

# Based on Adaptive Backstepping Error Control for Permanent Magnet Synchronous Motor

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Received 2 February 2016; accepted 30 April 2016; published 3 May 2016

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## Abstract

Permanent Magnet Synchronous Motor (PMSM) displays chaotic phenomenon when PMSM in power on or power off. At present, there are many methods to control chaos in PMSM. However, there appears oscillation in course of control chaos in PMSM, which has an effect on practical application. This paper proposes error control based on adaptive backstepping to control chaos in PMSM; an error control item is added in each step virtual control design which has control effect of unknown dynamical error on system. This scheme can eliminate oscillation in course of control chaos. Finally, the simulation results show the effectiveness of theoretical analysis.

## Keywords

PMSM, Error Control, Adaptive Backstepping, Chaos Control

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## 1. Introduction

Research on PMSM has been going on for many years due to the fact that they have many advantages over the conventional internal combustion engine vehicle, such as independence from petroleum, reliability and quiet [1]-[3].

However there appear phenomena of chaos in PMSM when PMSM in turn on or turn off [4] [5]. Chaos of PMSM is harmful. Chaos can degrade performance of PMSM, even destroy PMSM and restrict the operating range of numerous electrical and mechanical devices. The high performance of PMSM depends on the absence of chaos so it is important for PMSM to control chaos [6] [7]. Due to the fact that PMSM is multivariable, non-linear and strongly coupled plant, controlling chaos of PMSM is very difficult [8].

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With the development of theory of chaos, there are many methods for control and analysis chaotic system [9] [10]. For example, the OGY is a basic methodology for controlling chaos. At the same time, there are variable structure control [11], entrainment and migration control, nonlinear feedback control [12], total sliding-mode control [13] and the backstepping nonlinear control, self-constructing fuzzy neural network speed control [14], dither chaos [15], hybrid control [16] and passivity control [17].

Various ways and techniques had been successfully used to control or suppress chaos in PMSM. For example, in 2009, M. Zribi *et al.* proposed to control chaos in PMSM by instantaneous Lyapunov exponent control algorithm [18]. In 2010, D. Li *et al.* proposed impulsive control for PMSM [19]. In 2010, S. C. Chang proposed synchronous and control chaos in a PMSM [20]. In 2011, J. Yu *et al.* proposed backstepping control for the chaotic permanent magnet synchronous motor drive system [21]. In 2011, S. C. Chang *et al.* proposed dither signal to quenching chaos of a permanent magnet synchronous motor in electric vehicles [22]. However, these methods appear oscillation in course of control chaos in PMSM which has an effect on practical application.

In this paper, a scheme is proposed to suppress oscillation in course of control chaos in PMSM. An error control item is added in the each step virtual control design which has control effect of unknown dynamical error on system. This scheme can gain more smoothly chaotic stabilization process and overcome oscillation in course of control chaos in PMSM. At the same time, all the signals in the system are bounded which based on Lyapunov function. This scheme has better transient response by simulation.

## 2. Problem Formulation

The dynamics PMSM, which model base on d-q axis, can be described as follows:

$$\begin{cases} \frac{di_d}{dt} = (u_d - R_1 + \omega L_q i_q) / L_d \\ \frac{di_q}{dt} = (u_q - R_1 i_q - \omega L_q i_d - \omega \psi_\gamma) / L_q \\ \frac{d\omega}{dt} = [n_p \psi_\gamma i_q + n_p (L_d - L_q) i_d i_q - T_L - \beta \omega] / J \end{cases} \quad (1)$$

where  $i_d$ ,  $i_q$  and  $\omega$  are state variables, which denote d-axis stator current, q-axis stator current and rotor angular speed respectively;  $u_d$ ,  $u_q$  and  $T_L$  are d-axis external voltage, q-axis external voltage and external torque;  $L_d$  and  $L_q$  are d-axis stator inductance and q-axis stator inductance.  $\psi_\gamma$  is permanent magnet fluxes,  $R_1$  is stator winding resistance,  $\beta$  is the viscous damping coefficient,  $J$  is rotor rotational inertia,  $n_p$  is the number of pole-pairs,  $R_1$ ,  $\beta$ ,  $J$ ,  $L_q$ ,  $L_d$ ,  $T_L$  are all positive. Applying transformation form,  $\mathbf{x} = \lambda \tilde{\mathbf{x}}$ , and a time scaling transformation,  $t = \tau \tilde{t}$ , where

