

Periodic Solutions of Cohen-Grossberg-Type BAM Neural Networks with Time-Varying Delays

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ABSTRACT

Sufficient conditions to guarantee the existence and global exponential stability of periodic solutions of a Cohen-Grossberg-type BAM neural network are established by suitable mathematical transformation.

Keywords: Cohen-Grossberg Neural Networks; BAM Neural Networks; Periodic Solution; Delay; Global Stability

1. Introduction

Many important results on the existence and global exponential stability of equilibria of neural networks with time delays have been widely investigated and successfully applied to signal processing system. However, the research of neural networks involves not only the dynamic analysis of equilibrium point but also that of periodic oscillatory solution. In practice, the dynamic behavior of periodic oscillatory solution is very important in learning theory [1,2], which is motivated by the fact that learning usually requires repetition, some important results for periodic solutions of Hopfield neural networks or Cohen-Grossberg neural networks with delays have been obtained in Refs. [3-15].

The objective of this paper is to study the existence and global exponential stability of periodic solutions of a class of Cohen-Grossberg-type BAM neural networks (CGBAMNNs) with time-varying delays by suitable mathematical transformation.

The rest of this paper is organized as follows: preliminaries are given in Section 2. Sufficient conditions which guarantee the existence and global exponential stability of periodic solutions for the CGBAMNNs are established in Section 3. An example is given in Section 4 to demonstrate the main results.

2. Preliminaries

Consider the following periodic CGNNs with time-

varying delays (see Equation (1)):

For $1 \leq i \leq n$, $t > 0$ and $Z^+ = \{1, 2, \dots, n\}$. $x_i(t)$ denote the state variables of the i th neuron, $f_j(\cdot)$ denote the signal functions of the j th neuron at time t ; $I_i(t)$ denote inputs of the i th neuron at time t ; $a_i(\cdot)$ represent amplification functions; $b_i(t, \cdot)$ are appropriately behaved functions; $p_{ij}(t)$ and $u_{ij}(t)$ are connection weights of the neural networks, respectively; ρ_j are positive constants which correspond to the neuronal gains associated with the neuronal activations; $\tau_{ij}(t)$ correspond to the finite speed of the axonal signal transmission at time t and there exist constants τ_{ij} such that $0 \leq \tau_{ij}(t) \leq \tau_{ij}$, $p_{ij}(t), u_{ij}(t), I_i(t)$ and $\tau_{ij}(t)$ are all continuously periodic functions on $[0, +\infty)$ with common period $T > 0$.

Throughout this paper, we assume for system (1) that

(H₁) Amplification functions $a_i(\cdot)$ are continuous and there exist constants $\underline{a}_i, \bar{a}_i$ such that $\underline{a}_i \leq a_i(\cdot) \leq \bar{a}_i$ for $1 \leq i \leq n$.

(H₂) $b_i(t, \cdot)$ are T -periodic about the first argument and there exist continuous T -periodic functions $\gamma_i(t)$ such that

$$\frac{b_i(t, x) - b_i(t, y)}{x - y} \geq \gamma_i(t) > 0,$$

$$\forall x \neq y \in \mathbb{R}, 1 \leq i \leq n$$

(H₃) For activation functions $f_j(\cdot)$, there exist positive constants L_j such that

$$\dot{x}_i(t) = -a_i(x_i(t)) \left[b_i(t, x_i(t)) - \sum_{j=1}^n p_{ij}(t) f_j(\rho_j x_j(t)) - \sum_{j=1}^n u_{ij}(t) f_j(\rho_j x_j(t - \tau_{ij}(t))) - I_i(t) \right] \quad (1)$$

$$L_j = \sup_{x \neq y} \left| \frac{f_j(x) - f_j(y)}{x - y} \right|, \forall x \neq y \in R, 1 \leq j \leq n$$

For any continuous function $S(t)$ on $[0, T]$, \underline{S} and \overline{S} denote $\min_{t \in [0, T]} \{S(t)\}$ and $\max_{t \in [0, T]} \{S(t)\}$, respectively.

