Scientific Research Publishing

Int. J. Communications, Network and System Sciences, 2017, 10, 191-201
http://www.scirp.org/journal/ijcns
ISSN Online: 1913-3723
ISSN Print: 1913-3715

Single-Channel Compressive Sensing for DOA Estimation via Sensing Model Optimization

Hongtao Li, Zeshi Yuan

Nanjing University of Science and Technology, Nanjing, China
Email: liht@njust.edu.cn, yzs644613931@gmail.com

How to cite this paper: Li, H.T. and Yuan, Z.S. (2017) Single-Channel Compressive Sensing for DOA Estimation via Sensing Model Optimization. *Int. J. Communications, Network and System Sciences*, **10**, 191-201.
https://doi.org/10.4236/ijcns.2017.105B019

Received: April 11, 2017
Accepted: May 23, 2017
Published: May 26, 2017

Abstract

The performance of multi-channel Compressive Sensing (CS)-based Direction-of-Arrival (DOA) estimation algorithm degrades when the gains between Radio Frequency (RF) channels are inconsistent, and when target angle information mismatches with system sensing model. To solve these problems, a novel single-channel CS-based DOA estimation algorithm via sensing model optimization is proposed. Firstly, a DOA sparse sensing model using single-channel array considering the sensing model mismatch is established. Secondly, a new single-channel CS-based DOA estimation algorithm is presented. The basic idea behind the proposed algorithm is to iteratively solve two CS optimizations with respect to target angle information vector and sensing model quantization error vector, respectively. In addition, it avoids the loss of DOA estimation performance caused by the inconsistent gain between RF channels. Finally, simulation results are presented to verify the efficacy of the proposed algorithm.

Keywords

Compressive Sensing, Direction-of-Arrival Estimation, Single-Channel, Mismatching Error

1. Introduction

Compressive Sensing (CS) theory, deduced from signal processing and information theories [1] [2] [3], has been widely applied in radar, image processing, wireless communication and many other engineering fields [4] [5] [6] [7] [8]. The CS theory indicates that the solution of a norm optimization problem can rebuild a sparse signal with comparatively high accuracy by adopting finite nonadaptive random projected measure value [9].

The strong scatter centers of target in interested area only occupy finite angle resolution cells and the target is sparse in space-domain, so that CS theory has been widely applied in Direction-of-Arrival (DOA) estimation [10]-[19]. A major advantage of CS-based algorithms over conventional super-resolution algorithms [10] is that the CS-based algorithm can offer higher resolution with reduced antenna elements and Radio Frequency (RF) channels. For example, [11] presents a CS-based DOA estimation method, which reduces the sampling number by making use of the sparsity of radar echo signals to perform compressive sampling in time-domain. [12] adopts an array element randomly distributed antenna to perform compressive sampling in space domain to reduce the number of RF channels of the system. However, these two algorithms treat the over-complete based matrices as the redundant dictionaries, obtained from the angle interval of uniform quantization interested area, which cannot ensure that the corresponding sensing matrix meets the Restricted Isometry Property (RIP) condition [13]. [14] uses random Gauss matrix to perform compressive sampling in space-domain and adopts Regularized Multi-vectors Focal Undetermined System Solver (RMFOCUSS) algorithm to achieve high-resolution DOA estimation. However, the algorithm has computational complexity increasing dramatically with the increasing of snapshots, while at the same time is unsuitable for low signal-to-noise ratio (SNR) situations. Furthermore, the authors in [15] investigate the CS-based DOA estimation in the presence of sensing model mismatching errors, proving that the performance of CS-based DOA estimation algorithm degrades dramatically in the presence of sensing model mismatching. [16] [17] [18] present a DOA estimation model under sensing model mismatching, and then use Bayesian method to realize DOA estimation. [19] proposes a joint Least-Absolute Shrinkage and Selection Operator (LASSO) algorithm to achieve DOA estimation in the presence of mismatching.

In addition, all the aforementioned algorithms utilize multi-channel data so that the estimation performance degrades seriously in the presence of inconsistent gain between RF channels of the array.

In this paper, we derive a single-channel CS-based DOA estimation algorithm via sensing mode optimization to solve the above mentioned problems. Firstly, a DOA estimation model is set up considering mismatch error between system sensing model and target angle information. Secondly, a single-channel array system, which can avoid gain inconsistency between RF channels, is introduced. Meanwhile, it can be proved that the sensing matrix of the single-channel array system meets the RIP condition. Finally, on the basis of Robust Smooth L_0 (RSL0) algorithm [20] and LASSO algorithm [21], a new DOA estimation algorithm is presented to achieve high resolution DOA estimates.