$$\tilde{\mathbf{x}} = [\tilde{i}_d \quad \tilde{i}_q \quad \tilde{\omega}]^T, \quad \lambda = \begin{bmatrix} \lambda_d & 0 & 0 \\ 0 & \lambda_q & 0 \\ 0 & 0 & \lambda_\omega \end{bmatrix} = \begin{bmatrix} bk & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1/\tau \end{bmatrix}, \quad b = L_q/L_d, \quad k = \beta/n_p \tau \psi_\gamma, \quad \tau = L_q/L_{R_1}.$$

The system (1) can be changed into nondimensionalized form as follows:

$$\begin{cases} \dot{\tilde{i}}_d = -\frac{L_q}{L_d} \tilde{i}_d + \tilde{\omega} \tilde{i}_q + \tilde{u}_d \\ \dot{\tilde{i}}_q = -\tilde{i}_q - \tilde{\omega} \tilde{i}_d + \gamma \tilde{\omega} + \tilde{u}_q \\ \dot{\tilde{\omega}} = \sigma (\tilde{i}_q - \tilde{\omega}) + \xi \tilde{i}_d \tilde{i}_q - \tilde{T} \end{cases} \quad (2)$$

where

$$\begin{aligned} \gamma &= n_p \psi_\gamma / R_1 \beta, \quad \sigma = L_q \beta / R_1 J, \quad \tilde{u}_q = n_p L_q \psi_\gamma u_q / R_1^2 \beta, \quad T_L = L_q^2 T_L / R_1^2 J, \quad \tilde{u}_d = n_p L_q \psi_\gamma u_d / R_1^2 \beta, \\ \xi &= L_q \beta^2 (L_d - L_q) / L_d J n_p \psi_\gamma, \quad n_p = 1. \end{aligned}$$

System (2) is smooth air-gap when  $L_d = L_q$ . In order to describe conveniently, assuming  $i_d = \tilde{i}_d$ ,  $i_q = \tilde{i}_q$ ,

$\omega = \tilde{\omega}$ ,  $u_d = \tilde{u}_d$ ,  $u_q = \tilde{u}_q$ . The model can be simplified as follows:

$$\begin{cases} \dot{i}_d = -i_d + \omega i_d + u_d \\ \dot{i}_q = -i_q - \omega i_d + \gamma \omega + u_q \\ \dot{\omega} = \sigma(i_q - \omega) - T_L \end{cases} \quad (3)$$

Now, for model of PMSM of smooth air-gap (3), research motor without external force which can be considered PMSM no-load running and power fail interrupt, namely,  $u_d = u_q = T_L = 0$ . The system (3) can be shows as follows:

$$\begin{cases} \dot{i}_d = -i_d + \omega i_d \\ \dot{i}_q = -i_q - \omega i_d + \gamma \omega \\ \dot{\omega} = \sigma(i_q - \omega) \end{cases} \quad (4)$$

the parameters value of system (4),  $\sigma$  and  $\gamma$ , can effect on chaotic motion of PMSM greatly. Theoretically, there are many values of  $\sigma$  and  $\gamma$  which can cause chaos occurred in system (4). For system (4),

$$\Delta V = \frac{\partial \Delta \omega}{\partial \omega} \frac{\partial \Delta i_q}{\partial i_q} \frac{\partial \Delta i_d}{\partial i_d} = -(\sigma + 2).$$

Due to  $\sigma > 0$ ,  $\Delta V < 0$ , so the system (4) is dissipative system base on dissipation theory. System (4) is chaos when  $\gamma = 25$  and  $\sigma = 4$  base on above analysis [22]. The system (4) have three equilibrium point:  $(0, 0, 0)$ ,  $(\gamma - 1, -\sqrt{\gamma - 1}, -\sqrt{\gamma - 1})$ ,  $(\gamma - 1, \sqrt{\gamma - 1}, \sqrt{\gamma - 1})$ .

