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Extraction of Buried Signals in Noise: Correlated Processes

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Abstract

In this paper, we propose a method for extraction of signals correlated with noise in which they are buried. The proposed extraction method uses no a-priori information on the buried signal and works independently of the nature of noise, correlated or not with the signal, colored or white, Gaussian or not, and locations of its spectral extent. Extraction of buried correlated signals is achieved without averaging in the time or frequency domain.

Keywords: Extraction, Buried Signals, Spectral Analysis, Colored Noise, White Noise, Correlated Processes

1. Introduction

Extraction of buried signals remains to date an important challenge for investigation purposes in different areas of science as underwater acoustics, wave propagation, transmission, astronomical observations, earth observation, data mining, etc (see, for example, [1-4] and a number of references in [5,6]). Signals taken in extremely poor conditions or corrupted by various natures of noise in different systems are encountered in practice. This noise addition correlated or not with the desired signal degrades significantly the quality of information. In some situations, the signal is totally buried by various sources of noise.

The effect of noise can be reduced but at the expense of the bandwidth and/or resolution which is, in most cases, undesired. Over-sampling and multi-sampling techniques and their properties are well known by a long time and are applied in several applications where detail preservation under largely unpredictable noise statistics is mandatory (seismics, evoked potentials and so on). Such methods in some practical settings (images with linear patterns, for example) can remove noise without significantly impacting the desired signal. Multi-channel and multi-dimensional signals have a lot of features to be exploited.

It is crucial to notice that, in fact, these features are not suited to buried signals. Moreover, no a-priori information on the buried signal and the nature of noise with which it is correlated, is known. Notice also that by *buried signals*, we mean signals defined for *low or extreme low* signal-to-noise ratio and the terms *extraction of signals buried in noise*, mean extraction of *clean* spectra of buried signals in noise.

We proposed in [5,6] two *non-parametric* and *equivalent* extraction methods of buried signals assumed uncorrelated with noise. These equivalent methods are called respectively "modified frequency extent denoising (MFED)" and "constant frequency extent denoising (CFED)". Here, the frequency extent is the interval $[0, f_e]$ where f_e is the sampling frequency of the *buried* continuous-time signal to be extracted. The first procedure (MFED) is based on modifying the sampling frequency and the second one (CFED) is suited to a collection of available sample noisy processes.

On the other hand, extraction of buried signals *correlated with noise* is of questionable interest (see, for example, [7-11] in different areas of science). In this work, we extend results of the aforementioned extraction method, CFED, to the presence of noise correlated with buried signals, maybe non-Gaussian, which is a fact and an issue. Notice that CFED extraction method is chosen for its implementation simplicity. Advantages of proposed extraction of buried signals are:

1) no a-priori information on the buried signal is used,

2) extraction works without averaging or smoothing in the time or frequency domain,

3) extraction is achieved independently of the nature of noise, colored or white, Gaussian or not, correlated or not with the signal, and locations of its spectral extent and,

4) straightforward extraction of signals buried in cor-

related noise (noise whose samples are correlated).

Performances of the extraction method via examples of buried signals correlated with white and/or colored noise are given. Comparative results with other methods [3,4] are included.

2. Fundamentals

In this section, we recall some definitions and principal results reported in [5,6].

2.1. Definitions

2.1.1. Signal Representation

Consider a band-limited signal $s(t)$ buried in zero-mean wide sense stationary noise $b(t)$ observed by means of $z(t)$, defined by,

$$z(t) = s(t) + b(t). \tag{1}$$

We denote by $P(f)$ the band-limited spectrum of $s(t)$, *i.e.*,

$$P(f) = 0, \quad f_{\min} \geq |f| \geq f_{\max}, \tag{2}$$

where $f_{\min}$ and $f_{\max}$ are bounds of the spectral support of $P(f)$.