The paper is organized as follows. Section II formulates the problem of interest. Section III develops the proposed algorithm. Section IV provides the simulation results that demonstrate the efficacy of the proposed algorithm. Section V concludes the paper.

2. Signal Model

2.1. Space Signal Model

Consider K far-field narrow-band signals impinging upon a uniform linear array

(ULA) of L elements. The receive signal can be represented as

$$\boldsymbol{x}(t)=\sum_{k=1}^{K}s_k(t)\boldsymbol{\alpha}(\theta_k)+\boldsymbol{e}(t) \tag{1}$$

where $\boldsymbol{\alpha}(\theta_k)=\left[1,e^{j2\pi f_{\theta_k}},\cdots,e^{j2\pi(L-1)f_{\theta_k}}\right]^{\mathrm{T}}$ is steering vector of the kth signal, $f_{\theta_k}=d\sin\theta_k/\lambda$, d is the distance between two adjacent elements, λ is the wavelength of carrier wave, $\boldsymbol{e}(t)=\left[e_1(t),e_2(t),\cdots,e_L(t)\right]^{\mathrm{T}}$ is array noise vector, and $s_k(t)$ is waveform of the kth signal.

For CS processing, the angle field-of-view of the interested area is sampled uniformly at $\tilde{\theta}_1,\tilde{\theta}_2,\cdots,\tilde{\theta}_N$. Denoting $\tilde{\boldsymbol{\theta}}=\left[\tilde{\theta}_1,\tilde{\theta}_2,\cdots,\tilde{\theta}_N\right]$ as angle information vector, where $\tilde{\theta}_n$, $n=1,2,\cdots,N$ is defined as quantization angle, N is the length of vector $\tilde{\boldsymbol{\theta}}$, and $\varphi=\tilde{\theta}_2-\tilde{\theta}_1$ is the angle resolution cell, then if target angle θ_k matches with one of the quantization angles, Equation (1) can be rewritten as

$$\boldsymbol{x}(t)=A(\tilde{\boldsymbol{\theta}})\boldsymbol{s}(t)+\boldsymbol{e}(t) \tag{2}$$

where $\boldsymbol{s}(t)=\left[s_1(t),s_2(t),\cdots,s_N(t)\right]^{\mathrm{T}}$ is denoted as the target waveform information vector, and $\boldsymbol{A}(\tilde{\boldsymbol{\theta}})=\left[\boldsymbol{\alpha}(\tilde{\theta}_1),\boldsymbol{\alpha}(\tilde{\theta}_2),\cdots,\boldsymbol{\alpha}(\tilde{\theta}_N)\right]$ is the steering vector matrix of all sampled angles. In practical, targets in interested area only occupy finite angle resolution cells, so that $\|\boldsymbol{s}(t)\|_0=K\ll N$, with $\|\bullet\|_0$ denoting L_0 norm. Therefore, the receive signal vector $\boldsymbol{x}(t)$ is K sparse signal, $\boldsymbol{A}(\tilde{\boldsymbol{\theta}})$ is the sparsity based matrix, and K is the sparsity of target angle information vector.

2.2. DOA Estimation Model under Sensing Model Mismatching

Obviously, since N is finite, the target angle θ_k might not match exactly with one of the quantization angles. This phenomenon is called mismatching between sensing model and target angle information. According to CS theory, sensing model mismatching will lead to the angle information vector failing to represent target angle precisely, increasing the estimation error of target angles through conventional CS-based DOA estimation method.

Let $\tilde{\theta}_{n_k}\in\tilde{\boldsymbol{\theta}},n_k\in[1,2,\cdots,N]$ be the quantization angle that is nearest to the target angle θ_k. Approximating the steering vector of the kth target $\boldsymbol{a}(\theta_k)$ by its first-order Taylor series expansion with respect to the variable θ_k, about $\tilde{\theta}_{n_k}$, we have

$$\boldsymbol{a}(\theta_k)\approx\boldsymbol{a}(\tilde{\theta}_{n_k})+\boldsymbol{b}(\tilde{\theta}_{n_k})(\theta_k-\tilde{\theta}_{n_k}) \tag{3}$$

where $\boldsymbol{b}(\tilde{\theta}_{n_k})=\dfrac{\mathrm{d}\left(\boldsymbol{a}(\tilde{\theta}_{n_k})\right)}{\mathrm{d}\tilde{\theta}_{n_k}}$. Expressing in matrix form, we have

