (α,β)-cut is $[2.956, 3.066]$; and finally

For $(6,1,1)$, (α,β)-cut is $[5.978, 6.022]$.

Hence, the associated FLPP becomes the following:

$$\max\ f_1 = [1.9784, 2.0431]x_1 + [2.978, 3.022]x_2$$

or,

$$\max\ f_1^* = \frac{1}{2}(1.9784 + 2.0431)x_1 + \frac{1}{2}(2.978 + 3.022)x_2 = 2.01075x_1 + 3.0x_2$$

subject to the constraints

$$[0.978, 1.044]x_1 + [1.978, 2.066]x_2 \le [3.956, 4.022] \tag{11}$$

$$\mathrm{AI}\left(\tilde{A}^I < \tilde{B}^I\right) = \frac{\dfrac{3.956+4.022}{2} - \dfrac{0.978x_1 + 1.978x_2 + 1.044x_1 + 2.066x_2}{2}}{2\left[\dfrac{1.044x_1 + 2.066x_2 - 0.978x_1 - 1.978x_2}{2} + \dfrac{4.022-3.956}{2}\right]} \ge 0$$

i.e.,

$$3.989 - 1.011x_1 - 2.022x_2 \ge 0$$

i.e.,

$$1.011x_1 + 2.022x_2 \le 3.989$$

&

$$[2.956, 3.066]x_1 + [0.978, 1.044]x_2 \le [5.978, 6.022]$$

i.e.,

$$3.011x_1 + 1.011x_2 \le 6 \tag{12}$$

Hence, the LPP assumes the form:

$$\begin{aligned} & \max\ f_1^* = 2.01075x_1 + 3.0x_2 \\ \text{s.t.}\ & 1.011x_1 + 2.022x_2 \le 3.989 \\ & 3.011x_1 + 1.011x_2 \le 6.0 \\ & x_1, x_2 \ge 0 \end{aligned}$$

The solution of the IFLPP for different values of α, β is presented in Table 1 and Table 2.

Example 2: Let us consider another intuitionistic fuzzy LPP as in the following:

$$\begin{aligned} & \max\ f(x_1, x_2) = (1,1,1)^I x_1 + (1,1,2)^I x_2 \\ \text{s.t.}\ & (1,1,2)^I x_1 + (2,2,1)^I x_2 \le (3,2,1)^I \\ & (2,1,2)^I x_1 + (3,1,2)^I x_2 \le (4,1,2)^I \end{aligned} \tag{13}$$

The solution of this IFLPP for different values of α, β is presented in Table 3.

5. Comparative Study

In Section 4, instead of TIFN if we take both the co-efficient matrix and cost co-efficient as TFN, then according to the proposed method with the help of α-cut

Table 1. Solution of example 1 in case I of TIFN.

Sr. No.	$\frac{1-\beta}{1-u_a}$	α	β	w_a	u_a	x_1	x_2	z
1	0.5667	0.50	0.49	0.890	0.10	1.575082	0.7672818	5.793250
2	0.6220	0.55	0.44	0.890	0.10	1.578146	0.8135342	5.895164
3	0.6670	0.55	0.40	0.890	0.10	1.580664	0.8528314	5.983002
4	0.6800	0.57	0.32	0.860	0.00	1.581395	0.8644747	6.009238
5	0.7000	0.60	0.30	0.860	0.00	1.582524	0.8826509	6.050380
6	0.7202	0.51	0.35	0.890	0.10	1.583771	0.9030222	6.096753
7	0.7550	0.63	0.32	0.860	0.10	1.585652	0.9343457	6.168583
8	0.7780	0.60	0.30	0.880	0.10	1.586969	0.9567404	6.220313
9	0.8670	0.60	0.35	0.720	0.25	1.592100	1.0474000	6.433050
10	0.9420	0.80	0.19	0.850	0.14	1.596500	1.1312700	6.633199
11	0.9647	0.80	0.18	0.840	0.15	1.597889	1.1576960	6.697068
12	0.9733	0.70	0.27	0.720	0.25	1.598000	1.1681000	6.722000
13	0.9750	0.70	0.22	0.720	0.20	1.598504	1.1698840	6.726640
14	0.9780	0.90	0.09	0.925	0.07	1.598683	1.1734580	6.735325
15	1.0000	1.00	0.00	1.000	0.00	1.600000	1.2000000	6.800000

Table 2. Solution of example 1 in case II of TIFN.