A finite observation of $z(t)$ in the interval of length *T*, chosen so that $Tf_{\max} \gg 1$, available at the output of a low-pass filter of cut-off frequency $f_{\max}$ yields,

$$z_T(t) = \begin{cases} b_T(t) + s_T(t), & t \in [0, T] \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

where $b_T(t)$ and $s_T(t)$ represent respectively the additive noise (white or colored) and the signal observed in the time interval of length *T*.

By considering the instants $t_n = n/f_e$ where $f_e \geq 2 f_{\max}$ is the sampling frequency, we can define the discrete-time process $z_N(n)$ with $N = Tf_e$.

2.1.2. The Sample Power Spectral Density (SPSD)[5]

Given $\{z_N(0), z_N(1), \cdots, z_N(N-1)\}$, we can form the estimate,

$$\Phi(f, f_e, T) = \left| \frac{1}{T} \mathrm{D}FT(z_N(n)) \right|^2, \tag{4}$$

where DFT $(z_N(n))$ denotes Discrete Fourier Transform of $z_N(n)$. The estimate $\Phi(f, f_e, T)$ depends on the frequency, *f*, the sampling frequency, f_e, and the length of the observation interval, *T*.

It is crucial to notice that (4) is not a power spectral density in the usual sense. Here (4) is defined as the "*Sample*" Power Spectral Density or the sample spectrum and in [5], we reported conditions under which (4) can be used literally without ensemble averaging or smoothing for extraction of buried signals independently of the nature of noise and locations of its spectral extent.

2.2. CFED Extraction Method

Here, we have a collection of β realizations of duration *T* of a noisy process so that the length of the total observation interval is βT. These β realizations denoted $z_T^{(p)}(t)$ where $p = 0, \cdots, \beta - 1$ of the process are *concatenated* in order to form the process,

$$z_{\beta T}(t) = \sum_{p=0}^{\beta-1} z_T^{(p)}(t - pT). \tag{5}$$

2.2.1. Sample Spectrum of Noise

We found that the sample spectrum of noise obtained by Fourier transformation of (5) is given by,

$$\rho(f, f_e, \beta T) = \sum_{p=0}^{\beta-1} \alpha_p(\beta) \rho(f - p/(\beta T), f_e, T), \tag{6}$$

where $\rho(f - p/(\beta T), f_e, T)$ are translated copies of the *original* sample spectrum of noise whose components are spaced with the mutual distance $1/T$ on the frequency axis.

It is crucial to notice that scaling multiplication factors $\alpha_p(\beta)$ in (6) are defined by [5],

$$\sum_{p=0}^{\beta-1} \alpha_p(\beta) = 1. \tag{7}$$

Clearly, $\alpha_p(\beta)$ are *reduction* factors since $\forall p, \alpha_p(\beta) < 1$. Here (7) is bounded by,

$$\beta \min(\alpha_p(\beta)) \leq \sum_{p=0}^{\beta-1} \alpha_p(\beta) \leq \beta \max(\alpha_p(\beta)), \tag{8}$$

where $\min(\alpha_p(\beta))$ and $\max(\alpha_p(\beta))$ denote respectively the minimum and the maximum values of $\alpha_p(\beta)$.

Since $\alpha_p(\beta)$ are arbitrary reduction factors, we can for the sake of simplicity and without loss of generality, consider that, $\forall p$, $\alpha_p(\beta) = 1/\beta$. Factors $\alpha_p(\beta)$ reduce indifferently translated copies of the original spectrum of noise independently of their nature (white or colored, Gaussian or not) and act indifferently at all frequencies.

2.2.2. Spectral Distribution

As translated copies of the original spectrum of noise $\rho(f - p/(\beta T), f_e, T)$ are shifted by $1/(\beta T)$ with respect to each other (see (6)), the resulted sample spectrum $\rho(f, f_e, \beta T)$ will exhibit spectral lines separated by the mutual distance $1/(\beta T)$. Hence original spectral lines of noise separated by the mutual distance $1/T$ are now distributed in new β frequency locations created in each original frequency interval.