For any

$$x(t) = (x_1(t), x_2(t), \dots, x_k(t))^T \in R^k, t > 0,$$

define

$$\|x(t)\| = \sum_{i=1}^k |x_i(t)|,$$

and for any

$$\begin{aligned} \varphi(s) &= (\varphi_1(s), \varphi_2(s), \dots, \varphi_k(s))^T \in R^k, \\ s &\in (-\sigma, 0], \end{aligned}$$

define

$$\|\varphi(s)\| = \sup_{s \in [-\sigma, 0]} \sum_{i=1}^k |\varphi_i(s)|$$

in which

$$0 < \sigma < +\infty.$$

Denote

$$C([-\sigma, 0], R^k) = \{\psi : [-\sigma, 0] \rightarrow R^k \mid \psi(s)\}$$

is continuous on $[-\sigma, 0]$.

Then $C([-\sigma, 0], R^k)$ is a Banach space with respect to $\|\cdot\|$.

The initial conditions of system (1) are given by

$$\begin{aligned} x_i(s) &= \varphi_i(s), -\tau \leq s \leq 0, \\ \tau &= \max_{1 \leq i, j \leq n} \{\tau_{ij}\}, 1 \leq i \leq n \end{aligned} \quad (2)$$

where

$$\varphi(s) = (\varphi_1(s), \varphi_2(s), \dots, \varphi_n(s)) \in C([-\tau, 0], R^n).$$

Let $x(t, \varphi) = (x_1(t, \varphi), x_2(t, \varphi), \dots, x_n(t, \varphi))^T$ denotes any solution of the system (1) with initial value $\varphi \in C([-\tau, 0], R^n)$.

Definition 1. An solution $x(t, \varphi)$ of system (1) is said to be globally exponentially stable, for any solutions $x(t, \phi)$ of the system (1), if there exist positive constant $\lambda > 0$ and $M > 0$ such that

$$\|x(t, \phi) - x(t, \varphi)\| \leq M \|\phi - \varphi\| e^{-\lambda t}, t \geq 0 \quad (3)$$

Lemma 1. Under assumptions (H₁)-(H₃), system (1) has a T -periodic solution which is globally exponentially stable, if the following conditions hold.

(H₄) Assume that there exist constants $\hat{\tau}_{ij}^+$ such that $\hat{\tau}_{ij}^-(t) \leq \hat{\tau}_{ij}^+ < 1, 1 \leq i, j \leq n$.

(H₅) $M_1 = A_1 - C_1$ is a nonsingular M -matrix, where

$$A_1 = \text{diag}(\gamma_1, \gamma_2, \dots, \gamma_n), C_1 = (c_{ij})_{n \times n},$$

$$c_{ij} = \left(\bar{p}_{ij} + \bar{u}_{ij} \frac{1}{1 - \hat{\tau}_{ij}^+} \right) L_j \rho_j$$

Proof. If $v_{ijl} = 0$, the model (2.1) in [14] reduces to the system (1), we know that Lemma 1 holds from Theorem 3.1 with $r = 1$ in [14].

3. Periodic Solutions of CGBAMNNs with Time Varying Delays

Consider the following CGBAMNNs with time-varying delays:

$$\begin{aligned} \dot{x}_i(t) &= -a_i(x_i(t)) \left[b_i(t, x_i(t)) - \sum_{j=1}^m p_{ij}(t) f_j(\rho_j y_j(t)) \right. \\ &\quad \left. - \sum_{j=1}^m u_{ij}(t) f_j(\rho_j y_j(t - \tau_{ij}(t))) - I_i(t) \right] \\ \dot{y}_j(t) &= -c_j(y_j(t)) \left[d_j(t, y_j(t)) - \sum_{i=1}^n q_{ji}(t) g_i(\tilde{\rho}_i x_i(t)) \right. \\ &\quad \left. - \sum_{i=1}^n v_{ji}(t) g_i(\tilde{\rho}_i x_i(t - \tilde{\tau}_{ji}(t))) - J_j(t) \right] \end{aligned} \quad (4)$$