Sr. No.	$\frac{1-\beta}{1-u_a}$	α	β	w_a	u_a	x_1	x_2	z
1	0.6444	0.57	0.42	0.850	0.100	1.580866	0.8560424	5.990228
2	0.7071	0.60	0.30	0.800	0.010	1.585366	0.9295393	6.157520
3	0.9420	0.40	0.35	0.420	0.310	1.597158	1.1433040	6.662241
4	0.9444	0.70	0.15	0.720	0.100	1.598337	1.1665580	6.718564
5	0.9800	0.80	0.02	0.810	0.001	1.599243	1.1850090	6.763429
6	0.9800	0.90	0.02	0.910	0.000	1.599341	1.1866550	6.767442

Table 3. Solution of example 2 in case of TIFN.

Sr. No.	$\frac{1-\beta}{1-u_a}$	α	β	w_a	u_a	x_1	x_2	z
1	0.5667	0.50	0.49	0.890	0.10	1.902323	0.0	1.902323
2	0.6220	0.55	0.44	0.890	0.10	1.913659	0.0	1.913659
3	0.6670	0.55	0.40	0.890	0.10	1.923148	0.0	1.923148
4	0.6800	0.57	0.32	0.860	0.00	1.925926	0.0	1.925926
5	0.7000	0.60	0.30	0.860	0.00	1.930233	0.0	1.930233
6	0.7220	0.51	0.35	0.890	0.10	1.935016	0.0	1.935016
7	0.7550	0.63	0.32	0.860	0.10	1.942285	0.0	1.942285
8	0.7780	0.60	0.30	0.880	0.10	1.947418	0.0	1.947418
9	0.8441	0.60	0.35	0.740	0.23	1.962487	0.0	1.962487
10	0.9420	0.80	0.19	0.850	0.14	1.985707	0.0	1.985707
11	0.9647	0.80	0.18	0.840	0.15	1.991252	0.0	1.991252
12	0.9733	0.70	0.27	0.720	0.25	1.993369	0.0	1.993369
13	0.9750	0.70	0.22	0.720	0.20	1.993789	0.0	1.993789
14	0.9780	0.90	0.09	0.925	0.07	1.994530	0.0	1.994530
15	1.000	1.00	0.00	1.000	0.00	2.000000	0.0	2.000000

we can reformulate it as a crisp LPP and find a solution of the problem. For that let us consider the same Example 1. The solution of the said problem is given in Table 4.

6. Result and Discussion

For $\alpha \geq 0.5$ and $\beta \leq 0.5$ example 1 and example 2 approaches towards a limiting solution as $\frac{1-\beta}{1-u_a}$ tend to 1 as shown in first and third tables.

Moreover, solution of our approach is convergent. The solution obtained from our proposed approach for solving a LPP in an intuitionistic fuzzy environment is better for same values of α than the fuzzy environment which is shown from the first, second and fourth tables. If we defuzzify this IFLPP then we obtain optimal solution of the corresponding crisp LPP. The solution of our IFLPP is quite close to the optimal solution of the associated crisp LPP.

Actually, when available information is not sufficient, the evaluation of the membership and non-membership functions together gives satisfactory result than considering any one of the membership value or the non-membership value. In which case, there remains a part indeterministic on which hesitation survives. Certainly, fuzzy optimization is unable to deal with such hesitation since in this case membership and non-membership functions are complement to each other. Here, in our proposed (α,β)-cut technique, sum of membership degree and non-membership degree is always taken as strictly less than one and hence hesitation is considered. Consequently, in our proposed method for solving IFLPP converge rapidly than fuzzy environment as seen in Figure 1. Next, Table 5 and Table 6 shows that our proposed technique for solving IFLPP yield, a better solution for the Example 1 and Example 2 respectively than the existing Decisive set method [32] for solving FLPP, Modified subgradient method [32] for solving FLPP as well as Zimmermann's extended approach [15] for solving IFLPP.

Table 4. Solution of example 1 in case of TFN.