On the other hand, the spectrum of the signal $s_T(t)$

as given by the transformation of concatenated realizations (5), is specified by $\Gamma(f, f_e, \beta T)$. Since β zeros are distributed in β frequency locations created in each interval of length $1/T$ (see [5]) then,

$$\Gamma(f, f_e, \beta T) = \Gamma(f, f_e, T). \quad (9)$$

2.2.3. Extraction Properties

Extraction of the sample spectrum of the buried signal is obtained by *decimation*. This decimation by the factor β is applied in the frequency domain to the Fourier transformation of (5), *i.e.*,

$$D_\beta[\Phi(f, f_e, \beta T)] = \Phi_D(f, f_e, T), \quad (10)$$

where $D_\beta[\Phi]$ represents the decimation by β applied to Φ.

The signal-to-noise ratio $\widehat{\Upsilon}$ of the decimated spectrum written as a function of the signal-to-noise ratio of the original spectrum Υ is given by,

$$\widehat{\Upsilon} = \beta\Upsilon \quad (11)$$

We have shown in [5] that increasing β (the number of collected sample processes) increases the signal-to-noise ratio Υ of the original noisy spectrum $\Phi(f, f_e, T)$. Moreover, the variance of *extracted* sample spectral estimates tends to zero as β increases, *i.e.*,

$$Var[\Phi_D(f, f_e, T)] = \frac{1}{\beta^2} Var[\Phi(f, f_e, T)]. \quad (12)$$

3. Buried Correlated Processes

In [5,6], we assumed that the signal and additive noise, independently of its nature, are uncorrelated. In this section, we introduce correlation between the signal and noise in which it is buried by setting,

$$\eta_T(t) = (s_T(t) + n_T(t)) \star h(t), \quad (13)$$

where $h(t)$ represents the transfer function of a filtering system whose input is the stationary process $s_T(t) + n_T(t)$ and its output is the $\eta_T(t)$. Here the symbol $\star$ denotes the convolution.

Let us assume that we have β sample processes. By concatenating these realizations, we form the process of duration βT,

$$\eta_{\beta T}(t) = \sum_{p=0}^{\beta-1} \eta_T(t - pT). \quad (14)$$

We propose hereafter to find the sample spectrum of (14).

3.1. Expression of the CFED Sample Spectrum

Let $H(f, f_e, T)$ be the Fourier transform of the impulse response of the filter $h(t)$. By using (14), the sample spectrum $\Phi(f, f_e, \beta T)$ yielded by (4) is composed of respectively the filtered sample spectrum of noise and the filtered sample spectrum of the signal defined by,

$$\phi(f, f_e, \beta T) = |H(f, f_e, \beta T)|^2 \rho(f, f_e, \beta T),$$

and,

$$\Gamma(f, f_e, \beta T) = |H(f, f_e, \beta T)|^2 P(f, f_e, \beta T),$$

and the cross-products of their amplitude spectra. This means that the *sample* power spectrum $\Phi(f, f_e, \beta T)$ as defined by (4) is given by,

$$\begin{aligned}\Phi(f, f_e, \beta T) &= \Gamma(f, f_e, \beta T) + \phi(f, f_e, \beta T) \\ &+ S^*(f, f_e, \beta T)\varphi(f, f_e, \beta T) \\ &+ S(f, f_e, \beta T)\varphi^*(f, f_e, \beta T),\end{aligned} \quad (15)$$

where $S^*(f, f_e, \beta T)$ and $\varphi^*(f, f_e, \beta T)$ are amplitude spectra of $s_T(t) \star h(t)$ and $b_T(t) \star h(t)$. Here x^* denote the complex conjugate of x.

