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Bulk Viscous Bianchi Type V Space-Time with Generalized Chaplygin Gas and with Dynamical G and Λ

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Abstract

In this paper, bulk viscous Bianchi type V cosmological model with generalized Chaplygin gas, dynamical gravitational and cosmological constants has been investigated. We are assuming the condition on metric potential $\frac{\dot{R}_1}{R_1} = \frac{\dot{R}_2}{R_2} = \frac{m}{t^n}$. To obtain deterministic model, we have considered physically plausible relations like $P = p + \Pi$, $\eta = \eta_0 \rho^r$ and the generalized Chaplygin gas is described by equation of state $p = \frac{-B}{\rho^\alpha}$. A new set of exact solutions of Einstein's field equations has been obtained in Eckart theory, truncated theory and full causal theory. Physical behavior of the models has been discussed.

Keywords

1. Introduction

Recent cosmology is on Fridman-Lemaitra-Robertson-Walkar (FLRW) which is completely homogeneous and

isotropic. But it is widely believed that FLRW model does not give a correct matter description in the early stage of universe. The theoretical argument [1] and the recent experimental data support the existence of an aniso-tropic phase, which turns into an isotropic one during the evolution of the universe. Anisotropic model plays significant role in description of evolution of the early phase of the universe and also helps in finding more general cosmological models than the isotropic FRW models. This motivates researcher for obtaining exact aniso-tropic solution for Einstein's field equations as a cosmologically accepted physical models for the universe (in the early stages). The study of Bianchi type V cosmological model being anisotropic generalization of open FRW models is important to study old universe. A number of authors have investigated Bianchi type V cosmological model in general relativity in different context [2]-[15]. Rajbali and Seema Tinkar have discussed Bianchi type V bulk viscous Barotropic fluid cosmological model with variable G and Λ. Recently Yadav and Sharma [16] and Yadav [17] have discussed about transit universe in Bianchi type V space-time with variable G and Λ.

It has been widely discussed in the literature that during the evolution of the universe, bulk viscosity can arise in many circumstances and can lead to an effective mechanism of galaxy formation [18]. It is known that real fluids behave irreversibly and therefore it is important to consider dissipative processes both in cosmology and in astrophysics. To consider more realistic models, one must take in to account the viscosity mechanism. Bulk viscosity leading to an accelerated phase of the universe today has been studied by Fabris *et al.* [19]. Very recently Kotambkar *et al.* [20] have investigated anisotropic cosmological models with quintessence considering the effect of bulk viscosity.

A wide range of observations strongly suggest that the universe possesses non zero cosmological term [21]. The astronomical observations [22] [23] support that the expansion of the universe is accelerated. It suggests that there exists a new component in universe named as dark energy with negative pressure. A natural explanation for the accelerated expansion is due to a positive small cosmological constant. An attention has been paid to cosmological models with non zero cosmological term Λ [21] [24], whose existence is favored by supernovae SNe Ia observations (refer to [22] [23]) which are consistent with the recent anisotropy measurements of the cosmic microwave background (CMB) made by the WAMAP experiment [25]. Sahni and Starobinski [26] have presented detailed discussion on current observational situation focusing on cosmological tests on Λ.

Time varying G has many interesting consequences in astrophysics. Cunuto and Narlikar [27] have shown that G-varying cosmology is consistent with what so ever cosmological observations available at present. A new approach is appealing; it assumes the conservation of the energy momentum tensor which consequently gives G and Λ as coupled fields similar to the case of G in original Brans-Dicke theory. The cosmological model with variable G and Λ has been investigated by several researchers [28]-[32]. A number of researchers have discussed various anisotropic cosmological models with variable G and Λ [33]-[37].

According to recent observational evidence, the expansion of the universe is accelerated, which is dominated by a smooth component with negative pressure, the so called dark energy. To avoid problems associated with Λ and quintessence models, recently, it has been shown that Chaplygin gas may be useful. The unification of the dark matter and dark energy component creates a considerable theoretical interest, because on the one hand, model building becomes reasonably simpler, and on the other hand such unification implies existence of an era during which the energy densities of dark matter and dark energy are strikingly similar. For representation of such a unification, the generalized Chaplygin gas (GCG) with exotic condition of state $p = \dfrac{-B}{\rho^{\alpha}}$ is considered, where constant B and α satisfy $B > 0$ and $0 < \alpha \leq 1$ respectively. Due to observational evidence, cosmological models based on CG-EOS are very encouraging. Chaplygin gas and generalized Chaplygin gas cosmological models are first time proposed by Kamenshchik *et al.* [38]. WMAP constraints on the generalized Chaplygin gas model have been investigated by Bento *et al.* [39].

Motivated by above work we thought that it was worthwhile to study bulk viscous Bianchi type V space-time with generalized Chaplygin gas and dynamical G and Λ.