Sr. No.	α	x_1	x_2	z
1	0.10	1.553863	0.4879819	5.218856
2	0.20	1.562298	0.5942085	5.427466
3	0.30	1.571429	0.7142857	5.678571
4	0.40	1.397908	0.9442094	5.861195
5	0.57	1.579151	0.8290688	5.929755
6	0.60	1.397908	0.9442094	5.861195
7	0.70	1.586969	0.9567404	6.220313
8	0.80	1.593472	1.0729320	6.493382
9	0.90	1.594648	1.0950200	6.546117
10	0.99	1.599460	1.1890700	6.773329

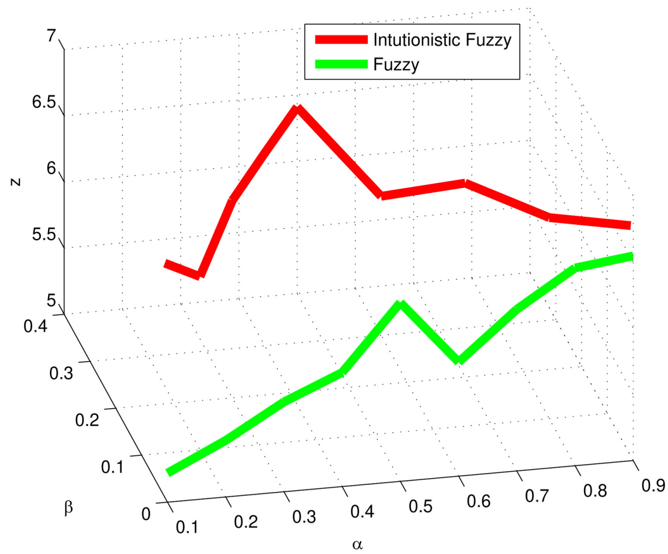

Figure 1. Comparison of Example 1 in case of TIFN and TFN.

Table 5. Solution of example 1 using different techniques.

Decisive set method [32] A fuzzy approach	Modified subgradient method [32] A fuzzy approach	Zimmermann's extended approach [15] An IF approach	Our proposed approach An IF approach
$x_1 = 1.1474$	$x_1 = 1.1475$	$x_1 = 1.470526$	$x_1 = 1.598683$
$x_2 = 0.7508$	$x_2 = 0.7514$	$x_2 = 0.9410526$	$x_2 = 1.173458$
		$\alpha = 0.5491$	$\alpha = 0.90$
		$\beta = 0.08421$	$\beta = 0.09$
$Z^* = 4.5474$	$Z^* = 4.5492$	$Z^* = 5.7642098$	$Z^* = 6.735325$

Table 6. Solution of example 2 using different techniques.

Decisive set method [32] A fuzzy approach	Modified subgradient method [32] A fuzzy approach	Zimmermann's extended approach [15] An IF approach	Our proposed approach An IF approach
$x_1 = 1.690331$	$x_1 = 1.45804$	$x_1 = 1.333333$	$x_1 = 1.99453$
$x_2 = 0.0$	$x_2 = 7.8 \times 10^{-8}$	$x_2 = 0.0$	$x_2 = 0.0$
		$\alpha = 0.5556$	$\alpha = 0.90$
		$\beta = 0.142$	$\beta = 0.09$
$Z^* = 1.690331$	$Z^* = 1.45804$	$Z^* = 1.333333$	$Z^* = 1.99453$

7. Conclusions

Concept of an intuitionistic fuzzy set can be viewed as an alternative approach to define a fuzzy set in cases where available information is not sufficient for the definition of an imprecise concept. In general, the theory of intuitionistic fuzzy set is a generalization of the theory of fuzzy set. Therefore, it is expected that intuitionistic fuzzy sets would perform effectively the task of simulation of human decision-making processes and any activities requiring human expertise and knowledge, which are inevitably imprecise or not totally reliable. As proposed we have tried to obtain a solution of an intuitionistic fuzzy LPP using (α, β)-cut method. With different simple problems it is tested and significant improvements over existing techniques have been noticed in each case. However, an analytical proof of the same could not be possible to be constructed because of the subjective nature of membership or non-membership functions of TIFN's used in the representation of the original problem.

There is considerable scope for research in this domain. This includes, in particular, an attempt to find solution for a class of IFLPP without converting them to crisp LPP and to compare other existing fuzzy and intuitionistic fuzzy optimization techniques.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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