for $1 \leq i \leq n, 1 \leq j \leq m, t > 0$ and $Z^+ = \{1, 2, \dots\}$. $x_i(t)$ and $y_j(t)$ denote the state variables, $f_j(\cdot)$ and $g_i(\cdot)$ denote the signal functions, $I_i(t)$ and $J_j(t)$ denote inputs; $a_i(\cdot)$ and $c_j(\cdot)$ represent amplification functions; $b_i(t, \cdot)$ and $d_j(t, \cdot)$ are appropriately behaved functions; $p_{ij}(t), q_{ji}(t), u_{ij}(t)$ and $v_{ji}(t)$ are the connection weights and $\rho_j, \tilde{\rho}_i$ are positive constants, which correspond to the neuronal gains associated with the neuronal activations; Time delays $\tau_{ij}(t)$ and $\tilde{\tau}_{ji}(t)$ correspond to the finite speed of the axonal signal transmission at time t and there exist constants τ_{ij} and $\tilde{\tau}_{ji}$ such that $0 \leq \tau_{ij} \leq \tau_{ij}, 0 \leq \tilde{\tau}_{ji}(t) \leq \tilde{\tau}_{ji}; u_{ij}(t), v_{ji}(t), p_{ij}(t), q_{ji}(t), I_i(t), J_j(t), \tau_{ij}(t)$ and $\tilde{\tau}_{ji}(t)$ are all continuously periodic functions on $[0, +\infty)$ with common period $T > 0$.

Throughout this paper, we assume for system (4) that (H₆) Amplification functions $a_i(\cdot)$ and $c_j(\cdot)$ are continuous and there exist positive constants $\underline{a}_i, \bar{a}_i$ and $\underline{c}_j, \bar{c}_j$ such that $\underline{a}_i \leq a_i(x_i(t)) \leq \bar{a}_i, \underline{c}_j \leq c_j(y_j(t)) \leq \bar{c}_j, 1 \leq i \leq n, 1 \leq j \leq m$.

(H₇) $b_i(t, \cdot), d_j(t, \cdot)$ are T -periodic about the first argument and there exist continuous T -periodic functions $\gamma_i(t)$ and $\delta_j(t)$ such that

$$\begin{aligned} \frac{b_i(t, x) - b_i(t, y)}{x - y} &\geq \gamma_i(t) > 0, \frac{d_j(t, x) - d_j(t, y)}{x - y} \\ &\geq \delta_j(t) > 0, \forall x \neq y \in R, 1 \leq i \leq n, 1 \leq j \leq m \end{aligned}$$

(H₈) For activation functions $f_j(\cdot)$ and $g_i(\cdot)$, there

exist constants L_j and \tilde{L}_i such that

$$L_j = \sup_{x \neq y} \left| \frac{f_j(x) - f_j(y)}{x - y} \right|, \tilde{L}_i = \sup_{x \neq y} \left| \frac{g_i(x) - g_i(y)}{x - y} \right|,$$

$$\forall x \neq y \in \mathbb{R}, 1 \leq i \leq n, 1 \leq j \leq m$$

The initial conditions of system (4) are given by

$$x_i(s) = \varphi_i(s), y_j(s) = \phi_j(s), -\tilde{\tau} \leq s \leq 0,$$

$$\tilde{\tau} = \max_{1 \leq i \leq n, 1 \leq j \leq m} \{ \tau_{ij}, \tilde{\tau}_{ji} \}, 1 \leq i \leq n, 1 \leq j \leq m$$

where

$$(\varphi, \phi) \in C([-\tilde{\tau}, 0], \mathbb{R}^{n+m}),$$

$$\varphi(s) = (\varphi_1(s), \dots, \varphi_n(s)), \phi(s) = (\phi_1(s), \dots, \phi_m(s)).$$

Theorem 1. Under assumptions (H₆)-(H₁₀), system (4) has a T -periodic solution which is globally exponentially stable, if the following condition holds.

(H₉) Assume that there exist constants $\tilde{\tau}_{ij}^+$ and $\tilde{\tau}_{ji}^+$ such that $\dot{\tau}_{ij}(t) \leq \tilde{\tau}_{ij}^+ < 1$ and $\dot{\tau}_{ji}(t) \leq \tilde{\tau}_{ji}^+ < 1$ hold for $1 \leq i \leq n, 1 \leq j \leq m$.