2. Metric and Field Equations

The spatially homogeneous and anisotropic space-time metric is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + R_1^2 \mathrm{d}x^2 + R_2^2 \mathrm{e}^{2kx} \mathrm{d}y^2 + R_3^2 \mathrm{e}^{2kx} \mathrm{d}z^2 \tag{1}$$

where R_1, R_2, R_3 are functions of t alone.

Einstein field equation with time dependent Λ and G may be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G T_{ij} + \Lambda g_{ij} \tag{2}$$

where G and Λ are time dependent gravitational and cosmological constants. T_{ij} is energy momentum tensor of cosmic fluid in the presence of bulk viscosity defined as

$$T_{ij} = (\rho + P)u_i u_j - (P)g_{ij} \tag{3}$$

$$P = p + \Pi \tag{4}$$

where p is equilibrium pressure, Π is bulk viscous stress together with $u_i u^j = 1$.

Einstein's field Equation (2) for the metric (1) takes form

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1}{R_1}\frac{\dot{R}_3}{R_3} - \frac{1}{R_1^2} = -8\pi G (p + \Pi) + \Lambda, \tag{5}$$

$$\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3} - \frac{1}{R_1^2} = -8\pi G (p + \Pi) + \Lambda \tag{6}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2} - \frac{1}{R_1^2} = -8\pi G (p + \Pi) + \Lambda \tag{7}$$

$$\frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2} + \frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3} + \frac{\dot{R}_3}{R_3}\frac{\dot{R}_1}{R_1} - \frac{3}{R_1^2} = 8\pi G \rho + \Lambda, \tag{8}$$

$$2\frac{\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} = 0. \tag{9}$$

By the divergence of Einstein's tensor *i.e.* $\left(R_{ij} - \frac{1}{2}Rg_{ij}\right)_{;j} = 0$ which lead to

$(8\pi G T_{ij} - \Lambda g_{ij})_{;j} = 0$, then yields

$$8\pi \dot{G}\rho + \dot{\Lambda} + 8\pi G \left[\dot{\rho} + (\rho + p + \Pi)\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right)\right] = 0 \tag{10}$$

The energy momentum conservation equation $(T^{ij}_{;j} = 0)$ splits Equation (10) into two equations.

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right) = 0, \tag{11}$$

$$8\pi \dot{G}\rho + \dot{\Lambda} = -8\pi G \Pi \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right). \tag{12}$$

For the full causal non-equilibrium thermodynamics the causal evolution equation for bulk viscosity is given by [40]

$$\Pi + \tau\dot{\Pi} = -\eta\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right) - \frac{\varepsilon\tau\Pi}{2}\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} + \frac{\dot{\tau}}{\tau} - \frac{\dot{\eta}}{\eta} - \frac{\dot{T}}{T}\right). \tag{13}$$

$T \geq 0$ absolute temperature, η is bulk viscosity coefficient which cannot become negative, τ denote the relaxation time for transient bulk viscous effects. Causality requires $\tau > 0$. When $\varepsilon = 0$, Equation (13) reduces to evolution equation for truncated theory. For $\varepsilon = 1$ Equation (13) reduces to evolution equation for full caus-

al theory and for $\tau = 0$ Equation (13) reduces to evolution equation for non-causal theory (Eckart's theory).

3. Cosmological Solutions

It can be easily seen that we have five Equations (5)-(9) with eight unknowns $R_1, R_2, R_3, \rho, p, G, \Lambda$ and η. Hence to solve the system of equations completely we need three additional physically plausible relations among these variables.

3.1. Case I: Non-Causal Cosmological Solution

For non causal solution $\tau = 0$, therefore the evolution Equation (13) takes the form of

$$\Pi = -\eta \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right) = -3\eta H \tag{14}$$

To find the complete solution of the system of equations, following relations are taken into consideration. The power law relation for bulk viscosity is taken as

$$\eta = \eta_0 \rho^r, \tag{15}$$

where $\eta_0 \geq 0$ and *r* is a constant.

We consider an exotic background fluid, the Chaplygin gas, described by the equation of state

$$p = \frac{-B}{\rho^\alpha}, \tag{16}$$

where *B* is constant and $0 < \alpha \leq 1$

To obtain the deterministic scenario of the universe, we assume the condition

$$\frac{\dot{R}_1}{R_1} = \frac{\dot{R}_2}{R_2} = \frac{m}{t^n} \tag{17}$$

From Equation (9) and (17), one can get

$$\frac{\dot{R}_3}{R_3} = \frac{m}{t^n}, \tag{18}$$

From Equations (17)-(18), one can easily calculate

$$R_1 = K_1 e^{\frac{mt^{1-n}}{1-n}}, \quad R_2 = K_2 e^{\frac{mt^{1-n}}{1-n}}, \quad R_3 = K_3 e^{\frac{mt^{1-n}}{1-n}}. \tag{19}$$

Using Equations (17) and (18), Equation (11) yields

$$\dot{\rho} + \left(\rho - \frac{B}{\rho^\alpha} \right) \left(\frac{3m}{t^n} \right) = 0. \tag{20}$$

By solving Equation (20), we get

$$\rho = \left[B + C e^{-Dt^{1-n}} \right]^{\frac{1}{1+\alpha}}. \tag{21}$$

where $D = \frac{3m(1+\alpha)}{1-n}$, and where *C* is constant of integration.

