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General Concave Integral Control

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ABSTRACT

In this paper, a class of fire-new general integral control, named general concave integral control, is proposed. It is derived by normalizing the bounded integral control action and concave function gain integrator, introducing the partial derivative of Lyapunov function into the integrator and originating a class of new strategy to transform ordinary control into general integral control. By using Lyapunov method along with LaSalle's invariance principle, the theorem to ensure regionally as well as semi-globally asymptotic stability is established only by some bounded information. Moreover, the highlight point of this integral control strategy is that the integrator output could tend to infinity but the integral control action is finite. Therefore, a simple and ingenious method to design general integral control is founded. Simulation results showed that under the normal and perturbed cases, the optimum response in the whole domain of interest can all be achieved by a set of the same control gains, even under the case that the payload is changed abruptly.

1. Introduction

Integral control [1] plays an important role in control system design because it ensures asymptotic tracking and disturbance rejection. In the presence of the parametric uncertainties and unknown constant disturbances, integral control can still preserve the stability of the closed-loop system and create an equilibrium point at which the tracking error is zero. The main task of the integral controller is to stabilize this point, which is challenging because it depends on uncertain parameters and unknown disturbances.

1.1. Classical Integral Control

The simplest controllers that achieve integral action are of the proportional integral derivative (PID) form that introduces integral action by integrating the error. It is well known that integral-action controllers with this class of integrator often suffer a serious loss of performance due to integrator windup, which occurs when the actuators in the control loop saturate. Actuator saturation not only deteriorates the control performance, causing large overshoot and large settling time, but can also lead to instability, since the feedback loop is broken for such saturation. To disguise this drawback, various anti-

windup schemes have been proposed to deal with integrator windup or to improve transient performance. These are classified into three different approaches: 1) conditional integration and/or integrator limiting [2-7], in which the integrator value is frozen or restricted when certain conditions are verified; 2) back-calculation [8-11], in which the difference between the controller output and the actual plant input is fed back to the integrator; and 3) a nonlinear integrator [12-16], whose output is shaped by a nonlinear error function before it enters the controller. Some conditional integration and/or integrator limiting may not guarantee a zero steady error and could result in an oscillatory system for the step-referent input when an estimated limitation is embedded in the controller. In the back-calculation approach, the compensation for integrators is active whenever actuators are saturated; integrator windup cannot be completely avoided. For nonlinear integrators, the output still goes to infinity and integrator windup may occur. In addition, the universal integral continuous sliding mode control (CISMC) first reported by [1] has the same problem as a PID controller because it applies the same integrator. An improved version was proposed by [7], in which the integrator is modified to provide integral action only inside the boundary layer and the derivative of the error introduced

into the integrator. All these integrators, except for the one proposed by [7], were designed by using the error as the indispensable element. So, all of them is called classical integral control.

1.2. General Integral Control

In 2009, general integral control, which uses all available state variables to design the integrator, is originated in [17], where presents a unified framework for general integral control, some general integrator and controller, the necessary conditions and basic principles for designing a general integrator, however, their justification was not verified by strictly mathematical analysis. In 2012, based on linear system theory, we present a systematic design method for general integral control [18] with a linear integrator on all the state of dynamics. The results, however, were local. The regionally as well as semiglobally results were proposed in [19], where presents a nonlinear integrator shaped by sliding mode manifold, and then general integral control design is achieved by sliding mode technique and linear system theory. Therein, the sprout of concave function gain integrator appeared. In 2013, based on feedback linearization technique, a class of nonlinear integrator which is shaped by diffeomorphism, and a systematic design method for general integral control are presented by [20] and the conditions to ensure regionally as well as semiglobally asymptotic stability are provided.

This paper is not a simple extension of the work [19], but it is developed as a class of fire-new general integral control, named general concave integral control in such a way of normalization. The main contributions are as follows: 1) the partial derivative of a class of general Lyapunov function is firstly introduced into the integrator design; 2) the bounded integral control action and concave function gain integrator are normalized; 3) a general strategy to transform ordinary control into general integral control is proposed; iv) by using Lyapunov method and LaSalle's invariance principle, the theorem to ensure regionally as well as semi-globally asymptotic stability is established only by some bounded information. Moreover, the highlight point of this integral control strategy is that the integrator output could tend to infinity but the integral control action is finite. Therefore, a simple and ingenious method to design general integral control is founded.