3.2. Decimated CFED Sample Spectrum

The decimated N-point sample spectrum applied to $\Phi(f, f_e, \beta T)$, as depicted by (15), yields,

$$\begin{aligned}D_\beta[\Phi(f, f_e, \beta T)] &= D_\beta[\Gamma(f, f_e, \beta T)] + D_\beta[\phi(f, f_e, \beta T)] \\ &+ D_\beta[S^*(f, f_e, \beta T)\varphi(f, f_e, \beta T) \\ &+ S(f, f_e, \beta T)\varphi^*(f, f_e, \beta T)],\end{aligned} \quad (16)$$

where $D_\beta[\Phi]$ is the β-decimation applied to Φ.

Here, it is crucial to notice that our aim is to find the *optimal* form under which expression of the CFED sample spectrum is written only as a function of the sample spectrum of the signal and noise independently of any correlation between the signal and noise and without averaging in the time or frequency domain.

Since the sample spectra of the signal and noise are given by,

$$\begin{aligned}D_\beta[\Gamma(f, f_e, \beta T)] &= \Gamma(f, f_e, T) \\ D_\beta[\varphi(f, f_e, \beta T)] &= \frac{1}{\beta}\varphi(f, f_e, T),\end{aligned} \quad (17)$$

then (16) becomes,

$$\begin{aligned}D_\beta[\Phi(f, f_e, \beta T)] &= \Gamma(f, f_e, T) + \frac{1}{\beta}\varphi(f, f_e, \beta T) \\ &+ D_\beta[S^*(f, f_e, \beta T)\phi(f, f_e, \beta T)] \\ &+ D_\beta[S(f, f_e, \beta T)\phi^*(f, f_e, \beta T)].\end{aligned} \quad (18)$$

Now, let us write the cross-products as a function of their Fourier coefficients. Let λ_k be the Fourier coefficient of the amplitude spectrum $S(f, f_e, T)$ of the signal and let ξ_k be the Fourier coefficient of the amplitude noise spectrum $\varphi(f, f_e, T)$. As,

$$\Gamma(f,f_e,T)=S(f,f_e,T)S^*(f,f_e,T)$$
$$\phi(f,f_e,T)/\beta=\varphi(f,f_e,T)\varphi^*(f,f_e,T). \tag{19}$$

Since γ_k and c_k are respectively Fourier coefficients of $\Gamma(f,f_e,T)$ and $\phi(f,f_e,T)$, (19) becomes,

$$\gamma_k=\lambda_k\lambda_k^*$$
$$c_k/\beta=\xi_k\xi_k^*. \tag{20}$$

By using (20) and noting that,

$$\Gamma(f)=\sum_{k=0}^{N-1}\gamma_k\delta(f-k/T), \tag{21}$$

the sample spectrum, as given by (18), yields therefore explicitly,

$$D_\beta[\Phi(f,f_e,\beta T)]=\frac{1}{\beta}\varphi(f,f_e,T)$$
$$+\sum_{k=0}^{N-1}(\gamma_k+\lambda_k\xi_k^*+\lambda_k^*\xi_k)\delta(f-k/T). \tag{22}$$

3.2.1. The Optimal Reduction Factor

In the following, we derive the expression of optimal reduction factor representing the optimal number β of *concatenated* sample processes under which contribution of cross-products in (18) are made negligible. This means that extracted spectrum consists, under this condition, only of the spectra of the signal and noise.

Coefficients of the last right-hand side of (22) can be put under the form,

$$\gamma_k+\lambda_k\xi_k^*+\lambda_k^*\xi_k=\gamma_k\times\left[1+\left(\frac{\xi_k}{\lambda_k}\right)^*+\frac{\xi_k}{\lambda_k}\right]. \tag{23}$$

Let $\psi_k=\xi_k/\lambda_k+(\xi_k/\lambda_k)^*$ and note that,

$$|1+\psi_k|\le 1+|\psi_k|, \tag{24}$$

where ψ_k rewritten as a function of ξ_k and λ_k yields,

$$\left|\frac{\xi_k}{\lambda_k}+\left(\frac{\xi_k}{\lambda_k}\right)^*\right|\le 2\frac{|\xi_k|}{|\lambda_k|}. \tag{25}$$