(H₁₀) The following M_3 is a nonsingular M -matrix, and

$$M_3 = \begin{pmatrix} A_3 & -C_3 \\ -\tilde{C}_3 & \tilde{A}_3 \end{pmatrix} \quad (5)$$

in which

$$A_3 = \text{diag}(\underline{\gamma}_1, \underline{\gamma}_2, \dots, \underline{\gamma}_n), \tilde{A}_3 = \text{diag}(\underline{\delta}_1, \underline{\delta}_2, \dots, \underline{\delta}_{n+m})$$

$$\tilde{C}_3 = (\tilde{c}_{ji})_{m \times n}, \tilde{c}_{ji} = \left(\bar{q}_{ji} + \bar{v}_{ji} \frac{1}{1 - \tilde{\tau}_{ij}^+} \right) \tilde{L}_i \tilde{\rho}_j,$$

$$C_3 = (\hat{c}_{ij})_{n \times m}, \hat{c}_{ij} = \left(\bar{p}_{ij} + \bar{u}_{ij} \frac{1}{1 - \tilde{\tau}_{ij}^+} \right) L_j \rho_j$$

Proof. Let

$$x_{n+j}(t) = y_j(t), a_{n+j}(t, x_{n+j}(t)) = c_j(t, y_j(t)),$$

$$b_{n+j}(t, x_{n+j}(t)) = d_j(t, y_j(t))$$

$$p_{n+j,i}(t) = q_{ji}(t), p_{i,n+j}(t) = p_{ij}(t),$$

$$u_{n+j,i}(t) = v_{ji}(t), u_{i,n+j}(t) = u_{ij}(t)$$

$$S_i(x_i(t)) = g_i(x_i(t)), \quad (6)$$

$$S_{n+j}(x_{n+j}(t)) = f_j(x_j(t)),$$

$$\varphi_{n+j}(s) = \phi_j(s), I_{n+j} = J_j(t)$$

$$\tau_{n+j,i}(t) = \tilde{\tau}_{ji}(t), \tau_{i,j}(t) = \tilde{\tau}_{i,n+j}(t),$$

$$\gamma_{n+j}(t) = \delta_j(t), \tilde{L}_{n+j} = L_j, \tilde{\rho}_{n+j} = \rho_j,$$

It follows that system (4) can be rewrote as

$$\dot{x}_i(t) = -a_i(x_i(t)) \left[b_i(t, x_i(t)) - \sum_{j=1}^m p_{i,n+j}(t) S_{n+j}(\tilde{\rho}_{n+j} x_{n+j}(t)) - \sum_{j=1}^m u_{i,n+j}(t) S_{n+j}(\tilde{\rho}_{n+j} x_{n+j}(t - \tau_{i,n+j}(t))) - I_i(t) \right],$$

$$\dot{x}_{n+j}(t) = -a_{n+j}(x_{n+j}(t)) \left[b_{n+j}(t, x_{n+j}(t)) - \sum_{i=1}^n p_{n+j,i}(t) S_i(\tilde{\rho}_i x_i(t)) - \sum_{i=1}^n u_{n+j,i}(t) S_i(\tilde{\rho}_i x_i(t - \tau_{n+j,i}(t))) - I_{n+j}(t) \right], \quad (7)$$

for $1 \leq i \leq n, 1 \leq j \leq m$.

Initial conditions are given by

$$x_i(s) = \varphi_i(s), s \in (-\tilde{\tau}, 0], i = 1, 2, \dots, (n+m) \quad (8)$$

