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A Rank-One Fitting Method with Descent Direction for Solving Symmetric Nonlinear Equations*

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ABSTRACT

In this paper, a rank-one updated method for solving symmetric nonlinear equations is proposed. This method possesses some features: 1) The updated matrix is positive definite whatever line search technique is used; 2) The search direction is descent for the norm function; 3) The global convergence of the given method is established under reasonable conditions. Numerical results show that the presented method is interesting.

Keywords: Rank-One Update, Global Convergence, Nonlinear Equations, Descent Direction

1. Introduction

Consider the following system of nonlinear equations:

$$F(x) = 0, x \in R^n, \quad \textbf{(1)}$$

where $F: R^n \to R^n$ is continuously differentiable and the Jacobian $\nabla F(x)$ of $F(x)$ is symmetric for all $x \in R^n$. Let $\theta(x)$ be the norm function defined by

$$\theta(x) = \frac{1}{2}\|F(x)\|^2$$

then the nonlinear Equation (1) is equivalent to the following global optimization problem

$$\min \theta(x), x \in R^n \quad \textbf{(2)}$$

The following iterative method is used for solving (1)

$$x_{k+1} = x_k + \alpha_k d_k \quad \textbf{(3)}$$

where x_k is the current iterative point, d_k is a search direction, and a_k is a positive step-size.

It is well known that there are many methods [1–9] for the unconstrained optimization problems

$$\min_{x \in R^n} f(x) (UP) \ ,$$

where the BFGS method is one of the most effective quasi-Newton methods [10–17]. These years, lots of modified BFGS methods (see [18–23]) have been proposed for UP. Different from their techniques, Xu [24] presented a rank-one fitting algorithm for UP and the numerical examples are very interesting. Motivated by their idea, we give a new rank-one fitting algorithm for (1) which possesses the global convergence, the method can ensure that the updated matrices are positive definite without carrying out any line search, the search direction is descent for the normal function, and the numerical results is more competitive than those of the BFGS method for the test problem.

For nonlinear equations, the global convergence is due to Griewank [25] for Broyden's rank one method. Fan [1], Yuan [26], Yuan, Lu andWei [27], and Zhang [28] presented the trust region algorithms for nonlinear equations. Zhu [29] gave a family of nonmonotone backtracking inexact quasi-Newton algorithms for solving smooth nonlinear equations. In particular, a Gauss-Newton-based BFGS method is proposed by Li and Fukushima [30] for solving symmetric nonlinear equations, and the modified methods [31,32] are studied.

The line search rules play an important role for solving the optimization problems. In the following, we briefly review some line search technique to obtain the stepsize a_k.

Brown and Saad [33] proposed the following line search method:

$$\theta(x_k + \alpha_k d_k) - \theta(x_k) \le \sigma \alpha_k \nabla \theta(x_k)^T d_k \quad \textbf{(4)}$$

where

$$\nabla \theta(x_k)^T d_k = F(x)^T \nabla F(x_k) d_k ,$$

$$\sigma \in (0,1), \alpha_k = r^{i_k}, r \in (0,1) ,$$

i_k is the smallest nonnegative integer i such that (4). Zhu

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[29] gave the nonmonotone line search technique:

$$\theta(x_k+\alpha_k d_k)-\theta(x_{l(k)})\le\sigma\alpha_k\nabla\theta(x_k)^T d_k$$

$$\theta(x_{l(k)})=\max_{0\le j\le m(k)}\{\theta(x_{k-j})\},m(0)=0,$$

$$m(k)=\min\{m(k-1),M\},k\ge1$$

and M is a nonnegative integer. Yuan and Lu [32] presented a new backtracking inexact technique to obtain the stepsize a_k:

$$\|F(x_k+\alpha_k d_k)\|^2-\|F(x_k)\|^2\le\delta\alpha_k F(x_k)^T d_k \quad \textbf{(5)}$$