$$\boldsymbol{\Omega}(\tilde{\boldsymbol{\theta}})=\boldsymbol{A}(\tilde{\boldsymbol{\theta}})+\boldsymbol{B}(\tilde{\boldsymbol{\theta}})\boldsymbol{\Lambda} \tag{4}$$

where $\boldsymbol{B}(\tilde{\boldsymbol{\theta}})=\left[\boldsymbol{b}(\tilde{\theta}_1),\boldsymbol{b}(\tilde{\theta}_2),\cdots,\boldsymbol{b}(\tilde{\theta}_N)\right]$, $\boldsymbol{\Lambda}=diag(\boldsymbol{\beta})$, $\boldsymbol{\beta}=[\beta_1,\beta_2,\cdots,\beta_N]^{\mathrm{T}}$ is a vector representing the sensing model mismatching errors, and

$$\beta_n = \begin{cases} \theta_k - \tilde{\theta}_{n_k} & n = n_k, k \in [1, 2, \cdots, K] \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

is denoted as angular quantization error.

Considering the quantization error, Equation (2) can be modified as

$$\boldsymbol{x}(t) = \boldsymbol{\Omega}(\tilde{\theta})\boldsymbol{s}(t) + \boldsymbol{e}(t) \tag{6}$$

3. Proposed Model and Algorithm

3.1. Compressive Sensing Model Based on RF Single-Channel Array

The introduced RF single-channel array is shown in **Figure 1**. Because the array has only one RF channel, it is characterized by low-power and small-size compared with those RF multi-channel arrays. Moreover, the RF single-channel array can effectively avoid gain inconsistency between RF channels along with the influence on subsequent signal processing caused by imbalance of amplitude and phase, and hence, it plays an important role in practical applications [22].

Unlike previously developed multi-channel CS-based algorithms, this paper will, for the first time, derive a single-channel CS based algorithm for DOA estimation. First, a $0/\pi$ phase shifter is connected to each array element and random sampling in space-domain is accomplished through randomly adjusting the phase of phase shifters. Second, an L-combiner is used to combine the signals from L paths through phase-shifter to one signal. Finally, the digital signal $y_s(t)$ is obtained through single RF channel and A/D converter. The channel output signal can be expressed as

$$y_s(t) = \boldsymbol{\psi}\boldsymbol{x}(t) = \boldsymbol{\psi}\left[\boldsymbol{\Omega}(\tilde{\theta})\boldsymbol{s}(t) + \boldsymbol{e}(t)\right] \tag{7}$$

where $\boldsymbol{\psi} = [\psi_1, \psi_2, \cdots, \psi_L]$ denotes random weighting vector, which is generated by L sets of $0/\pi$ phase shifters. The value of ψ_i is either +1 or −1 for $i = 1, 2, \cdots, L$.

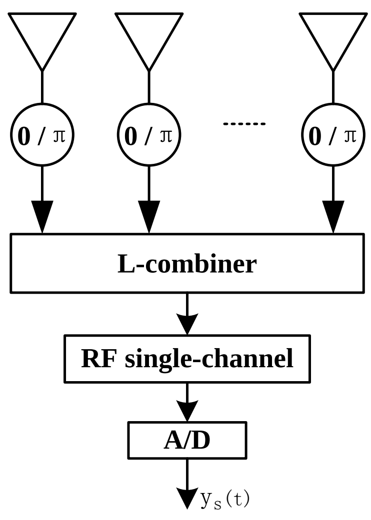

Figure 1. Single-channel array system.

As the target is sparse in space, we assume that the target does not cross angle resolution cells within M snapshots. Then, the M snapshots measured value of the target can be denoted as

$$\boldsymbol{y}_s(t) = \boldsymbol{\Psi}\left[\boldsymbol{\Omega}(\tilde{\theta})\boldsymbol{s}(t) + \boldsymbol{e}(t)\right] = \boldsymbol{\Theta}\boldsymbol{s}(t) + \boldsymbol{n}(t) \quad (8)$$

where $\boldsymbol{\Psi} = \left[\boldsymbol{\psi}_1^{\mathrm{T}}, \boldsymbol{\psi}_2^{\mathrm{T}}, \cdots, \boldsymbol{\psi}_M^{\mathrm{T}}\right]^{\mathrm{T}}$ is $M \times L$ weighting coefficient matrix. Since the element of ψ_i is randomly generated, the elements of $\boldsymbol{\Psi}$, are independent identically distributed Bernoulli random variables. $\boldsymbol{\Theta} = \boldsymbol{\Psi}\boldsymbol{\Omega}(\tilde{\theta})$ is an $M \times L$ sensing matrix and $\boldsymbol{n} = \boldsymbol{\Psi}\boldsymbol{e}(t)$ is noise vector.