Throughout this paper, we use the notation $\lambda_m(A)$ and $\lambda_M(A)$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive define bounded matrix $A(x)$, for any $x \in R^n$. The norm of vector x is defined as $\|x\| = \sqrt{x^T x}$, and that of matrix A is defined as the corresponding induced norm $\|A\| = \sqrt{\lambda_M(A^T A)}$.

The remainder of the paper is organized as follows: Section 2 describes the system under consideration, assumption, and definition. Section 3 addresses the control design. Simulation is provided in Section 4. Conclusions are presented in Section 5.

2. Problem Formulation

Consider the following nonlinear system,

$$\begin{cases} \dot{x} = f(x,w) + g(x,w)u \\ y = h(x,w) \end{cases} \tag{1}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $y \in R^m$ is the controlled output, $w \in R^l$ is a vector of unknown constant parameter and disturbance. The functions $f(x,w)$, $g(x,w)$ and $h(x,w)$ are continuous in (x,w,u) on the domain $D_x \times D_u \times D_w \subset R^n \times R^m \times R^l$. In this study, the function $f(x,w)$ does not necessarily vanish at the origin; *i.e.*, $f(0,w) \neq 0$. Let $r \in D_r \subset R^m$ be a vector of constant reference. Set $v \equiv (r,w) \in D_v$ and $D_v \equiv D_r \times D_w$. We want to design a feedback control law u such that $y(t) \to r$ as $t \to \infty$.

Assumption 1: For each $v \in D_v$, there is a unique pair (x_0, u_0) that depends continuously on v and satisfies the equations,

$$\begin{cases} 0 = f(x_0, w) + g(x_0, w)u_0 \\ y = r = h(x_0, w) \end{cases} \tag{2}$$

so that x_0 is the desired equilibrium point and u_0 is the steady-state control that is needed to maintain equilibrium at x_0, where $y = r$.

For convenience, we state all definitions, assumptions and theorems for the case when the equilibrium point is at the origin of R^n, that is, $x_0 = 0$. There is no loss of generality in doing so because any equilibrium point can be shifted to the origin via a change of variables.

Assumption 2: No loss of generality, suppose that the function $g(x,w)$ satisfies,

$$g(x,w) > g_0 > 0, \quad \forall w \in D_w, \forall x \in D_x \tag{3}$$

$$\|g(x,w) - g(0,w)\| \leq l_g^x \|x\|, \quad \forall w \in D_w, \forall x \in D_x. \tag{4}$$

where l_g^x is a positive constant.

Assumption 3: Suppose that there exists a control law $u_x(x)$ such that $x = 0$ is an exponentially stable equilibrium point of the system,

$$\dot{x} = f(x,w) - f(0,w) + g(x,w)u_x(x) \tag{5}$$

and there exists a Lyapunov function $V_x(x)$ that satisfies,

$$c_1 \|x\|^2 \leq V_x(x) \leq c_2 \|x\|^2 \tag{6}$$

$$\frac{\partial V_x(x)}{\partial x}\big(f(x,w)-f(0,w)+g(x,w)u_x(x)\big)\le -c_3\|x\|^2$$

$$(7)$$

$$\left\|\frac{\partial V_x(x)}{\partial x}\right\|\le c_4\|x\| \tag{8}$$

for all $x\in D_x$, $r\in D_r$ and $w\in D_w$. Where c_1, c_2, c_3 and c_4 are all positive constants.

For the purpose of this note, we introduce the following definition and property, which is proposed by [13].

Definition 1: $F(\alpha,\beta,x)$ with $1\ge\alpha>0$, $\beta>0$ and $x\in R^n$ denotes the set of all continuous differential increasing bounded functions,

$$\varphi(x)=\begin{bmatrix}\varphi_1(x_1) & \varphi_2(x_2) & \cdots & \varphi_n(x_n)\end{bmatrix}^T \text{ such that}$$

$$|x|\ge|\varphi(x)|\ge\alpha|x| \qquad \forall x\in R:|x|<\beta$$

$$\beta\ge|\varphi(x)|\ge\alpha\beta \qquad \forall x\in R:|x|\ge\beta$$

$$1\ge\mathrm{d}\varphi(x)\mathrm{d}x\ge 0 \qquad \forall x\in R$$

where $|\cdot|$ stands for the absolute value.