where $\delta\in(0,1)$ is a constant, and d_k is a solution of the system of linear Equation (9). Li and Fukashima [11] give a line search technique to determine a positive step-size a_k satisfying

$$\begin{aligned}&\|F(x_k+\alpha_k d_k)\|^2-\|F(x_k)\|^2\\&\le\delta_1\|\alpha_k F(x_k)\|^2-\delta_2\|\alpha_k d_k\|^2+\varepsilon_k\|F(x_k)\|^2\end{aligned} \quad \textbf{(6)}$$

where δ_1 and δ_2 are positive constants, and $\{\varepsilon_k\}$ is a positive sequence such that

$$\sum_{k=0}^{\infty}\varepsilon_k<\infty \quad \textbf{(7)}$$

The Formula (7) means that $\{F(x_k)\}$ is approximately norm descent when k is sufficiently large. Gu, Li, Qi, and Zhou [14] presented a descent line search technique as follows

$$\begin{aligned}&\|F(x_k+\alpha_k d_k)\|^2-\|F(x_k)\|^2\\&\le\delta_1\|\alpha_k F(x_k)\|^2-\delta_2\|\alpha_k d_k\|^2\end{aligned} \quad \textbf{(8)}$$

where δ_1 and δ_2 are positive constants. In this paper, we also use the Formula (8) as line search to find the step-size a_k:

The search direction d_k: play a main role in line search methods for solving optimization problems too, and d_k: is a solution of the system of linear equation

$$B_k d_k+F(x_k)=0 \quad \textbf{(9)}$$

where B_k is often generated by BFGS update formula

$$B_{k+1}=B_k+\frac{y_k y_k^T}{y_k^T s_k}-\frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} \quad \textbf{(10)}$$

where

$y_k=g_{k+1}-g_k$ and $s_k=x_{k+1}-x_k$ Is there another way to determine the update formula? Accordingly the search direction d_k is determined by the way. In this paper, the updated matrix B_k is generated by the following rank-one updated formula

$$B_{k+1}=B_k+v_k^T v_k^T \quad \textbf{(11)}$$

$$H_{k+1}=H_k-\frac{H_k v_k v_k^T H_k}{1+v_k^T B_k v_k} \quad \textbf{(12)}$$

where, as k = 0, B_0 is the given symmetric positive definite matrix,

$B_k^{-1}=H_k$ and $v_k=\delta_0\alpha_k F(x_k),\delta_0$

is a positive constant. Then we use the following formula to get the search direction,

$$B_k d+q(\alpha_{k-1})=0 \quad \textbf{(13)}$$

$$q(\alpha_{k-1})=\frac{F(x_k+\alpha_{k-1}F_k)-F(x_k)}{\alpha_{k-1}} \quad \textbf{(14)}$$

B_k follows (11), a_{k-1} is the steplength used at the previous iteration, and the Equation (14) is inspired by [34]. Throughout the paper, we use these notations: $\|.\|$ is the Euclidean norm, and $F(x_k)$ and $F(x_{k+1})$ are replaced by F_k and F_{k-1}, respectively.

This paper is organized as follows. In the next section, the algorithm is stated. The global convergence convergence is established in Section 3. The numerical results are reported in Section 4.

2. Algorithm

In this section, we state our new algorithm based on Formulas (3), (8), (11), (12) and (13) for solving (1).

Rank-One Updated Algorithm (ROUA).

Step 0: Choose an initial point $x_0\in R^n$ constants

$$r\in(0,1),0<\delta_0,\delta_1,\delta_2<1,\alpha_{-1}>0,$$

symmetric positive definite matrices B_0 and $B_0^{-1}=H_0$. Let: k = 0;

Step 1: If $\|F(x_k)\|=0$, stop. Otherwise, solving linear Equation (13) to get d_k;

Step 2: Find a_k is the largest number of $\{1,r,r^2,\ldots\}$ such that (8);

Step 3: Let the next iterative point be $x_{k+1}=x_k+a_k d_k$;

Step 4: Update B_{k+1} and H_{k+1} by the Formulas (11) and (12) respectively;

Step 5: Set k: = k + 1. Go to Step 1.