By observing (8), we can conclude that sampling of space-domain signals through single channel array can be regarded as performing random projection of measurement matrix $\boldsymbol{\Psi}$ on receive signal $\boldsymbol{x}(t)$, thus converting the multiple measurement vectors (MMV) problem to a single measurement vector (SMV) problem [23]. In addition, the sensing matrix $\boldsymbol{\Theta}$ is the product of matrix $\boldsymbol{\Psi}$, whose elements are Bernoulli distributed, and sparsity based matrix $\boldsymbol{\Omega}(\tilde{\theta})$, which can be generated by discrete Fourier transform (DFT) matrix of space-domain signal. Therefore, $\boldsymbol{\Theta}$ meets the RIP condition with great probability, thus ensuring the effectiveness and robustness of using compressive sensing reconstruction algorithm to perform DOA estimation.

3.2. Derivation of the Proposed Algorithm

It is found from (8) that the influences of measurement noise and sensing model mismatching error on DOA estimation can be summed up to two parts: "additive" disturbance and "productive" disturbance. Conventional CS-based algorithms only have constraint on "additive" disturbance, but do not consider the influence of "productive" disturbance on the accuracy of target angle information reconstruction. Therefore, these algorithms are not robust in the presence of sensing model mismatching since they cannot effectively reduce the effect of quantization errors.

To overcome these problems, we present a novel CS-based DOA estimation algorithm using single-channel array. The insight of the proposed algorithm is to combine RSL0 algorithm and LASSO algorithm to achieve valid DOA estimates by performing alternative iterative optimization separately on target angle information vector and sensing model quantization error. The basic step of the proposed algorithm can be summarized as follows. The parameters to be optimized are separated into two sets: target angle information set and quantization error set. Each time, a CS cost function that depends only on one set is minimized. With the solution of this CS problem, the subsequent stages of the proposed algorithm consist of applying the same principle on another set of parameter. The algorithm iterates, changing from one set to the next, until the variation of the cost function or of the parameters is less than a predefined convergence criterion.

To initiate the algorithm, we set the sensing model quantization error $\boldsymbol{\beta} = \mathbf{0}$. Then, according to CS theory, by solving the optimization problem expressed in

(8) with L_0 norm optimization, we can obtain the estimate of target waveform information vector $\hat{\boldsymbol{s}}(t)$. Mathematically,

$$\hat{\boldsymbol{s}}(t) = \arg\min \left\|\boldsymbol{s}(t)\right\|_0 \text{, subject to } \left\|\boldsymbol{y}_s(t) - \boldsymbol{\Theta}\boldsymbol{s}(t)\right\|_2 < \varepsilon \tag{9}$$

where constant ε is relevant to noise variance. This optimization problem can be perfectly solved by RSL0 algorithm.

Insert the estimate $\hat{\boldsymbol{s}}(t)$ obtained from (9) into (8), we can get

$$\boldsymbol{y}_s(t) = \boldsymbol{\Psi}[\boldsymbol{\Omega}(\tilde{\theta}) \bullet \hat{\boldsymbol{s}}(t) + \boldsymbol{e}(t)] \tag{10}$$

$$= \boldsymbol{\Psi}\boldsymbol{A}(\tilde{\theta})\hat{\boldsymbol{s}}(t) + \boldsymbol{\Psi}\boldsymbol{B}(\tilde{\theta})\boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{n}(t) \tag{11}$$

where $\boldsymbol{X} = diag(\hat{\boldsymbol{s}}(t))$. Equation (11) can be transformed to

$$\boldsymbol{f}(t) = \nu\boldsymbol{\beta} + \boldsymbol{n}(t) \tag{12}$$

where $\boldsymbol{f}(t) = \boldsymbol{y}_s(t) - \boldsymbol{\Psi}\boldsymbol{A}(\tilde{\theta})\hat{\boldsymbol{s}}(t)$, and $\nu = \boldsymbol{\Psi}\boldsymbol{B}(\tilde{\theta})\boldsymbol{X}$.