By setting $c_k/\beta=\xi_k\xi_k^*$, (25) becomes,

$$\left|\frac{\xi_k}{\lambda_k}+\left(\frac{\xi_k}{\lambda_k}\right)^*\right|\le 2\sqrt{\frac{c_k}{\beta\gamma_k}}. \tag{26}$$

Now, let us find the condition that defines the minimum value of β under which (26) is smaller than unity, *i.e.*,

$$2\sqrt{\frac{c_k}{\beta\gamma_k}}\ll 1. \tag{27}$$

We propose to find β as a function of the signal-to-noise ratio of the β collection of processes. The signal-to-noise ratio defined by $\Upsilon=p_s/\sigma^2$, where p_s and σ^2 are respectively the mean power of the signal and the variance of noise, can be written under the form,

$$\Upsilon=p_s/\sigma^2$$
$$=\frac{\gamma_k}{c_k}\times\frac{I_\gamma}{I_c}. \tag{28}$$

where γ_k and c_k are arbitrary chosen coefficients and,

$$I_\gamma=1+\sum_{s=0,s\neq k}^{S-1}\gamma_s/\gamma_k$$
$$I_c=1+\sum_{q=0,q\neq k}^{N-1}c_q/c_k. \tag{29}$$

where S and N represent respectively the number of spectral components of the signal and noise.

It is easy to see that I_γ is bounded by,

$$\forall k,\ S\min\{\gamma_s/\gamma_k\}\le I_\gamma\le S\max\{\gamma_s/\gamma_k\}, \tag{30}$$

where $\min\{\gamma_s/\gamma_k\}$ and $\max\{\gamma_s/\gamma_k\}$ denote respectively the minimum and the maximum values of the set formed by γ_s/γ_k, for $s=0,1,\cdots,S-1$ and $\forall k$.

Since γ_k is an arbitrary chosen coefficient and according to (30), we can consider that,

$$\forall k,\ I_\gamma=S. \tag{31}$$

Similarly, $I_c=N$. The signal-to-noise ratio, as depicted by (28), becomes,

$$\Upsilon=\frac{\gamma_k}{c_k}\frac{S}{N}. \tag{32}$$

Now, the expression (27) is satisfied if,

$$\beta\gg\frac{4S}{N\Upsilon}. \tag{33}$$

For a useful interpretation of (33), let us express the optimal reduction factor β only as a function of Υ, the signal-to-noise ratio. By setting $\beta_{\min}=4S/(N\Upsilon)$, one finds that since $S/N<1$ two conditions have to be considered: $4S/N\le 1$ and $4S/N\ge 1$. This gives,

$$4S/N\le 1,\quad 1<\beta_{\min}\le\frac{1}{\Upsilon}$$
$$4S/N\ge 1,\quad \beta_{\min}\ge\frac{1}{\Upsilon}. \tag{34}$$

According to (33), since $\beta\gg\beta_{\min}$, we have,

$$\beta>\frac{1}{\Upsilon}. \tag{35}$$

Here (35) depicts the optimal reduction factor β as a function of the signal-to-noise ratio Υ for which the condition (27) is fulfilled.

3.2.2. Optimal Sample Spectrum

According to (35), (23) yields,

$$\forall \beta > 1/\Upsilon,\ |\gamma_k + \lambda_k \zeta_k^* + \lambda_k^* \zeta_k| \le \gamma_k. \quad (36)$$

By using (36), (22) becomes,

$$\forall \beta > 1/\Upsilon,$$
$$D_\beta[\Phi(f, f_e, \beta T)] \le \Gamma(f, f_e, T) + \frac{1}{\beta}\varphi(f, f_e, T), \quad (37)$$

which yields,

$$\forall \beta > 1/\Upsilon,$$
$$\widehat{\Phi}_{\max}(f, f_e, T) = \Gamma(f, f_e, T) + \frac{1}{\beta}\varphi(f, f_e, T). \quad (38)$$