From **Figure 1** one can easily see that energy density is decreasing with evolution of the universe.

On differentiating Equation (21), we get

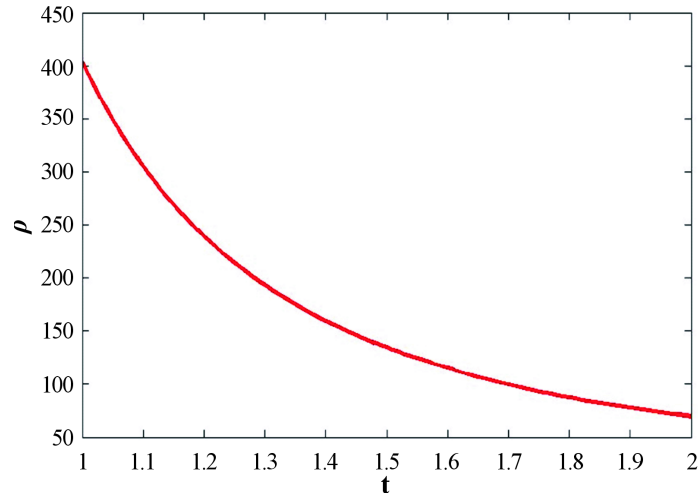$$\dot{\rho} = -\frac{3mC}{t^n} e^{-Dt^{1-n}} \left[B + C e^{-Dt^{1-n}} \right]^{\frac{1}{1+\alpha}-1}. \tag{22}$$

Figure 1. This figure shows variation of energy density ρ with respect to cosmic time t. Here we consider $B = 1$, $C = 1$, $n = 1.5$, $m = 2$ and $\alpha = 1$.

Now with the help of Equations (17)-(19) and (21), Equation (8) becomes

$$8\pi G \rho + \Lambda = \frac{3m^2}{t^{2n}} - \frac{3}{K_1^2} \exp\left\{\frac{-2mt^{1-n}}{1-n}\right\}. \tag{23}$$

Which on differentiation yields

$$8\pi \dot{G} \rho + 8\pi G \dot{\rho} + \dot{\Lambda} = \frac{-6m^2 n}{t^{2n+1}} + \frac{6m}{K_1^2 t^n} \exp\left\{\frac{-2mt^{1-n}}{1-n}\right\}, \tag{24}$$

With the help of Equations (12), (14), (17)-(18) and (21), Equation (24) becomes

$$8\pi G \left[\dot{\rho} + \frac{9m^2 \eta}{t^{2n}}\right] = \frac{6m}{t^n} \left[\frac{1}{K_1^2} \exp\left\{-\frac{2mt^{1-n}}{1-n}\right\} - \frac{mn}{t^{n+1}}\right], \tag{25}$$

By use of Equations (15), (21) and (22) in Equation (25), we get

$$G = \frac{1}{4\pi} \left[\frac{1}{K_1^2} \exp\left\{\frac{-2mt^{1-n}}{1-n}\right\} - \frac{mn}{t^{n+1}}\right] \left[-Ce^{-Dt^{1-n}} \left(B + Ce^{-Dt^{1-n}}\right)^{\frac{1}{1+\alpha}-1} + \frac{3m\eta_0}{t^n}\left(B + Ce^{-Dt^{1-n}}\right)^{\frac{r}{\alpha+1}}\right]^{-1}, \tag{26}$$

From **Figure 2** it can be seen that G is increasing with evolution of the universe.
Now using Equations (21) and (26) in Equation (23) gives

$$\Lambda = \frac{3m^2}{t^{2n}} - \frac{3}{K_1^2} \exp\left\{\frac{-2mt^{1-n}}{1-n}\right\} - 2\left[\frac{1}{K_1^2} \exp\left\{\frac{-2mt^{1-n}}{1-n}\right\} - \frac{mn}{t^{n+1}}\right]$$

$$\cdot \left[-Ce^{-Dt^{1-n}} \left(B + Ce^{-Dt^{1-n}}\right)^{-1} + \frac{3m\eta_0}{t^n}\left(B + Ce^{-Dt^{1-n}}\right)^{\frac{r-1}{\alpha+1}}\right]^{-1} \tag{27}$$

Figure 3 shows that cosmological constant is decreasing with the evolution of the universe.
On solving Equations (21) and (15) we can obtain the expression for bulk viscosity coefficient as