Figure 1 depicts the region allowed for all the functions belonging to function set $F(\alpha,\beta,x)$. For instance, the hyperbolic tangent, arc tangent functions and so on.

An important property of function $\varphi(x)$ belonging to function set $F(\alpha,\beta,x)$ is that the Euclidean norm of $\varphi(x)$ satisfies for all $x\in R^n$,

$$\|\varphi(x)\|\le\beta\sqrt{n} \tag{9}$$

3. Control Design

For achieving asymptotic regulation and disturbance rejection, we need to include "integral action" in the control law u. Thus, general integral controller are proposed as follows,

$$\begin{cases}u=u_x(x)-K_\sigma\varphi(\sigma)\\ \dot\sigma^T=\big(\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma\big)^{-1}\partial V_x(x)/\partial x\end{cases} \tag{10}$$

where $\dot\sigma_i=\big(\mathrm{d}\varphi_i(\sigma_i)/\mathrm{d}\sigma_i\big)^{-1}\partial V_x(x)/\partial x_i$, $i=1,2,\cdots,m$;
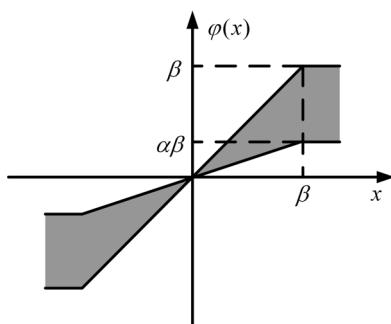

Figure 1. $F(\alpha,\beta,x)$ functions.

$\varphi(\cdot)$ belongs to function set $F(\alpha,\beta,x)$. K_σ is a positive define diagonal $m\times m$ matrix.

Thus, substituting (10) into (1) to obtain the augmented system,

$$\begin{cases}\dot x=f(x,w)+g(x,w)u_x(x)-g(x,w)K_\sigma\varphi(\sigma)\\ \dot\varphi^T(\sigma)=\partial V_x(x)/\partial x\end{cases} \tag{11}$$

By Assumption 1 and choosing K_σ to be nonsingular and large enough, and then set $\dot x=0$ and $x=0$ of the Equation (11), we obtain,

$$g(0,w)K_\sigma\varphi(\sigma_0)=f(0,w) \tag{12}$$

Therefore, we ensure that there is a unique solution σ_0, and then $(0,\sigma_0)$ is a unique equilibrium point of the closed-loop system (11) in the control domain of interest. At the equilibrium point, $y=r$, irrespective of the value of w.

Now, the design task is to provide the conditions on the positive constants c_3, c_4 and matrix K_σ such that $(0,\sigma_0)$ is an asymptotically stable equilibrium point of the closed-loop system (11) in the control domain of interest, which is not a trivial task because the closed-loop system depends on the unknown vector w. This is established in the following theorem.

Theorem 1: Under Assumptions 1-3, if there exists a positive define diagonal matrix K_σ such that the the following inequalities,

$$\lambda_m\big(g_0 K_\sigma\varphi(\alpha\beta)\big)\ge\|f(0,w)\| \tag{13}$$

$$c_3>c_4 l_g^x\beta\sqrt{m}\|K_\sigma\| \tag{14}$$

hold, and then $(0,\sigma_0)$ is an exponentially stable equilibrium point of the closed-loop system (11). Moreover, if all assumptions hold globally, and then it is globally exponentially stable.

Proof: To carry out the stability analysis, we consider the following Lyapunov function candidate,

$$\begin{aligned}V\big(x,\varphi(\sigma)-\varphi(\sigma_0)\big)&=V_x(x)\\ &+\big(\varphi(\sigma)-\varphi(\sigma_0)\big)^T g(0,w)K_\sigma\big(\varphi(\sigma)-\varphi(\sigma_0)\big)/2\end{aligned} \tag{15}$$