### 3. Theory and Method

Set

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad Y = AX.$$

So system (4) can be change as follows:

$$\begin{cases} \dot{y}_1 = \sigma(y_2 - y_1) \\ \dot{y}_2 = \gamma y_1 - y_2 - y_1 y_3 \\ \dot{y}_3 = y_1 y_2 - y_3 \end{cases} \quad (5)$$

To realize stability of system (4), we may add controller to the third equation of system (4), system (4) can be changed as follows:

$$\begin{cases} \dot{y}_1 = \sigma(y_2 - y_1) \\ \dot{y}_2 = \gamma y_1 - y_2 - y_1 y_3 \\ \dot{y}_3 = y_1 y_2 - y_3 + u \end{cases} \quad (6)$$

Definite three error variables:

$$\begin{cases} e_1 = y_1 \\ e_2 = y_2 - \alpha_1 \\ e_3 = y_3 - \alpha_2 \end{cases} \quad (7)$$

**Step 1:** Base on system (7), the first derivative of  $e_1$  is

$$\dot{e}_1 = \dot{y}_1 = \sigma(y_2 - y_1) = \sigma(\alpha_1 + e_2 - e_1) = -\sigma e_1 + \sigma e_2 + \sigma \alpha_1. \quad (8)$$

Choose the Lyapunov function candidate as:

$$V_1 = \frac{1}{2}e_1^2, \quad (9)$$

then the time derivative of  $V_1$  is computed,

$$\dot{V}_1 = e_1\dot{e}_1 = e_1(-\sigma e_1 + \sigma e_2 + \sigma\alpha_1) = -\sigma e_1^2 + \sigma e_1 e_2 + \sigma\alpha_1 e_1. \quad (10)$$

The virtual control  $\alpha_1$  is constructed as

$$\alpha_1 = p_1 e_1 - p_2 e_2, \quad (11)$$

where  $p_1$  and  $p_2$  are control parameters,  $\frac{1}{\sigma} \leq p_1 \leq 1$ ,  $0 \leq p_2 \leq 1$ , substituting Equation (11) into Equation (10) that

$$\dot{V}_1 = -\sigma e_1^2 + \sigma e_1 e_2 + \sigma e_1 (p_1 e_1 - p_2 e_2) = -\sigma(1-p_1)e_1^2 + \sigma(1-p_2)e_1 e_2. \quad (12)$$

**Step 2:** Derivative of  $e_2$ , we have equation,

$$\dot{e}_2 = \dot{y}_2 - \dot{\alpha}_1 = \gamma y_1 - y_2 - y_1 y_3 - p_1 \dot{e}_1 + p_2 \dot{e}_2 = \gamma e_1 - (e_2 + \alpha_1) - e_1(e_3 + \alpha_2) - p_1 \dot{e}_1 + p_2 \dot{e}_2, \quad (13)$$

substituting Equation (8) into Equation (13), the Equation (13) expression is given by

$$\begin{aligned} \dot{e}_2 &= \frac{1}{1-p_2} \left[ (-\alpha_2 - p_1)e_1 + (-1 + p_2 + \sigma p_1 p_2 - \sigma p_1)e_2 - e_1 e_3 + \sigma p_1 e_1 - \sigma p_1^2 e_1 + \gamma e_1 \right] \\ &= \frac{1}{1-p_2} \left[ (-\alpha_2 - p_1)e_1 - (1-p_2)(\sigma p_1 + 1)e_2 - e_1 e_3 + p_1(1-p_1)\hat{\gamma}e_1 + \hat{\gamma}e_1 - p_1(1-p_1)\tilde{\sigma}e_1 - \hat{\gamma}e_1 \right], \end{aligned} \quad (14)$$

where  $\hat{\sigma}$ ,  $\hat{\gamma}$  are  $\sigma$ ,  $\gamma$  estimated value,  $\tilde{\sigma} = \hat{\sigma} - \sigma$ ,  $\tilde{\gamma} = \hat{\gamma} - \gamma$ ,  $\tilde{\sigma}$  and  $\tilde{\gamma}$  are parameters estimation error.