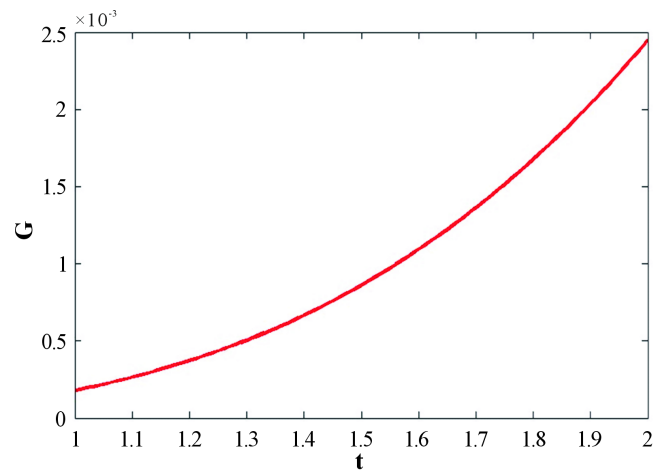$$\eta = \eta_0 \left[B + Ce^{-Dt^{1-n}}\right]^{\frac{r}{\alpha+1}}. \tag{28}$$

Figure 2. This figure shows variation of gravitational constant with respect to cosmic time *t*. Here we consider $B = 1$, $C = 1$, $n = 1.5$, $m = 2$, $\alpha = 1$, $r = 1.5$, $a = 1$ and $\eta_0 = 1$.
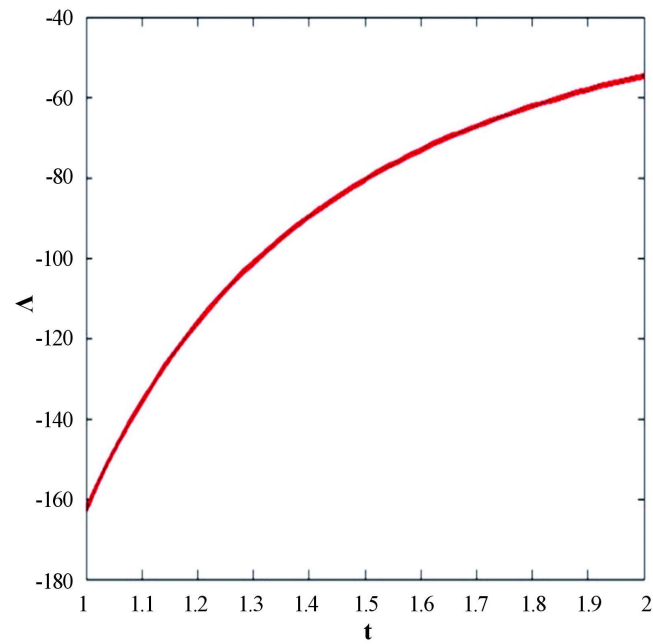

Figure 3. This figure shows variation of cosmological constant with respect to cosmic time *t*. Here we consider $B = 1$, $C = 1$, $n = 1.5$, $m = 2$, $\alpha = 1$, $r = 1.5$, $a = 1$ and $\eta_0 = 1$.

Figure 4 shows that bulk viscosity coefficient is decreasing with evolution of the universe. Thus the metric (1) reduces into the form

$$ds^2 = -dt^2 + K^2 \exp\left\{\frac{2mt^{1-n}}{1-n}\right\}\left(dx^2 + e^{2k}dy^2 + e^{2k}dz^2\right). \tag{29}$$

The deceleration parameter is given by

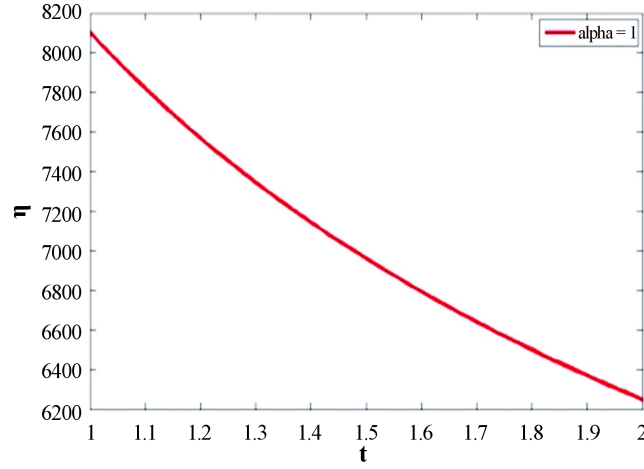$q = -1 - \dfrac{\dot{H}}{H^2}$, for this model deceleration parameter is

Figure 4. This figure shows variation of bulk viscosity coefficient with respect to cosmic time t. Here we consider $B = 1$, $C = 1$, $n = 1.5$, $m = 2$, $\alpha = 1$, $r = 1.5$ and $\eta_0 = 1$.