Hence system (7) is a special case of system (1) in mathematical form in which there are $n+m$ neurons and connection weights $p_{ij} = u_{ij} = 0$ for $1 \leq i, j \leq n$ and $n+1 \leq i, j \leq m+n$. Under conditions (H₆)-(H₁₀), from Lemma 1, we obtain that system (7) has a T -periodic solution which is globally exponentially stable, if the following matrix M' is a M -matrix, and

$$M' = A' - C', \quad (9)$$

where

$$A' = \text{diag}(\underline{\gamma}_1, \underline{\gamma}_2, \dots, \underline{\gamma}_{n+m})$$

$$C' = \begin{pmatrix} 0 & \dots & 0 & w'_{1,n+1} & \dots & w'_{1,n+m} \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \dots & 0 & w'_{n,n+1} & \dots & w'_{n,n+m} \\ w'_{n+1,1} & \dots & w'_{n+1,n} & 0 & \dots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ w'_{n+m,1} & \dots & w'_{n+m,n} & 0 & \dots & 0 \end{pmatrix}$$

$$\text{in which } w'_{ij} = \left(\bar{p}_{ij} + \bar{u}_{ij} \frac{1}{1 - \tilde{\tau}_{ij}^+} \right) \tilde{\rho}_j \tilde{L}_j$$

Then, we know from (6) and (9) that Theorem 1 holds.

4. An Example

Consider the following CGBAMNNs with time delays:

$$\dot{x}_1(t) = -(1 + 0.2 \cos(x_1(t))) \cdot \left[2.1x_1(t) - 2 \sin \pi t |y_1(t - e^{-0.0001x_1 t})| - 1 \right], \quad (10)$$

$$\dot{y}_1(t) = -(1 - 0.2 \sin(y_1(t))) \cdot \left[2.1y_1(t) - \cos \pi t |2x_1(t - 4)| - 1 \right],$$

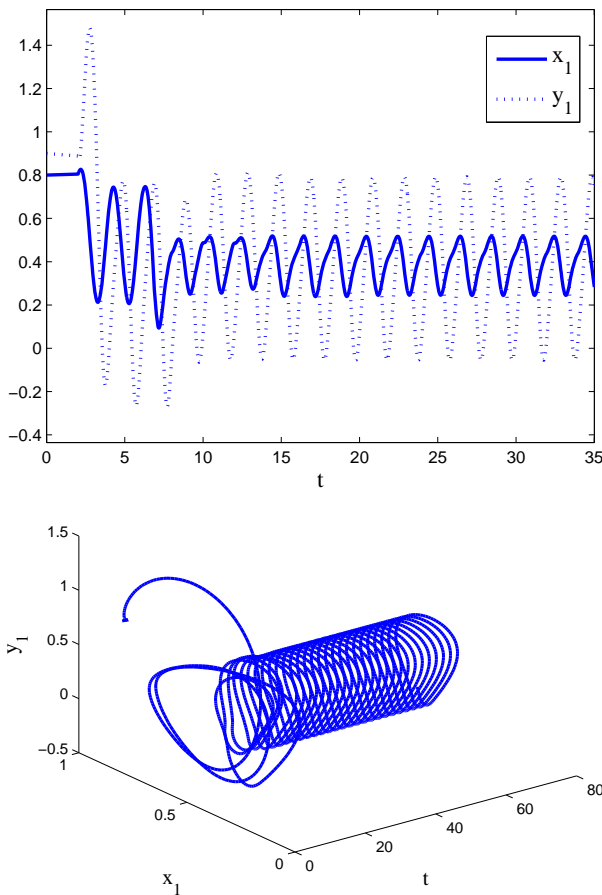


Figure 1. Time response of state variables x_1 , y_1 and phase plot in space (t, x_1, y_1) for system (10).

It is easy to verify system (10) satisfies (H_6) - (H_9) . In addition, system (10) satisfies (H_{10}) because

$$M_1 = \begin{pmatrix} 2.1 & -1 \\ -2 & 2.1 \end{pmatrix}$$

is a nonsingular M -matrix. According to Theorem 1, system (10) has a 2-periodic solution which is globally exponentially stable. **Figure 1** shows the dynamic behaviors of system (10) with initial conditions $(0.8, 0.9)$.

Remark 1 The results in [3,15] have more restrictions than the results in this paper because conditions for the results in [3,15] are relevant to amplification functions. In addition, in view of proof of Theorem 1, since CGBAMNNs with time-varying delays is a special case of CGNNs time-varying delays in form as BAM neural networks is a special case of Hopfield neural networks, many results of CGBAMNNs can be directly obtained from the ones of CGNNs, needing no repetitive discussions, which coincide with the conclusion in [16,17].

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