Choose the Lyapunov function as follows,

$$V_2 = V_1 + (e_2^2 + \tilde{\sigma} + \tilde{\gamma}^2)/2,$$

the time derivative of  $V_2$  is given by

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + e_2 \dot{e}_2 + \tilde{\sigma} \dot{\tilde{\sigma}} + \tilde{\gamma} \dot{\tilde{\gamma}} \\ &= -\sigma(1-p_1)e_1^2 - \frac{e_1 e_2 e_3}{1-p_2} - \frac{p_1}{1-p_2} e_1 e_2 - (\sigma p_1 + 1)e_2^2 + \tilde{\sigma} \left[ \dot{\tilde{\sigma}} - (1-p_2)e_1 e_2 - \frac{p_1(1-p_1)}{1-p_2} e_1 e_2 \right] \\ &\quad + \tilde{\gamma} \left( \dot{\tilde{\gamma}} - \frac{e_1 e_2}{1-p_2} \right) - \frac{e_1 e_2}{1-p_2} \left\{ \alpha_2 - \hat{\sigma} \left[ (1-p_2)^2 + p_1(1-p_1) \right] - \hat{\gamma} \right\}. \end{aligned} \quad (15)$$

Choose parameters adaptive rule:

$$\begin{cases} \dot{\hat{\sigma}} = \frac{p_1(1-p_1)}{1-p_2} e_1 e_2 + (1-p_2)e_1 e_2 - m\tilde{\sigma} \\ \dot{\hat{\gamma}} = \frac{1}{1-p_2} e_1 e_2 - n\tilde{\gamma} \end{cases} \quad (16)$$

where  $m > 0, n > 0$ .

Construct the virtual control  $\alpha_2$  as

$$\alpha_2 = \left[ (1-p_2)^2 + p_1(1-p_1) \right] \hat{\sigma} + p_2 \hat{\gamma} - p_3 e_3, \quad (17)$$

where  $p_3 \in \mathbb{R}$ ,  $p_3$  is a control parameter, substituting Equation (16) and Equation (17) into Equation (15), equation Equation (15) can be obtained as follows,

$$\dot{V}_2 = -\sigma(1-p_1)e_1^2 - \frac{1-p_3}{1-p_2} e_1 e_2 e_3 - \frac{p_1}{1-p_2} e_1 e_2 + e_1 e_2 (\gamma + \tilde{\gamma}) - (\sigma p_1 + 1)e_2^2 - m\tilde{\sigma}\dot{\tilde{\sigma}} - n\tilde{\gamma}\dot{\tilde{\gamma}}. \quad (18)$$

Base on Young inequality [21], inequality (19) can be obtained as follows

$$\begin{cases} -m\tilde{\sigma}\hat{\sigma} \leq -m\tilde{\sigma}(\tilde{\sigma} + \sigma) \leq -\frac{1}{2}m\tilde{\sigma}^2 + \frac{1}{2}m\sigma^2 \\ -n\tilde{\gamma}\hat{\gamma} \leq -n\tilde{\gamma}(\tilde{\gamma} + \gamma) \leq -\frac{1}{2}n\tilde{\gamma}^2 + \frac{1}{2}n\gamma^2 \end{cases} \quad (19)$$

so a straightforward calculation produces the following inequality

$$\begin{aligned} \dot{V}_2 \leq & -\sigma(1-p_1)e_1^2 - \frac{1-p_3}{1-p_2}e_1e_2e_3 - \frac{p_1}{1-p_2}e_1e_2 + e_1e_2(\gamma + \tilde{\gamma}) \\ & -(\sigma p_1 + 1)e_2^2 - \frac{1}{2}m\tilde{\sigma}^2 - \frac{1}{2}n\tilde{\gamma}^2 + \frac{1}{2}m\sigma^2 + \frac{1}{2}n\gamma^2. \end{aligned} \quad (20)$$