$$q = -1 + \frac{n}{mt^{1-n}} \tag{30}$$

Expansion scalar, Shear coefficient, relative anisotropy for this model is given by

$$\Theta = \frac{3\dot{H}}{H} = \frac{3m}{t^n} \tag{31}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{R}_1}{R_1}\right)^2 + \left(\frac{\dot{R}_2}{R_2}\right)^2 + \left(\frac{\dot{R}_3}{R_3}\right)^2\right] - \frac{\Theta^2}{6}$$

$$\sigma^2 = 0 \tag{32}$$

$$\text{Relative anisotropy} = \frac{\sigma^2}{\rho} = 0 \tag{33}$$

The critical energy density and the critical vacuum energy density are respectively given by

$$\rho_c = \frac{3H^2}{8\pi G}, \quad \rho_v = \frac{\Lambda}{8\pi G}$$

for the anisotropic Bianchi type V model can be expressed respectively as

$$\rho_c = \frac{3m^2 t^{-2n}}{2\left[\dfrac{1}{K_1^2}\exp\left\{\dfrac{-2mt^{1-n}}{1-n}\right\} - \dfrac{mn}{t^{n+1}}\right]\left[-Ce^{-Dt^{1-n}}\left(B + Ce^{-Dt^{1-n}}\right)^{\frac{1}{1+\alpha}-1} + \dfrac{3m\eta_0}{t^n}\left(B + Ce^{-Dt^{1-n}}\right)^{\frac{r}{\alpha+1}}\right]^{-1}}, \tag{34}$$

$$\rho_v =$$

$$\frac{\dfrac{3m^2}{t^{2n}} - \dfrac{3}{K_1^2}\exp\left\{\dfrac{-2mt^{1-n}}{1-n}\right\} - 2\left[\dfrac{1}{K_1^2}\exp\left\{\dfrac{-2mt^{1-n}}{1-n}\right\} - \dfrac{mn}{t^{n+1}}\right]\left[-Ce^{-Dt^{1-n}}\left(B + Ce^{-Dt^{1-n}}\right)^{-1} + \dfrac{3m\eta_0}{t^n}\left(B + Ce^{-Dt^{1-n}}\right)^{\frac{r-1}{\alpha+1}}\right]^{-1}}{2\left[\dfrac{1}{K_1^2}\exp\left\{\dfrac{-2mt^{1-n}}{1-n}\right\} - \dfrac{mn}{t^{n+1}}\right]\left[-Ce^{-Dt^{1-n}}\left(B + Ce^{-Dt^{1-n}}\right)^{\frac{1}{1+\alpha}-1} + \dfrac{3m\eta_0}{t^n}\left(B + Ce^{-Dt^{1-n}}\right)^{\frac{r}{\alpha+1}}\right]^{-1}}, \tag{35}$$

Mass density parameter and the density parameter of the vacuum are given by

$$\Omega_M = \frac{\rho}{\rho_c}, \ \ \Omega_\Lambda = \frac{\rho_v}{\rho_c}$$

for the anisotropic Bianchi type V model can be expressed respectively as

$$\Omega_M = \frac{2t^{2n}\left[B+Ce^{-Dt^{1-n}}\right]^{\frac{1}{1+\alpha}}\left[\frac{1}{K_1^2}\exp\left\{\frac{-2mt^{1-n}}{1-n}\right\}-\frac{mn}{t^{n+1}}\right]}{3m^2\left[-Ce^{-Dt^{1-n}}\left(B+Ce^{-Dt^{1-n}}\right)^{\frac{1}{1+\alpha}-1}+\frac{3m\eta_0}{t^n}\left(B+Ce^{-Dt^{1-n}}\right)^{\frac{r}{\alpha+1}}\right]} \tag{36}$$

$$\Omega_\Lambda = \frac{t^{2n}}{3m^2}\left(\frac{3m^2}{t^{2n}}-\frac{3}{K_1^2}\exp\left\{\frac{-2mt^{1-n}}{1-n}\right\}-2\left[\frac{1}{K_1^2}\exp\left\{\frac{-2mt^{1-n}}{1-n}\right\}-\frac{mn}{t^{n+1}}\right]\right.$$

$$\left.\cdot\left[-Ce^{-Dt^{1-n}}\left(B+Ce^{-Dt^{1-n}}\right)^{-1}+\frac{3m\eta_0}{t^n}\left(B+Ce^{-Dt^{1-n}}\right)^{\frac{r-1}{\alpha+1}}\right]^{-1}\right) \tag{37}$$

The State finder parameters $r = \dfrac{\ddot{R}}{RH^3}$ and $s = \dfrac{r-1}{3\left(q-\dfrac{1}{2}\right)}$.