In this paper, we also give the normal BFGS method for solving (1), the algorithm which has the same conditions to ROUA is stated as follows.

BFGS Algorithm(BFGSA).

In ROUA, the Step 4 is replaced by: Update Bk+1 by the Formula (10).

Remark 1. a) By the Step 0 of ROUA, there should exist constants $\lambda_1\geqslant\lambda_0>0$ such that

$$\begin{aligned}&\lambda_1\|d\|^2\ge d^T B_k d\ge\lambda_0\|d\|^2,\\&\frac{1}{\lambda_0}\|d\|^2\ge d^T H_k d\ge\frac{1}{\lambda_1}\|d\|^2,\forall d\in R^n\end{aligned} \quad \textbf{(15)}$$

b) By the Step 4 of ROUA, it is easy to deduce that the updated matrices are symmetric

3. Convergence Analysis

This section will establish the global convergence for ROUA. Let Ω be the level set defined by

$$\Omega = \{x \mid \|F(x)\| \le \|F(x_0)\|\} \quad \textbf{(16)}$$

In order to establish the global convergence of ROUA, the following assumptions are needed [30,34,35].

Assumption A 1) F is continuously differentiable on an open convex set Ω_1 containing Ω. **2)** The Jacobian of F is symmetric, bounded and uniformly nonsingular on Ω_1, i.e., there exist constants $M \geqslant m > 0$ such that

$$\|\nabla F(x)\| \le M, x \in \Omega_1 \quad \textbf{(17)}$$

and

$$\|\nabla F(x)d\| \ge m\|d\|, x \in \Omega_1, \forall d \in R^n \quad \textbf{(18)}$$

Remark Assumption A 2) implies that

$$M\|d\| \ge \|\nabla F(x)d\| \ge m\|d\|, x \in \Omega_1, \forall d \in R^n \quad \textbf{(19)}$$

$$M\|x-y\| \ge \|F9x) - F(y)\| \ge m\|x-y\|, x, y \in \Omega_1 \quad \textbf{(20)}$$

In particular, for all $x \in \Omega_1$, we have

$$M\|x-x^*\| \ge \|F(x)\| = \|F(x) - F(x^*)\| \ge m\|x-x^*\| \quad \textbf{(21)}$$

where x^* stand for the unique solution of (1) in Ω_1.

Lemma 3.1 Let Assumption A hold. Consider ROUA. Then for any $d \in R^n$, then there exist constants m_0 such that

$$d^T B_k d \ge m_0 \|d\|^2, \forall d \in R^n \quad \textbf{(22)}$$

i.e., the matrix B_k is positive for all k.

Proof. By ROUA, we know that the initial matrix B0 is symmetric positive, and then we have (15). Using (11), for $k \geqslant 1$, we have

$$\begin{aligned} d^T B_k d &= d^T B_{k-1} d + d^T v_k v_k^T d \\ &= d^T B_{k-1} d + \|d^T v_k\|^2 \\ &\ge d^T B_{k-1} d \ge \cdots \ge d^T B_0 d \ge \lambda_0 \|d\|^2 \end{aligned} \quad \textbf{(23)}$$

Let $m_0 = \lambda_0$. Then we get (22). The proof is complete.

Since B_k is positive definite, then d_k which is determined by (13) has the unique solution. The following lemma can found in [34], here we also give the process of this proof.