**Step 3:** Derivative of  $e_3$  results in the following differential equation,

$$\begin{aligned} \dot{e}_3 = & u + e_1(e_2 + \alpha_1) - (e_3 + \alpha_2) - \dot{\alpha}_2 \\ = & u + e_1e_2 + e_1\alpha_1 - (e_3 + \alpha_2) - \left[ (1-p_2)^2 + p_1(1-p_1) \right] \dot{\hat{\sigma}} - \dot{\hat{\gamma}} + p_3\dot{e}_3, \end{aligned} \quad (21)$$

choose  $p_3 \neq 1$ , Equation (21) can be written as follows,

$$\dot{e}_3 = \frac{1}{1-p_3} \left\{ u + e_1e_2 + \alpha_1e_1 - (e_3 + \alpha_2) - \left[ (1-p_2)^2 + p_1(1-p_1) \right] \dot{\hat{\sigma}} - \dot{\hat{\gamma}} \right\}, \quad (22)$$

choose the Lyapunov function candidate as

$$V_3 = V_2 + \frac{1}{2} \frac{1}{1-p_2} e_3^2. \quad (23)$$

The time derivative of  $V_3$  is

$$\begin{aligned} \dot{V}_3 = & \dot{V}_2 + \frac{1}{1-p_2} e_3 \dot{e}_3 \\ = & -\sigma(1-p_1)e_1^2 - \frac{1-p_3}{1-p_2}e_1e_2e_3 - \frac{p_1}{1-p_2}e_1e_2 + e_1e_2(\gamma + \tilde{\gamma}) - (\sigma p_1 + 1)e_2^2 - m\tilde{\sigma}\hat{\sigma} - n\tilde{\gamma}\hat{\gamma} \\ & + \frac{1}{1-p_2} e_3 \frac{1}{1-p_3} \left\{ u + e_1(e_2 + \alpha_1) - (e_3 + \alpha_2) - \left[ (1-p_2)^2 + p_1(1-p_1) \right] \dot{\hat{\sigma}} - p_2\dot{\hat{\gamma}} \right\} \\ = & -\sigma(1-p_1)e_1^2 - (\sigma p_1 + 1)e_2^2 - m\tilde{\sigma}\hat{\sigma} - n\tilde{\gamma}\hat{\gamma} + \frac{e_3}{(1-p_2)(1-p_3)} \left\{ u - \frac{p_1(1-p_3)}{e_3} e_1e_2 \right. \\ & \left. + \frac{\gamma + \hat{\gamma}}{e_3} e_1e_2 + e_1e_2 + e_1\alpha_1 - (e_3 + \alpha_2) - (1-p_3)^2 e_1e_2 - \left[ (1-p_2)^2 + p_1(1-p_1) \right] \dot{\hat{\sigma}} - p_2\dot{\hat{\gamma}} \right\}, \end{aligned} \quad (24)$$

setting

$$\begin{aligned} u = & - \left[ -\frac{p_1(1-p_3)}{e_3} e_1e_2 + \frac{\gamma + \hat{\gamma}}{e_3} e_1e_2 + e_1e_2 + e_1\alpha_1 - (e_3 + \alpha_2) \right. \\ & \left. - (1-p_3)^2 e_1e_2 - \left[ (1-p_2)^2 + p_1(1-p_1) \right] \dot{\hat{\sigma}} - p_2\dot{\hat{\gamma}} \right], \end{aligned} \quad (25)$$

substituting Equation (25) into Equation (24), we have the following equation.

$$V_3 = -\sigma(1-p_1)e_1^2 - (\sigma p_1 + 1)e_2^2 - p_4e_3^2 - m\tilde{\sigma}\hat{\sigma} - n\tilde{\gamma}\hat{\gamma}. \quad (26)$$