From (5), we know that the sensing model quantization error vector $\boldsymbol{\beta}$ should have the same sparsity as that of target waveform information vector $\boldsymbol{s}(t)$. Therefore, Equation (12) can be treated as another CS optimization problem by considering the sensing model quantization error as sparse signal. This CS optimization problem can be donated as

$$\hat{\boldsymbol{\beta}} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} \left\|\boldsymbol{f} - \nu\boldsymbol{\beta}\right\|_2^2, \text{ subject to } \left\|\boldsymbol{\beta}\right\|_1 \le \frac{1}{2}K\varphi \tag{13}$$

which can be perfectly solved by LASSO algorithm.

Finally, inserting the estimate $\hat{\boldsymbol{\beta}}$ obtained from (13) into (9), we perform the repetition of the above two CS optimization until the difference of two target waveform information vectors' Frobenius norm is less than a certain predefined threshold, *i.e.*,

$$\left\|\left[\hat{\boldsymbol{s}}(t)\right]^{(p+1)} - \left[\hat{\boldsymbol{s}}(t)\right]^{(p)}\right\|_2 \Big/ \left\|\left[\hat{\boldsymbol{s}}(t)\right]^{(p)}\right\|_2 \le \Delta \tag{14}$$

where $\left[\hat{\boldsymbol{s}}(t)\right]^{(p+1)}$ denotes the estimate of the target waveform information vector obtained at the *p*th iteration, and Δ is a predefined small value. The K largest values of $\left[\hat{\boldsymbol{s}}(t)\right]^{(p+1)}$ give the estimates of the target angles.

3.3. Implementation of the Proposed Algorithm

Assuming that the number of signals is known or correctly estimated, the proposed algorithm can be summarized as follows.

1) Initialize with $p = 0$ and $\boldsymbol{\beta}^{(0)} = 0$.

2) Solve (9) to get the estimate $\left[\hat{\boldsymbol{s}}(t)\right]^{(p+1)}$.

3) Solve (13) to get the estimate $\left[\hat{\boldsymbol{s}}(t)\right]^{(p+1)}$, and then set $p = p + 1$.

4) Stop the iteration if expression (14) is satisfied. Otherwise go back to step 2).

4. Simulations

Performance of the proposed algorithm is evaluated by comparing to the

CS-based algorithm in [11] [12] [18] and RMFOCUSS algorithm in [14]. A ULA with $L = 16$ elements, separated by $d = \lambda/2$ is considered. The angle resolution cell is set as $\varphi = 1^{\circ}$. Three independent narrowband signals are impinge upon the array from angles $\theta_1 = 31.5^{\circ}$, $\theta_2 = 43.1^{\circ}$ and $\theta_3 = 45.5^{\circ}$. The additive noise is assumed to be spatial white complex Gaussian, and the SNR is defined relative to each signal. The number of snapshots for each trial is set to be $M = 200$. The performance metrics used are the root mean squared error (RMSE), which for the unknown target is computed as

$$RMSE = \sqrt{\frac{1}{P}\sum_{k=1}^{P}(\hat{\Theta} - \Theta)^2} \tag{15}$$

where $P = 500$ is the number of independent Monte Carlo trials.

In the first simulation, we study the performance of the proposed algorithm in the presence of sensing model mismatching. **Figure 2** shows the RMSE of the proposed algorithm and its three competitors as functions of SNR varying from −15 dB to 10 dB. From the figure, we see that the proposed algorithm has an estimation accuracy that is higher than those of the other algorithms. The superiority of the proposed algorithm over the other algorithms is due to the fact that the proposed algorithm performs sensing model optimization by taking the quantization errors into account.

Next, we consider the presence of gain inconsistency between RF channels. The gains of RF channels are assumed to be of Gaussian distribution with mean value $\mu = 1$ and variance $\sigma_2 = 0.05$. The remaining parameters used are the same as those in plotting **Figure 2**. The RMSE of the algorithms versus SNR are shown in **Figure 3**. It is seen from the figure that the proposed algorithm does not suffer from gain inconsistency between RF channels due to the usage of single-channel array system, thus, providing better DOA estimation performance than the other multi-channel CS-based algorithms.

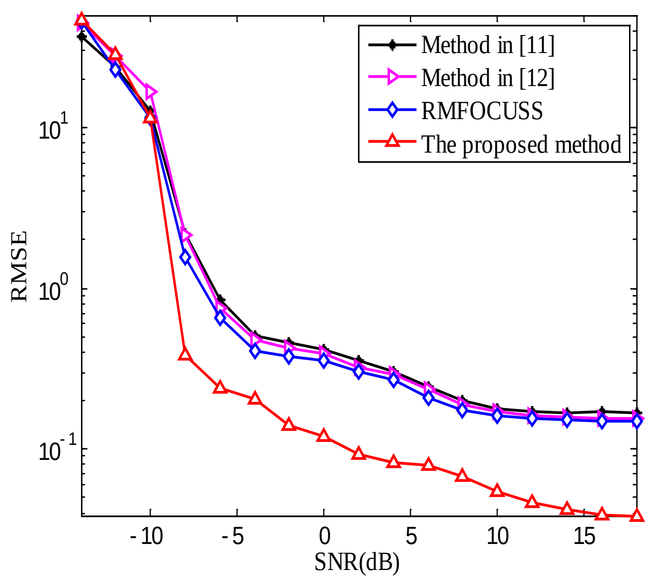

Figure 2. RMSE angle error performance versus SNR.