Lemma 3.2 Let Assumption A hold. If x_k is not a stationary point of (2), then there exists a constant $a' > 0$ depending on k such that when $a_{k-1} \in (0, a')$, the unique solution $d(a_{k-1})$ of (13) such that

$$\nabla\theta(x_k)^T d(\alpha_{k-1}) < 0 \quad \textbf{(24)}$$

Moreover, inequality

$$\begin{aligned} &\|F(x_k + \alpha_{k-1} d(\alpha_{k-1}))\|^2 - \|F(x_k)\|^2 \\ &\le -\delta_1 \|\alpha_{k-1} d(\alpha_{k-1})\|^2 - \delta_2 \|\alpha_{k-1} F(x_k)\|^2 \end{aligned} \quad \textbf{(25)}$$

Proof. By (14), we can deduce that

$$\lim_{\alpha_{k-1}\to 0} q(\alpha_{k-1}) = \nabla F(x_k) F(x_k) \quad \textbf{(26)}$$

From (13), we get

$$\begin{aligned} &\lim_{\alpha_{k-1}\to 0^+} \nabla\theta(x_k)^T d(\alpha_{k-1}) \\ &= -\lim_{\alpha_{k-1}\to 0^+} F(x_k).^T \nabla F(x_k) B_k^{-1} q_k(\alpha_{k-1}) \\ &= -F(x_k).^T \nabla F(x_k) B_k^{-1} \nabla F(x_k) F(x_k) \end{aligned} \quad \textbf{(27)}$$

Since x_k is not a stationary point of (2), we have $\nabla F(x_k)F(x_k) \neq 0$. By $\nabla F(x_k)$ is symmetric and B_k is positive. We obtain (24).

$$\begin{aligned} &\lim_{\alpha_{k-1}\to 0^+} \frac{\|F(x_k + \alpha_{k-1} d(\alpha_{k-1}))\|^2 - \|F(x_k)\|^2}{\alpha_{k-1}} \\ &= \lim_{\alpha_{k-1}\to 0^+} 2\nabla\theta(x_k)^T d(\alpha_{k-1}) \\ &= -2F(x_k).^T \nabla F(x_k) B_k^{-1} \nabla F(x_k) F(x_k) < 0 \end{aligned}$$

However, the right hand side of (25) is $O(a_{k-1})$. Thus, inequality (25) holds for all $a_{k-1} > 0$ sufficiently small. The proof is complete.

The above lemma shows that line search technique (8) is reasonable, and the given algorithm is well defined. Lemma 3.2 also shows that the sequence $\{\theta(x_k)\}$ is strictly decreasing. By Lemma 3.2, it is not difficult to get the following lemma.

Lemma 3.3 Let $\{x_k\}$ be generated by ROUA. Consider the line search (8). Then $\{x_k\} \in \Omega$ moreover, $\{\|F(x_k)\|\}$ converges.

Lemma 3.4 Let Assumption A hold and $\{\alpha_k, d_k, x_{k+1}, F_k\}$ be generated by ROUA. Then we have

$$\sum_{k=0}^{\infty} \|\alpha_k F_k\|^2 < \infty \quad \textbf{(28)}$$

and

$$\sum_{k=0}^{\infty} \|\alpha_k d_k\|^2 < \infty \quad \textbf{(29)}$$

Proof. By the line search (8), we get

$$\delta_1 \|\alpha_{k-1} d(\alpha_{k-1})\|^2 + \delta_2 \|\alpha_{k-1} F(x_k)\|^2 \le \|F_k\|^2 - \|F_{k+1}\|^2 \quad \textbf{(30)}$$

Summing these inequalities (30) for k from 0 to ∞ we obtain (28) and (29). Then we complete the proof of this Lemma.

Lemma 3.5 Let Assumption A hold. Consider ROUA. Then $\{\|B_k\|\}$ converges, for all k and any $d \in R^n$ then there exist constants m_0 and M_0 such that

$$d^T B_k d \le M_0 \|d\|^2, \forall d \in R^n \quad \textbf{(31)}$$

and

$$\frac{1}{M_0}\|d\|^2 \le d^T H_k d \ge \frac{1}{m_0}\|d\|^2, \forall d \in R^n \quad \textbf{(32)}$$

which mean that the updated matrices are all positive by ROUA.