For this model

$$r = 1 - \frac{3n}{mt^{1-n}} + \frac{n(n+1)}{m^2t^{2n-2}} \tag{38}$$

$$s = \frac{2n(n+1)t^{n-1}-6mn}{-9m^2t^{1-n}-6mn} \tag{39}$$

3.2. Case II: Causal Cosmological Solution

In addition to physically plausible relations (15)-(17), in this case we assume

$$\Lambda = \beta H^2. \tag{40}$$

where H is Hubble parameter, given by

$$H = \frac{\dot{R}}{R} \ \text{ and } \ R = \left(R_1R_2R_3\right)^{1/3}. \tag{41}$$

From Equation (17)-(19) and (41), the Hubble parameter is given by

$$H = \frac{m}{t^n} \tag{42}$$

Using equations (17)-(19), (40) and (42) in equation (8), we get

$$8\pi G\rho = \frac{(3-\beta)m^2}{t^{2n}} - \frac{3}{K_1^2}\exp\left\{\frac{-2mt^{1-n}}{1-n}\right\}, \tag{43}$$

From Equations (21) and (43),

$$G = \frac{1}{8\pi}\left[B+C\exp\left(-Dt^{1-n}\right)\right]^{\frac{-1}{\alpha+1}}\left[\frac{(3-\beta)}{t^{2n}}-\frac{3}{K_1^2}\exp\left(D_1t^{1-n}\right)\right] \tag{44}$$

where $D_1 = \dfrac{-2m}{1-n}$.

From **Figure 5** one can easily see that gravitational constant is increasing with cosmic time. Substitute the values from Equations (17)-(19), (40) and (44) in Equation (5), we get

$$\Pi = \left[\frac{(3-\beta)m^2}{t^{2n}} - \frac{2mn}{t^{n+1}} - \frac{1}{K_1^2} \exp\left(D_1 t^{1-n}\right) \right] \cdot \frac{-1}{8\pi G} + \frac{B}{\rho^\alpha}, \tag{45}$$

By use of Equation (21), Equation (44) gives

$$\Pi = \frac{B}{\left[B + C \exp\left(-Dt^{1-n}\right) \right]^{\frac{\alpha}{\alpha+1}}} - \frac{U_1(t)}{U_2(t)} \left[B + C \exp\left(-Dt^{1-n}\right) \right]^{\frac{1}{\alpha+1}} \tag{46}$$

where $U_1(t) = \left[\frac{(3-\beta)m^2}{t^{2n}} - \frac{2mn}{t^{n+1}} - \frac{1}{K_1^2} \exp\left(D_1 t^{1-n}\right) \right]$, $U_2(t) = \left[\frac{(3-\beta)}{t^{2n}} - \frac{3}{K_1^2} \exp\left(D_1 t^{1-n}\right) \right]$

Figure 6 shows that bulk viscous stress is decreasing with the evolution of the universe.

3.2.1. Sub Cease (i): Evaluation of Bulk Viscosity in Truncated Causal Theory

Now we study variation of bulk viscosity coefficient η and relaxation time τ with respect to the cosmic time. It has already been mentioned that for truncated theory $\varepsilon = 0$ and hence Equation (13) reduces to

$$\Pi + \tau \dot{\Pi} = -3\eta H. \tag{47}$$

In order to have exact solution of the system of equations one more physically plausible relation is required. Thus, we consider the well known relation

$$\tau = \frac{\eta}{\rho}. \tag{48}$$

Using Equations (17)-(19), (46) and (48) in Equation (47) one can obtain coefficient of bulk viscosity as

$$\eta = \frac{-B\left[B + C \exp\left(-Dt^{1-n}\right) \right]^{\frac{-\alpha}{\alpha+1}} + U_1(t)\left[U_2(t)\right]^{-1}\left[B + C \exp\left(-Dt^{1-n}\right) \right]^{\frac{1}{\alpha+1}}}{-\dfrac{U_1'(t)}{U_2(t)} + \dfrac{U_1(t)U_2'(t)}{\left[U_2(t)\right]^2} - \dfrac{U_1(t)}{U_2(t)}\dfrac{3mC}{t^n} \exp\left(-Dt^{1-n}\right)\left[B + C \exp\left(-Dt^{1-n}\right) \right]^{-1}}$$
$$- \frac{3\alpha BCm}{t^n} \exp\left(-Dt^{1-n}\right)\left[B + C \exp\left(-Dt^{1-n}\right) \right]^{-2} + \frac{3m}{t^n} \tag{49}$$
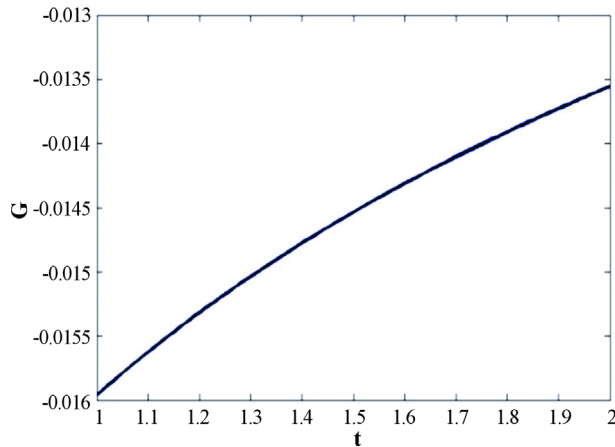