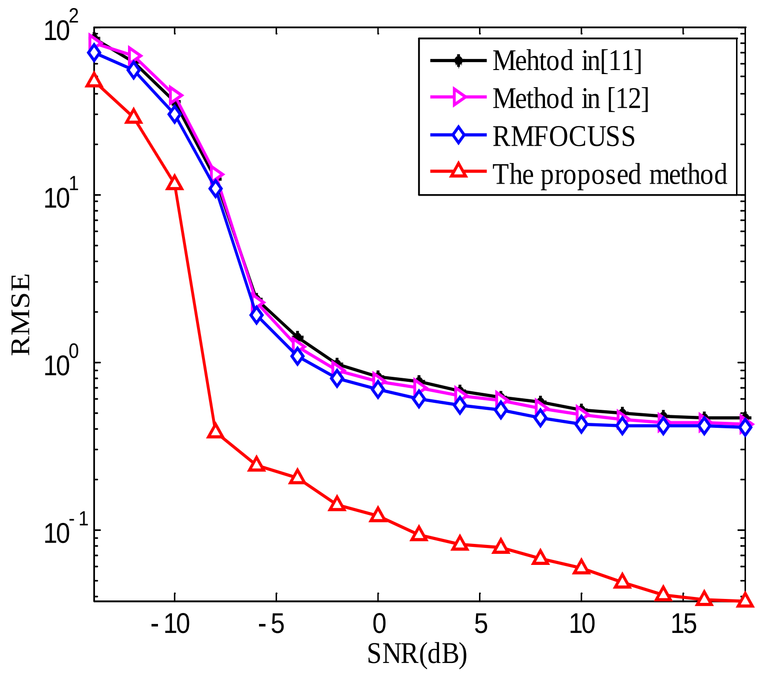

Figure 3. RMSE angle error performance versus SNR in the presence of inconsistent gain between RF channels.

Then, we study the performance on randomly generated DOAs. Suppose that the directions of the input three signals are uniformly generated within the intervals $\left[10^{\circ}, 30^{\circ}\right]$, the remaining parameters used are the same as those in plotting **Figure 2**. **Figure 4** shows the RMSE of different DOA estimation algorithms versus different SNR. From the figure, we see that the performance of the proposed method increases with the decrease of the certain predefined threshold, and when the certain predefined threshold $\Delta = 0.05$, the proposed method can achieve higher estimation accuracy compared with other off-grid CS-DOA methods.

In the last simulation, we consider the ability of the proposed method to represent the true signals with different angle resolution cells. The angle resolution cells are set as $\left[1^{\circ}, 3^{\circ}, 5^{\circ}\right]$. **Figure 5** shows the RMSE versus different angle resolution cells. It is seen from the figure that the performance of the proposed method increases with the decrease of the angle resolution cell.

5. Conclusion

We have proposed a novel CS-based DOA estimation algorithm via sensing model optimization using single-channel array to solve the problems of sensing model mismatching and channel gain inconsistency, from which most conventional multi-channel CS-based algorithms would suffer. The key idea of the proposed algorithm is to iteratively solve two CS optimizations with respect to target angle information vector and sensing model quantization error vector, respectively. Simulation results have also been presented to verify the efficacy of the proposed algorithm.