Similar to  $\dot{V}_2$ ,

$$\dot{V}_3 \leq -\sigma(1-p_1)e_1^2 - (\sigma p_1 + 1)e_2^2 - p_4e_3^2 - \frac{1}{2}m\tilde{\sigma}^2 - \frac{1}{2}n\tilde{\gamma}^2 + \frac{1}{2}m\sigma^2 + \frac{1}{2}n\gamma^2, \quad (27)$$

set

$$\beta \triangleq (m\sigma^2 + n\gamma^2)/2,$$

$$\tau = \min\{2\sigma(1-p_1), 2(\sigma p_1 + 1), 2p_4(1-p_2), m, n\},$$

inequality can be obtained as follows,

$$\dot{V}_3 \leq \left[ -\tau \left( e_1^2 + e_2^2 + \frac{e_3^2}{1-p_3} + \tilde{\sigma}^2 + \tilde{\gamma}^2 \right) + m\sigma^2 + n\gamma^2 \right] / 2 \leq -\tau V_3 + \beta. \quad (28)$$

#### 4. Stability Analysis

**Theorem 1.** Consider chaotic system (6) and parameter identification (16), for bounded initial conditions, the following conclusion was established:

(1) All the signals the consistent bounded in chaos system, state error  $e_i$  ( $i=1,2,3$ ) and parameter estimates error  $\tilde{\gamma}$ ,  $\tilde{\sigma}$  eventually converge to bounded sets:

$$\Omega \triangleq \{e_1, e_2, e_3, \tilde{\gamma}, \tilde{\sigma} | V < \beta/\tau\}.$$

(2) Reasonable choosing parameters  $m, n$  and  $p_i$  ( $i=1,2,3,4$ ), state of chaotic system  $y_1$ ,  $y_2$  and  $y_3$  can be stability in bounded point neighborhood  $(0,0,0)$ .

**Proof:** Choose Laypunov function  $V = V_3$ , by Equation (28) can be obtained as follows,

$$\dot{V} \leq -\tau V + \beta. \quad (29)$$

Equation (29) above both sides by the same  $e^{\tau t}$ , inequality can be obtained as follows

$$e^{\tau t} \dot{V} \leq -\tau e^{\tau t} V + \beta e^{\tau t},$$

namely

$$\frac{d}{dt}(V(t)e^{\tau t}) \leq \beta e^{\tau t}, \quad (30)$$

integral of formulas (30) in  $[0, t]$ ,

$$V(t) \leq V(0)e^{-\tau t} + \beta e^{-\tau t} \int_0^t e^{a\tau} da \leq V(0)e^{-\tau t} + \frac{\beta}{\tau}(1 - e^{-\tau t}) \leq V(0) + \frac{\beta}{\tau}. \quad (31)$$

For bounded initial conditions  $V(0)$ , we can draw a conclusion that  $V(t)$  is bounded base on theorem of Laypunov. We can get  $e_1$ ,  $e_2$ ,  $e_3$ ,  $\tilde{\sigma}$  and  $\tilde{\gamma}$  consistent bounded to inequality (28). Base on virtual control  $\alpha_j$  ( $j=1,2$ ),  $y_1$ ,  $y_2$  and  $y_3$  are all bounded. Control input  $u$  is bounded base on Equation (25), so all the signals in chaotic system are consistent bounded.

When  $t \rightarrow \infty$ ,

$$e^{-\tau t} \rightarrow 0, \quad V(t) \leq V(0)e^{-\tau t} + \frac{\beta}{\tau}(1 - e^{-\tau t}) < \frac{\beta}{\tau}.$$

So state error  $e_i$  ( $i=1,2,3$ ) and parameter estimation errors  $\tilde{\gamma}$ ,  $\tilde{\sigma}$  eventually converge to a bounded set