Figure 5. This figure shows variation of gravitational constant with respect to cosmic time t. Here we consider $B = 1$, $C = 1$, $n = 1.5$, $m = 2$, $\alpha = 1, \beta = 1$.
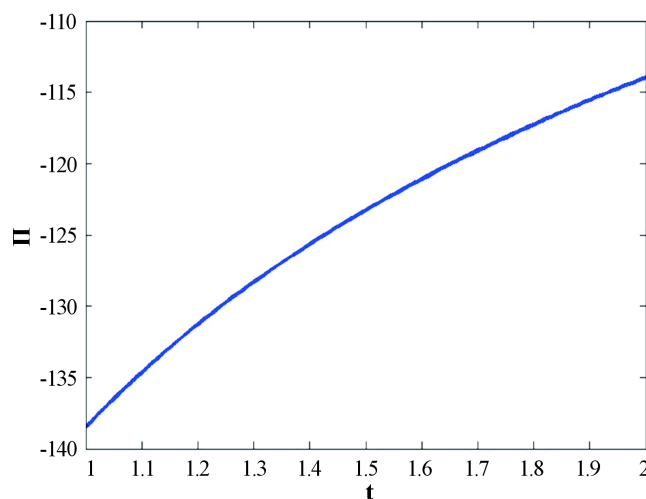
Figure 6. This figure shows variation of bulk viscous stress with respect to cosmic time *t*. Here we consider $B = 1$, $C = 1$, $n = 1.5$, $m = 2$, $\alpha = 1$, $\beta = 1$, and $K_1 = 1$.
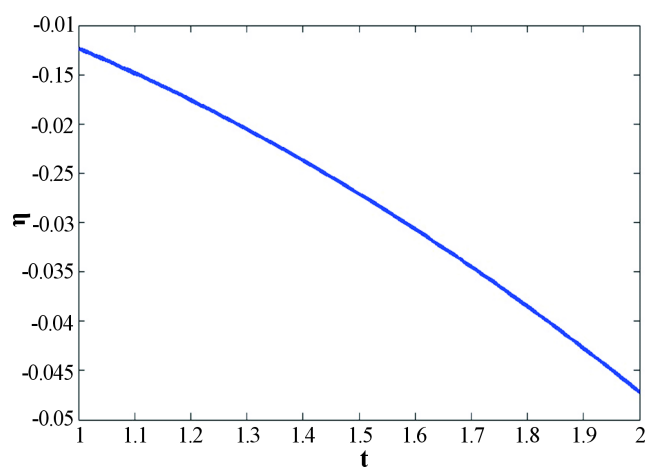

Figure 7. This figure shows variation of bulk viscosity coefficient with respect to cosmic time *t*. Here we consider $B = 1$, $C = 1$, $n = 1.5$, $m = 2$, $\alpha = 1$.

From **Figure 7** one can see that bulk viscosity coefficient is decreasing with time.

3.2.2. Sub Caese (ii). Evaluation of Bulk Viscosity in Full Causal Theory

It has already been mentioned that for full causal theory $\varepsilon = 1$ and hence Equation (13) reduces to

$$\Pi + \tau\dot{\Pi} = -3\eta H - \frac{\tau\Pi}{2}\left(3H - \frac{\dot{\tau}}{\tau} - \frac{\dot{\eta}}{\eta} - \frac{\dot{T}}{T}\right). \tag{50}$$

On the basis of Gibb's inerrability condition, Maartens [40] has suggested the equation of state for temperature as

$$T \propto \exp\int\frac{\mathrm{d}p}{\rho + p}, \tag{51}$$

which with the help of Equation (21) gives

$$T = T_0 \left[1 - B\rho^{-(\alpha+1)} \right]^{\frac{\alpha}{\alpha+1}}. \tag{52}$$