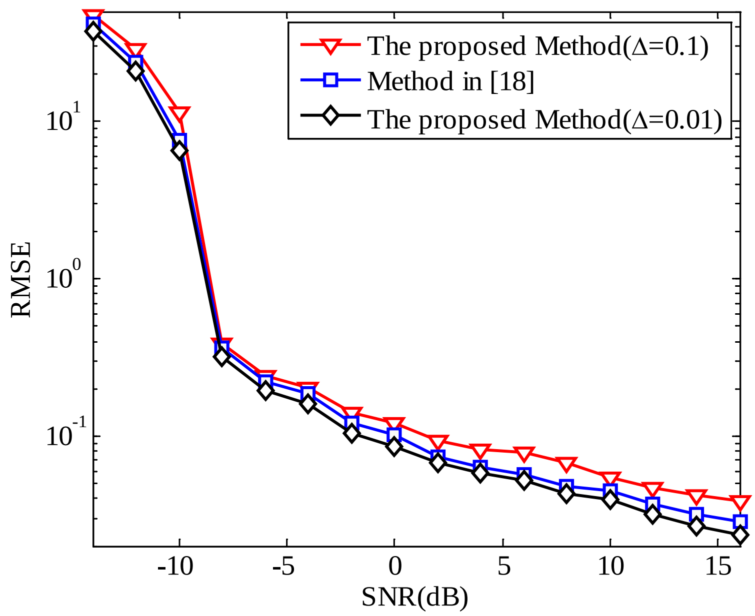

Figure 4. RMSE of the DOA estimates versus input SNR.

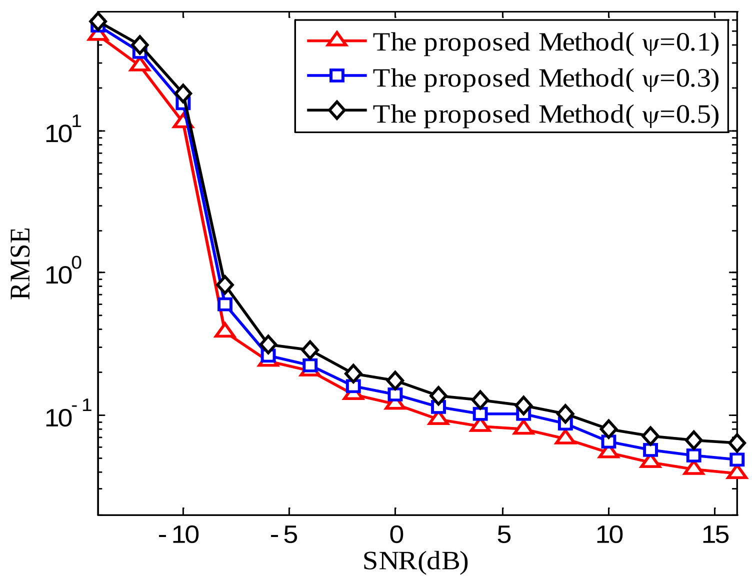

Figure 5. RMSE of the DOA estimates versus input SNR with different angle resolution cells.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant no. 61401204), Postdoctoral Science Foundation of Jiangsu Province (Grant no. 1501104C), Technology Research and Development Program of Jiangsu Province (Grant no. BY2015004-03), and the Fundamental Research Funds for the Central Universities (Grant no. 30916011319).

References

[1] Julio, M., Cuo, S. and Lawerence, C. (2013) Task-driven Adaptive Statistical Com-

pressive Sensing of Gaussian Mixture Models. *IEEE Trans. Signal Process*, **61**, 585-600. https://doi.org/10.1109/TSP.2012.2225054

[2] Candes, E.J. and Wakin, M.B. (2008) An Introduction to Compressive Sampling. *IEEE Signal Process. Mag.*, **25**, 21-30. https://doi.org/10.1109/MSP.2007.914731

[3] Donoho, D.L. (2006) Compressed Sensing. *IEEE Trans. Information Theory*, **52**, 1289-1306. https://doi.org/10.1109/TIT.2006.871582

[4] Romberg, J. (2008) Imaging via Compressive Sampling. *IEEE Signal Process. Mag.*, **25**, 14-20. https://doi.org/10.1109/MSP.2007.914729

[5] Li, H.T., Wang, C.Y., Wang, K., He, Y.P. and Zhu, X.H. (2015) High Resolution Range Profile of Compressive Sensing Radar with Low Computational Complexity. *IET Radar Sonar Navig.*, **9**, 984-990. https://doi.org/10.1049/iet-rsn.2014.0454

[6] Li, R.P., Zhao, Z.F., Zhang, Y., Palicot, J. and Zhang, H.G. (2014) Adaptive Multi-Task Compressive Sensing for Localisation in Wireless Local Area Networks. *IET Communications*, **8**, 1736-1744. https://doi.org/10.1049/iet-com.2013.1019

[7] Liu, Y., Mei, W.B., and Du, H.Q. (2014) Two Compressive Sensing-Based Estimation Schemes Designed for Rapidly Time-Varying Channels in Orthogonal Frequency Division Multiplexing Systems. *IET Signal Process*, **8**, 291-299. https://doi.org/10.1049/iet-spr.2013.0352