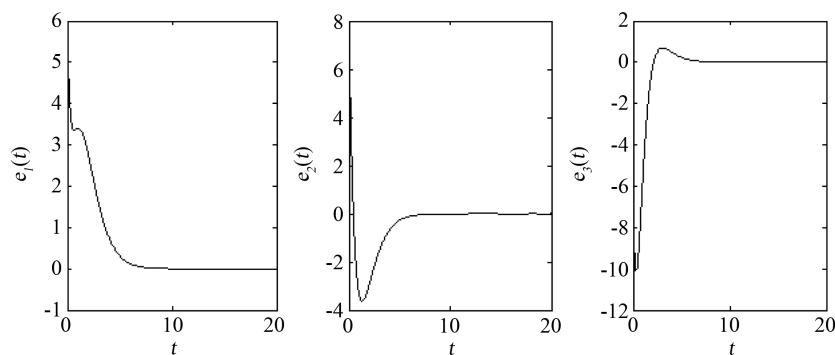
$$\Omega \triangleq \{e_1, e_2, e_3, \tilde{\gamma}, \tilde{\sigma} | V < \beta/\tau\}.$$

From inequality (31), inequality can be obtained as follows

$$\frac{1}{2} \sum_{k=1}^3 \frac{1}{g_k} e_k^2 + \frac{1}{2} \tilde{\Theta}^T \tilde{\Theta} \leq V(0)e^{-\tau t} + \frac{\beta}{\tau}, \quad (32)$$

where  $g_1 = g_2 = 1$ ,  $g_3 = 1 - p_2$ ,  $\tilde{\Theta} = [\tilde{\sigma}, \tilde{\gamma}]^T$ . setting  $g_m = \max\{g_1, g_2, g_3\}$ , base on Equation (32) inequality can be obtained as follows,

$$\sum_{k=1}^3 e_k^2 \leq 2g_m \left[ V(0)e^{-\tau t} + \frac{\beta}{\tau} \right]. \quad (33)$$



**Figure 1.** The synchronization errors.

Given constant  $\mu > 2g_m \beta / \tau$ , existing  $T > 0$ , for all  $t \geq T$ , error  $e_i$  ( $i=1,2,3$ ) satisfy  $\|e_i\| < \mu$ . We reasonable choose values of  $m, n$  and  $p_i$  ( $i=1,2,3,4$ ) which lead to the value of  $\mu$  can be decreased. So,  $e_i$  ( $i=1,2,3$ ) may eventually converge to a stable in bounded neighborhood  $(0, \alpha_1, \alpha_2)$ . Accordingly to Equation (10) and Equation (17),  $p_1, p_2$  and  $p_3$  are chosen smaller constant,  $(\alpha_1, \alpha_2)$  can be stabled in bounded neighborhood  $(0, 0)$ . So system (5) can be stable in bounded neighborhood  $(0, 0, 0)$ .

## 5. Simulation Results

Choose  $\gamma = 25$ ,  $\sigma = 4$ , the system (4) is chaos. Let  $y_0 = [8, 8, 12]^T$  due to  $Y = AX$ ,  $x_0 = [12, 8, 8]^T$ ,  $(\hat{\sigma}_0, \hat{y}_0) = (10, 10)$ ,  $p_1 = 0.5, p_2 = 0.56, p_3 = -0.3, p_4 = 1.0, (m, n) = (100, 100)$ . **Figure 1** shows the synchronization errors. From **Figure 1**, we can see that the proposed controller and the parameters update law are effective.

## 6. Conclusion

This paper puts forward error control for permanent magnet synchronous motor with uncertain parameter based on adaptive backstepping which can effectively eliminate oscillation during the course of control chaos in PMSM. An error control item is added in the each step virtual control design which has control effect of unknown dynamical error on system. This scheme can gain more smoothly chaotic stabilization process. At the same time, all the signals in the system are bounded base on Lyapunov function. This scheme has better transient response by simulation.

## Acknowledgements

This research is supported by the Sichuan Province Natural Science Foundation of China (Nos. 2014GZX0008, 2016JY0179), the Innovation Group Build Plan for the Universities in Sichuan (No. 15TD0024), the High-level Innovative Talents Plan of Sichuan University of Science and Engineering (2014), the Talents Project of Sichuan University of Science and Engineering (No. 2015RC50), the Cultivation Project of Sichuan University of Science and Engineering (Nos. 2012PY18, 2012PY19, 2012PY20), and the Project of Artificial Intelligence Key Laboratory of Sichuan Province (Nos. 2011RZY05, 2014RYY05, 2015RYY01).

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