Proof. By the updated Formula (11), we have

$$\begin{aligned} \|B_{k+1}\| &= \|B_k + v_k v_k^T\| \le \|B_k\| + \|v_k\|^2 \\ &= \|B_k\| + \delta_0^2 \|\alpha_k F_k\|^2 \\ &\le \|B_0\| + \delta_0^2 \sum_{i=0}^{k} \|\alpha_i F_i\|^2 \end{aligned} \quad \textbf{(33)}$$

By (28), we know that

$$\sum_{i=0}^{k} \|\alpha_i F_i\|^2$$

is convergent. Then we can deduce that $\{\|B_k\|\}$ is convergent. So there exists a constant M_0 such that

$$\|B_k\| \le M_0 \quad \text{for all k} \quad \textbf{(34)}$$

Accordingly, we get (28). By (32), (31), and the Remark 1(b), we can deduce that the updated matrices are all symmetric and positive. Consider $H_k = B_k^{-1}$ we obtain (32) immediately. So, we complete the lemma.
By (32), (31), and (34), we have

$$\|q_k(\alpha_{k-1})\| = \|B_k d_k\| \le M_0 \|d_k\|, \|d_k\| \le \frac{1}{m_0}\|q_k(\alpha_{k-1})\| \quad \textbf{(35)}$$

Now we establish the global convergence theorem of ROUA.

Theorem 3.1 Let Assumption A hold and $\{\alpha_k, d_{k,} x_{k+1}, F_k\}$ be generated by ROUA. Then the sequence $\{x_k\}$ converges to the unique solution x^* of (1) in sense of

$$\lim_{k\to\infty} \|F_k\| = 0 \quad \textbf{(36)}$$

Proof. By Lemma 3.3, we know that $\{\|F_k\|\}$ converges. By Lemma 3.4, we get

$$\lim_{k\to\infty} \|\alpha_k F_k\| = 0 \quad \textbf{(37)}$$

then, we have

$$\lim_{k\to\infty} \|F_k\| = 0 \quad \textbf{(38)}$$

or

$$\lim_{k\to\infty} \alpha_k = 0 \quad \textbf{(39)}$$

Therefore, we only discuss the case of (38). In this case, for all k sufficiently large and

$$\alpha_k' = \frac{\alpha_k}{r}$$

by (8), we obtain

$$\begin{aligned} &\|F(x_k + \alpha_k' d_k\|^2 - \|F(x_k)\|^2 \\ &> -\delta_1 \|\alpha_k F(x_k)\|^2 - \delta_2 \|\alpha_k d_k\| \end{aligned} \quad \textbf{(40)}$$

By Lemma 3.3, we know that $\{x_k\} \in \Omega$ is bounded, considering (35), it is easy to deduce that $\{q_k(a_{k-1})$ and $\{d_k\}$ are bounded. Let $\{x_k\}$ and $\{d_k(a)\}$ converge to x^* and dx^*, respectively. Then we have

$$\lim_{k\to\infty} q_k(\alpha_{k-1}) = \nabla\theta(x^*) \quad \textbf{(41)}$$

Let both sides of (40) be divided by a_k' and take limits as $k \to \infty$ we obtain

$$\nabla\theta(x^*)^T d^* \ge 0 \quad \textbf{(42)}$$

By (31) and (13), we have

$$\begin{aligned} 0 &= d_k^T B_k d_k + q_k(\alpha_{k-1})^T d_k \\ &\ge m_0 \|d_k\|^2 + q_k(\alpha_{k-1})^T d_k \end{aligned} \quad \textbf{(43)}$$

As $k \to \infty$ taking limits in both of (43) yields

$$\nabla\theta(x^*)^T d^* \le -m_0 \|d_k\|^2$$

This together with (42) implies $d^*=0$. From (35), we have

$$\lim_{k\to\infty} q_k(\alpha_{k-1}) = 0$$

which together with (41), we obtain

$$\nabla\theta(x^*) = 0 \quad \textbf{(44)}$$

By $\nabla\theta(x^*) = \nabla F(x^*) F(x^*)$ and using $\nabla F(x^*)$ is nonsingular, we have $F(x^*) = 0$. This implies (36). The proof is complete.