[8] Hawes, M.B. and Liu, W. (2014) Compressive Sensing-Based Approach to the Design of Linear Robust Sparse Antenna Arrays with Physical Size Constraint. *IET Microwaves, Antennas and Propag.*, **8**, 736-746. https://doi.org/10.1049/iet-map.2013.0469

[9] Julian, W., Simon, H. and Martin, K. (2013) Analysis Based Blind Compressive Sensing. *IEEE Signal Process. Lett.*, **20**, 491-494. https://doi.org/10.1109/LSP.2013.2252900

[10] Krim, H. and Viberg, M. (1996) Two Decades of Array Signal Processing Research: The Parametric Approach. *IEEE Signal Process. Mag.*, **13**, 67-94. https://doi.org/10.1109/79.526899

[11] Yu, Y., Petropulu, A.P. and Poor, H.V. (2010) MIMO Radar Using Compressive Sampling. *IEEE Journal of Selected Topic in Signal Process*, **4**, 146-163. https://doi.org/10.1109/JSTSP.2009.2038973

[12] Bilik, I. (2011) Spatial Compressive Sensing for Direction-Of-Arrival Estimation of Multiple Sources Using Dynamic Sensor Arrays. *IEEE Trans. Aerosp. Electron. Syst.*, **47**, 1757-1769. https://doi.org/10.1109/TAES.2011.5937263

[13] Liu, Z., Wei, X. and Li, X. (2013) Aliasing-Free Moving Target Detection in Random Pulse Repetition Interval Radar Based on Compressed Sensing. *IEEE Sensor Journal*, **13**, 2523-2534. https://doi.org/10.1109/JSEN.2013.2249762

[14] Cotter, S.F., Rao, B.D. and Engan, K. (2005) Sparse Solution to Linear Inverse Problems with Multiple Measurement Vectors. *IEEE Trans. Signal Process*, **53**, 2477-2488. https://doi.org/10.1109/TSP.2005.849172

[15] Ali, C.G., Volkan, C. and James, H.M. (2012) Bearing Estimation via Spatial Sparsity Using Compressive Sensing. *IEEE Trans. Aerosp. Electron. Syst.*, **48**, 1358-1369. https://doi.org/10.1109/TAES.2012.6178067

[16] Liu Z.M. and Zhou, Y.Y. (2013) A Unified Framework and Sparse Bayesian Perspective for Direction-Of-Arrival Estimation in the Presence of Array Imperfections. *IEEE Trans. Signal Process*, **61**, 3786-3798. https://doi.org/10.1109/TSP.2013.2262682

[17] Zhang, Y., Ye, Z., Xu, X. and Hu, N. (2014) Off-grid Doa Estimation Using Array Covariance Matrix and Block-Sparse Bayesian Learning. *Signal Process*, **98**,

197-201. https://doi.org/10.1016/j.sigpro.2013.11.022

[18] Yang, Z., Xie, L. and Zhang, C. (2013) Off-Grid Direction of Arrival Estimation Using Sparse Bayesian Inference. *IEEE Trans. Signal Process*, **61**, 38-43. https://doi.org/10.1109/TSP.2012.2222378

[19] Tan, Z. and Nehorai, A. (2014) Sparse Direction of Arrival Estimation Using Co-Prime Arrays with Off-Grid Targets. *IEEE Signal Process. Lett.*, **21**, 26-29. https://doi.org/10.1109/LSP.2013.2289740

[20] Mohimani, H., Massoud, B.Z. and Jutten, C. (2009) A Fast Approach for Overcomplete Sparse Decomposition Based on Smoothed L0 Norm. *IEEE Trans. Signal Process*, **57**, 289-301. https://doi.org/10.1109/TSP.2008.2007606

[21] Tibshirani, R. (1996) Regression Shrinkage and Selection via the Lasso. *Journal of the Royal Statistical Society*, **58**, 267-288.

[22] Nural, H.N., Tughrul, A. and Brian, W.F. (2013) Single-Channel Beamforming Algorithm for 3-Faceted Pahsed Array Antenna. *IEEE Antennas Wirel. Propag. Lett.*, **12**, 813-816. https://doi.org/10.1109/LAWP.2013.2271051

[23] Mishali, M. and Eldar, Y.C. (2008) Reduce and Boost: Recovering Arbitrary Sets of Jointy Sparse Vectors. *IEEE Trans. Signal Process*, **56**, 4692-4702. https://doi.org/10.1109/TSP.2008.927802