This is an important result since expression of the extracted spectrum, as depicted by (38), can be obtained without the requirement based on ensemble averaging (see, for example, [2]) and independently of the correlation between the signal and noise, Gaussian or not, white or colored, in which it is buried. Moreover, at the limit of large values of β, (38) becomes,

$$\lim_{\beta \gg 1/\Upsilon} \widehat{\Phi}_{\max}(f, f_e, T) = \Gamma(f, f_e, T). \quad (39)$$

The sample spectrum of noise, independently of its nature, vanishes. Extracted spectrum is *identical* to the filtered original deterministic spectrum of the signal. Results (39) and (38) achieve extraction of buried spectra of signals correlated with noise without any averaging or smoothing in the time or in the frequency domain.

4. Method and Results

4.1. Preliminary Notes

The extraction method CFED, as recalled above, is based on the collection of different realizations of a noisy process. This extraction method is often chosen in practice for simplicity of its implementation.

In one hand, it is crucial to notice that collection of different realizations of a process does not consist of multiple observations of the same deterministic signal in different noise realizations. Here a simple ensemble average would give the expected result. In order to see this, we consider, in the following, β different realizations of duration T/β "cut-out" from the *only* observed process of duration T. This example is motivated by the fact that in some real world applications, only one long realization of duration T of a buried signal is available. Clearly, we show that transformation of a simple ensemble average of these β realizations cut-out from the observed process does not extract the spectrum of the buried stationary signal (see **Figure 1(c)** and **Figure 2(c)** depicting spectra of the mean (ensemble average)).

On the other hand, it is easy to see that transformation of the available whole process of duration T followed by a decimation in the frequency domain by the factor β yields a sample spectrum defined by $\Upsilon\beta > 1$ in accordance with (38) where Υ is signal-to-noise ratio (SNR) (see **Figure 1(f)** and **Figure 2(f)** depicting decimated CFED sample spectra for white and colored noise). We show that the choice of β (the number of sequences cut-out from the only available long sequence of duration T) for decimation is a compromise between the desired extraction for which $\Upsilon > 1/\beta$ and the frequency resolution β/T. Comparative results with some PSD estimation methods as the Welch and the Thomson's multitaper method [3,4] are discussed.

4.2. Buried Correlated Signals with White Noise

Let us consider the process defined by,

$$y_N(n) = x_N(n) + w_N(n), \quad (40)$$

where $w_N(n)$ is a uniformly distributed white noise of length N and $x_N(n)$ consists of two sinusoids $\cos(2\pi f_0 n/f_e) + \cos(2\pi f_1 n/f_e)$ with $f_0 = 2$ Hz and $f_1 = 5$ Hz.

The sampling frequency is $f_e = 35$ Hz and the observation interval is given by $T = 99$ s. The process $y_N(n)$ is now present at the input of an averaging filter defined by its impulse response $h(n) = [1/8, 1/4, 1/4, 1/4]$. In the following we consider extraction of the buried signal correlated with noise by analyzing the signal yielded by the output of the filter $\eta_N(n)$, *i.e.*,

$$\eta_N(n) = y_N(n) \star h(n), \quad (41)$$

where $\star$ denotes the convolution.

Note that any other choice of the impulse filtering function is possible.

4.2.1. The Choice of β

It is crucial to keep in mind that we have here a single realization of duration T. When "cutting-out" β sub-processes from this available realization, we impose a resolution of β/T and extraction from noise is effective for $\Upsilon\beta > 1$ where Υ is the signal-to-noise ratio in accordance with (38). This means that if we choose $\beta = 50$ for $T = 99$ s, we have therefore a resolution given by 0.5 Hz and an extraction from noise for $\Upsilon > 0.025(-16)$ dB.