Table 1. Test results for ROUA.

x0	(5,5,…,5)	(20,20,…,20)	(–20,…, –20)	(–60, –60,…, –60)	(–100,…, –100)
Dim	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF
n=10	40/121/8.132565e-007	43/130/8.142272e-007	43/130/8.143256e-007	45/136/9.711997e-007	47/142/6.423242e-007
n=40	43/130/7.823362e-007	46/139/8.163385e-007	46/139/8.163465e-007	49/148/6.389598e-007	50/151/6.806303e-007
n=100	44/133/8.517388e-007	47/142/8.916340e-007	47/142/8.916354e-007	50/151/7.002112e-007	51/154/7.468255e-007
n=500	46/139/8.076481e-007	49/148/8.467259e-007	49/148/8.467260e-007	52/157/6.664491e-007	53/160/7.124612e-007
n=1000	47/142/7.340784e-007	50/151/7.698173e-007	50/151/7.698173e-007	52/157/9.480059e-007	54/163/6.502176e-007
x0	(5,0, 5,0,…)	(20, 0, 20, 0…)	(–20, 0, –20, 0…)	(–60, 0, –60, 0…)	(–100,0,–100,0…)
Dim	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF
n=10	39/118/6.440181e-007	42/127/6.456963e-007	42/127/6.458627e-007	44/133/7.690322e-007	45/136/8.088567e-007
n=40	41/124/9.581725e-007	44/133/9.998342e-007	44/133/9.998539e-007	47/142/7.823915e-007	48/145/8.333429e-007
n=100	43/130/6.657874e-007	46/139/6.969606e-007	46/139/6.969629e-007	48/145/8.555192e-007	49/148/9.121694e-007
n=500	44/133/9.861003e-007	48/145/6.615057e-007	48/145/6.615058e-007	50/151/8.129150e-007	51/154/8.675076e-007
n=1000	45/136/8.961735e-007	48/145/9.396191e-007	48/145/9.396192e-007	51/154/7.392479e-007	52/157/7.893927e-007
x0	(5, –5, 5, –5…)	(20, –20, 20, –20…)	(–20, 20, –20, 20…)	(–20, 20, –20, 20…)	(–100, 100…)
Dim	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF
n=10	30/91/8.710675e-007	33/100/7.545800e-007	33/100/7.545800e-007	35/106/8.150523e-007	36/109/8.146625e-007
n=40	31/94/8.893379e-007	34/103/8.687998e-007	34/103/8.687998e-007	37/112/6.403068e-007	38/115/6.691432e-007
n=100	31/94/8.918405e-007	34/103/8.713106e-007	34/103/8.713106e-007	37/112/6.423164e-007	38/115/6.713147e-007
n=500	31/94/8.923155e-007	34/103/8.717867e-007	34/103/8.717867e-007	37/112/6.426974e-007	38/115/6.717265e-007
n=1000	31/94/8.923306e-007	34/103/8.718018e-007	34/103/8.718018e-007	37/112/6.427095e-007	38/115/6.717395e-007

Table 2. Test results for BFGSA.