Figure 8 shows that temperature is decreasing with evolution of the universe. using Equations (21), (42), (48) and (52) in Equation (50) one can obtain

$$\Pi + \frac{\eta}{\rho}\dot{\Pi} = -\eta \frac{2m_1 + m_2}{t^n} - \frac{\eta\Pi}{2\rho}\left[\frac{2m_1 + m_2}{t^n} - \frac{\dot{\rho}}{\rho} - \frac{\dot{T}}{T} \right],$$

which on simplification yields the expression for bulk viscosity as

$$\eta = \frac{-B\left[B + C\exp\left(-Dt^{1-n}\right) \right]^{\frac{-\alpha}{\alpha+1}} + U_1(t)\left[U_2(t) \right]^{-1}\left[B + C\exp\left(-Dt^{1-n}\right) \right]^{\frac{1}{\alpha+1}}}{\frac{3m}{t^n} + \frac{\dot{\Pi}}{\rho} + \left[\frac{B}{\left(B + C\exp\left(-Dt^{1-n}\right) \right)} - \frac{U_1(t)}{U_2(t)} \right]\left[\frac{3m(1+\alpha)}{t^n} + \frac{3mC(1+\alpha)\exp\left(-Dt^{1-n}\right)}{t^n\left(B + C\exp\left(-Dt^{1-n}\right) \right)} \right]} \tag{53}$$

where

$$\frac{\dot{\Pi}}{\rho} = -\frac{U_1'(t)}{U_2(t)} + \frac{U_1(t)U_2'(t)}{\left[U_2(t) \right]^2} - \frac{U_1(t)}{U_2(t)}\frac{3mC}{t^n}\exp\left(-Dt^{1-n}\right)\left[B + C\exp\left(-Dt^{1-n}\right) \right]^{-1}$$

$$-\frac{3\alpha BCm}{t^n}\exp\left(-Dt^{1-n}\right)\left[B + C\exp\left(-Dt^{1-n}\right) \right]^{-2}$$

Figure 9 shows that bulk viscosity coefficient decreasing with evolution of universe.

4. Conclusion

In this paper, we have studied bulk viscous Bianchi type V space-time geometry with generalized Chaplygin gas and varying gravitational and cosmological constants. We have obtained a new set of exact solutions of Einstein's equations by considering $\frac{\dot{R_1}}{R_1} = \frac{\dot{R_2}}{R_2} = \frac{m}{t^n}$. For $n > 1$, the deceleration parameter $q < 0$ for $t > (n/m)^{\frac{1}{1-n}}$.

When $n \rightarrow 1$ considering present day limit for deceleration parameter $q = -0.53^{+0.17}_{-0.13}$ [40] suggests $1.56 \le m \le 2.94$. It is observed that in case I energy density, bulk viscosity and cosmological constant decrease where as gravitational constant G(t) is increasing with time. In case II, bulk viscosity η, bulk viscous stress Π and temperature T decrease with evolution of the universe which agrees with cosmic observations. In order to
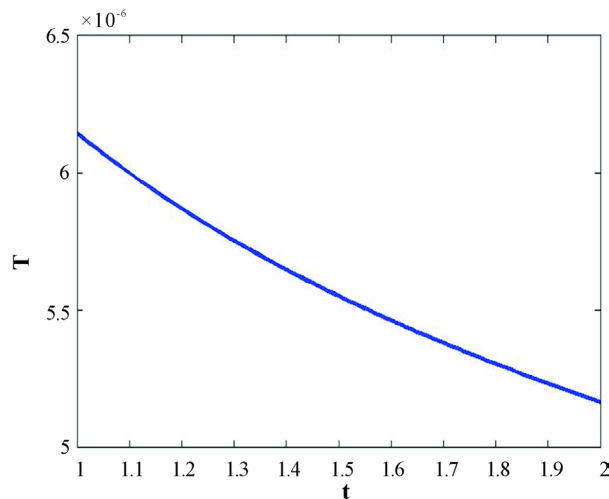

Figure 8. This figure shows variation of temparature with respect to cosmic time *t*. Here we consider $B = 1$, $C = 1$, $n = 1.5$, $m = 2$ and $\alpha = 1$.
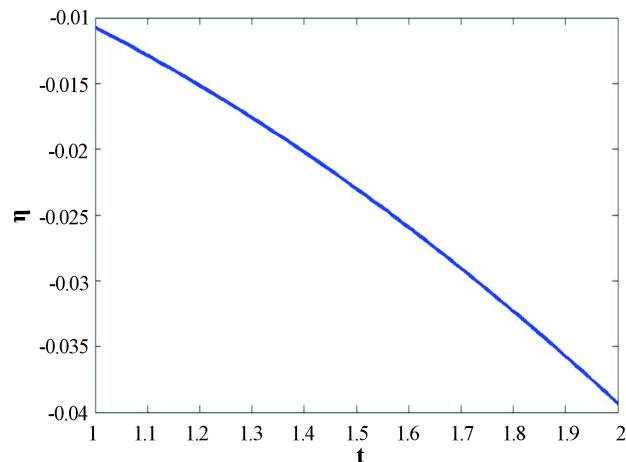
Figure 9. This figure shows variation of bulk viscosity coefficient with respect to cosmic time t. Here we consider B = 1, C = 1, n = 1.5, m = 2, $\alpha = 1$.

have clear idea of variation in behavior of cosmological parameters, relevant graphs have been plotted; all graphs are in fair agreement with cosmological observations.

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