4.2.2. Spectrum of the Mean

Let the signal-to-noise ratio be defined by $\Upsilon = 0.025(-16)$ dB. We cut-out from $\eta_N(n)$, $\beta = 50$ sample sequences $\{\eta_{N/\beta}^{(0)}(n), \cdots, \eta_{N/\beta}^{(49)}(n)\}$. In the **Figure 1(a)**, one finds the sample spectrum of true signal $x_{N/\beta}(n)$. The spectrum of the mean (ensemble average) of these β sample sequences is shown in **Figure 1(c)**. It can be seen that our sinusoids remain buried. As mentioned above, a simple ensemble average is not able to extracts buried sinusoids.

4.2.3. CFED and Other PSD Estimation Methods

Now, $\beta = 50$ filtered sample processes are concatenated (under the form given by (14)) in order to reform our original filtered sequence $\eta_N(n)$ consisting of $N = 3465$ points. We use for comparison the Welch method (see [12] or [13]) and the power spectral density using multitaper Thomson's method as described in [3] and the CFED denoising method proposed in this work. Results of the PSD Welch method are shown in **Figure 1(b)**. One can see that the used number of points is not sufficient for the PSD Welch method since depicted frequency range is smaller than $f_1 = 5$ Hz. One finds only the frequency $f_0 = 2$ Hz.

The Thomson's multiple window method [3] uses a bank of bandpass filters or windows instead of rectangular ones as in the periodogram method. These filters compute several periodograms of the entire signal and then averaging the resulting periodograms to construct a spectral estimate. In order to minimize bias and variance in each window, theses windows are chosen orthogonal. Optimal windows that satisfy these requirements are Slepian sequences or discrete prolate spheroidal sequences [4]. In **Figure 1(d)**, one finds the PSD yielded by the Thomson's multiple window method. It can be seen from this plot that for the SNR = −16 dB and $N = 3465$, no extraction of buried sinusoids (defined for $f_0 = 2$ Hz and $f_1 = 5$ Hz) is depicted.

In **Figure 1(e)** and **Figure 1(f)** results of CFED are represented in the frequency range [0,18] (Hz). Note that the spectrum is computed for the same number of points as in **Figure 1(d)** and **Figure 1(e)** ($N = 3465$). In **Figure 1(f)**, one finds the CFED decimated spectrum by $\beta = 50$. One can see that the two frequencies of our sinusoids are indeed extracted from uniformly distributed white noise with an excellent signal-to-noise ratio.

4.3. Buried Correlated Signals with Colored Noise

Here we consider the output of the filter,

$$z_N(n) = (x_N(n) + c_N(n)) \star h(n), \tag{42}$$

where colored noise $c_N(n)$ is given by,

$$c_N(n) = 0.45\, c_N(n-2) + 0.45\, c_N(n-1) + 0.23\, e_N(n) + 0.4\, e_N(n-1) + 0.2\, e_N(n-2), \tag{43}$$

where $e_N(n)$ is a Gaussian white noise sequence.

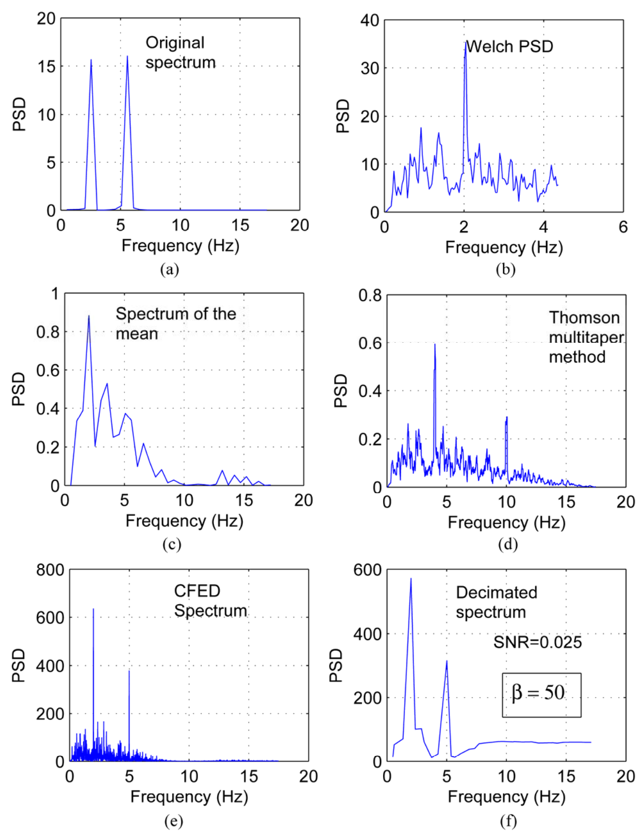

Figure 1. Extraction of buried correlated signals with white noise (SNR = −16 dB).