x0	(5,5,…,5)	(20,20,…,20)	(–20,…, –20)	(–60, –60,…, –60)	(–100,…, –100)
Dim	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF
n=10	26/62/4.019133e-007	28/67/7.629739e-007	26/62/7.836022e-007	28/67/7.942352e-007	29/69/8.843658e-007
n=40	53/141/8.955174e-007	56/151/9.298740e-007	54/145/7.883733e-007	57/152/9.506096e-007	61/162/6.146640e-007
n=100	89/247/6.293858e-007	93/258/6.009680e-007	95/263/4.620386e-007	95/263/4.877714e-007	103/283/6.719347e-007
n=500	121/347/9.502010e-007	129/371/9.550139e-007	129/371/9.550162e-007	136/391/9.229412e-007	140/402/8.368401e-007
n=1000	122/350/9.130277e-007	131/376/8.492495e-007	131/376/8.492495e-007	137/393/9.697413e-007	141/404/9.845929e-007
x0	(5,0, 5,0,…)	(20, 0, 20, 0…)	(–20, 0, –20, 0…)	(–60, 0, –60, 0…)	(–100,0,–100,0…)
Dim	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF
n=10	29/70/5.384995e-007	30/72/8.024920e-007	30/72/8.022076e-007	31/74/7.737379e-007	32/76/6.247863e-007
n=40	72/198/5.245237e-007	74/203/5.317215e-007	74/203/5.325755e-007	75/205/6.538916e-007	75/204/9.700355e-007
n=100	110/313/8.802791e-007	118/336/9.964184e-007	118/336/9.966396e-007	125/357/9.676773e-007	128/366/8.655033e-007
n=500	116/332/9.424860e-007	126/360/9.585718e-007	126/360/9.586065e-007	133/380/9.648650e-007	136/389/9.324697e-007
n=1000	113/325/8.970304e-007	122/351/8.659330e-007	122/351/8.659334e-007	129/371/8.270087e-007	132/380/8.530508e-007
x0	(5, –5, 5, –5…)	(20, –20, 20, –20…)	(–20, 20, –20, 20…)	(–20, 20, –20, 20…)	(–100, 100…)
Dim	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF	NI/NG/GF
n=10	29/71/5.110057e-007	28/69/4.091687e-007	28/69/4.091687e-007	29/70/7.916413e-007	28/68/7.221453e-007
n=40	68/183/9.825927e-007	69/188/5.723010e-007	69/188/5.722966e-007	69/185/9.294491e-007	69/189/8.093485e-007
n=100	87/239/7.675976e-007	92/254/9.416435e-007	92/254/9.413503e-007	92/255/9.920299e-007	98/269/9.349510e-007
n=500	98/281/9.381734e-007	106/304/9.843192e-007	106/304/9.843192e-007	113/324/9.911432e-007	116/333/9.971433e-007
n=1000	98/281/9.925145e-007	107/307/9.345099e-007	107/307/9.345099e-007	113/325/9.830913e-007	117/336/8.588496e-007

4. Numerical Results

In this section, we report results of some preliminary numerical experiments with ROUA. Problem. The discretized two-point boundary value problem is similar to the problem in [36]

$$F(x) = Ax + \frac{1}{(n+1)^2} T(x)$$

where A is the n×n tridiagonal matrix given by

$$A = \begin{bmatrix} 8 & -1 & & & & \\ -1 & 8 & -1 & & & \\ & -1 & 8 & -1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & \ddots & \ddots & -1 \\ & & & & -1 & 8 \end{bmatrix}$$

and

$$T(x) = (T_1(x), T_2(x), \cdots, T_n(x))$$

with $T_i(x) = \sin x_i - 1, i = 1,2,\cdots,n.$ In the experiments, the parameters in ROUA were chosen as $r = 0.1$, $\delta_0 = \delta_1 = \delta_2 = 10^{-4}$. The program was coded in MATLAB Subsection 6.5.1. We stopped the iteration when the condition $\|F(x)\| \le 10^{-6}$ was satisfied.

The columns of the tables have the following meaning:

Dim: the dimension of the problem.
NI: the total number of iterations.
NG: the number of the function evaluations.
GF: the function norm evaluations.

In the next table, the numerical results are to test ROUA.

In the Table 2, the numerical results are to test BFGSA.

From these two tables, we can see that the numerical results of the two methods are all interesting. The numerical results of the proposed method perform better, and more stationary than the method BFGSA. Moreover, for the method ROUA, the initial points and the dimension do not influence the number of iterations very much. However, for the BFGSA, the number of the iteration will increase quickly with the dimension becoming larger. One thing we like to point out is that δ_0 should be chosen in such a way that it is not too large. Overall, from the numerical results, we can see that the ROUA is one of the robust methods for symmetric nonlinear equations.

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