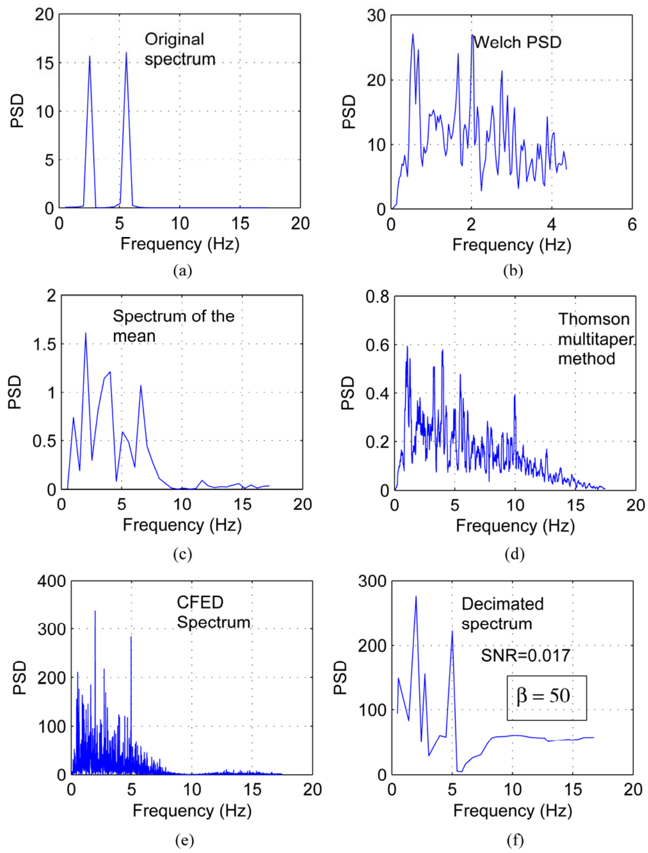

Figure 2. Extraction of buried correlated signals with colored noise (SNR = −17 dB).

Figure 2 summarizes obtained results for SNR = 0.017 (−17.7) dB. One finds in **Figure 2(c)** the spectrum of the mean (ensemble average). PSDs of the Welch, the Thomson's multitaper method (MTM) and CFED are shown respectively in **Figure 2(b)**, **Figure 2(d)**, **Figure 2(e)** and **Figure 2(f)**. Clearly, buried sinusoids correlated with colored noise are indeed extracted with an excellent signal-to-noise ratio in **Figure 2(e)** and **Figure 2(f)** for $\beta = 50$ whereas in **Figure 2(b)**, **Figure 2(c)** and **Figure 2(d)**, they remain buried.

Results of **Figures 1** and **2** show that CFED extraction method works for extraction of signals correlated with noise in which they are buried. This extraction, obtained without averaging, is independent of the nature of noise, white or colored, Gaussian or not. Extension of these results to extraction of signals in correlated noise independently of its nature is straightforward.

5. Conclusion

In this work, we proposed theoretical results on extraction of signals correlated with noise in which they are buried. We have shown that extraction is achieved without any averaging and using any a-priori information on the buried signal. Moreover, the proposed extraction method is independent of the nature of noise, correlated or not, correlated or not with the signal, colored or white, Gaussian or not, and locations of its spectral extent. Comparative results with other extraction methods are discussed and derived conclusions are in accordance with theoretical predictions.

6. References

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