Obviously, Lyapunov function candidate (15) is positive define. Therefore, our task is to show that its time derivative along the trajectories of the closed-loop system (11) is negative define, which is given by,

$$\begin{aligned}&\dot V\big(x,\varphi(\sigma)-\varphi(\sigma_0)\big)\\ &=\dot V_x(x)+\dot\varphi^T(\sigma)g(0,w)K_\sigma\big(\varphi(\sigma)-\varphi(\sigma_0)\big)\\ &=\frac{\partial V_x(x)}{\partial x}\big(f(x,w)+g(x,w)u_x(x)-g(x,w)K_\sigma\varphi(\sigma)\big)\\ &\quad+\frac{\partial V_x(x)}{\partial x}g(0,w)K_\sigma\big(\varphi(\sigma)-\varphi(\sigma_0)\big)\end{aligned}$$

$$(16)$$

Substituting (12) into (16), we obtain,

$$\dot{V}\left(x,\varphi(\sigma)-\varphi(\sigma_0)\right)$$

$$= \frac{\partial V_x(x)}{\partial x}\left(f(x,w)-f(0,w)+g(x,w)u_x(x)\right)$$

$$- \frac{\partial V_x(x)}{\partial x}\left(g(x,w)-g(0,w)\right)K_\sigma\varphi(\sigma) \qquad (17)$$

$$- \frac{\partial V_x(x)}{\partial x}g(0,w)K_\sigma\left(\varphi(\sigma)-\varphi(\sigma_0)\right)$$

$$+ \frac{\partial V_x(x)}{\partial x}g(0,w)K_\sigma\left(\varphi(\sigma)-\varphi(\sigma_0)\right)$$

Using (4), (7), (8) and (9), we get,

$$\dot{V}\left(x,\varphi(\sigma)-\varphi(\sigma_0)\right)$$

$$= \frac{\partial V_x(x)}{\partial x}\left(f(x,w)-f(0,w)+g(x,w)u_x(x)\right)$$

$$- \frac{\partial V_x(x)}{\partial x}\left(g(x,w)-g(0,w)\right)K_\sigma\varphi(\sigma) \qquad (18)$$

$$\leq -c_3\|x\|^2 + c_4 l_g^x \beta\sqrt{m}\|K_\sigma\|\|x\|^2$$

$$= -\left(c_3 - c_4 l_g^x \beta\sqrt{m}\|K_\sigma\|\right)\|x\|^2$$

Using the fact that Lyapunov function candidate (15) is a positive define function and its time derivative is a negative define function if the inequalities (13) and (14) hold, we conclude that the closed-loop system (11) is stable. In fact, $\dot{V}=0$ means $x=0$ and $\sigma=\sigma_0$. By invoking LaSalle's invariance principle [21], it is easy to know that the closed-loop system (11) is exponentially stable.

Corollary 1*:* If the function $g(x,w)$ is equal to a constant, and then the integrator can be taken as $\dot{\sigma}^T = \left(\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma\right)^{-1}\partial V_x(x)/\partial x$ or $\dot{\sigma}^T = \partial V_x(x)/\partial x$. Thus, under Assumptions 1 and 3, we only need to choose the gain matrix K_σ to be nonsingular and large enough such that the inequality (13) holds, and then $(0,\sigma_0)$ is an exponentially stable equilibrium point of the closed-loop system (11). Moreover, if all assumptions hold globally, and then it is globally exponentially stable. The proof can follow the similar argument and procedure. It is omitted because of the limited space.

Discussion 1: compared with the integral control proposed by [19], the main differences are as follows:

1) the integral control action is not confined to the hyperbolic tangent function and can be taken as any function belonging to function set $F(\alpha,\beta,x)$, and then the normalization of integral control action is achieved;

2) the indispensable element of integrator is not confined to sliding mode manifold and can be taken as the partial derivative of any Lyapunov function, which satisfies Assumption 3, and then not only the normalization of concave function gain integrator is achieved but also

the partial derivative of Lyapunov function firstly is introduced into the integrator design.

3) the control element $u_x(x)$ is not confined to sliding control and can be taken as any control, which satisfies the conditions of Assumption 3.

Remark 1: The proof of Theorem 1 seems to be very simple, in fact that is not the case because there are two tedious troubles to be concealed in the stability analysis, one is that integral control action must be bounded, another is how cancel the terms on $\varphi(\sigma)-\varphi(\sigma_0)$. Therefore, for solving these two troubles above, an ingenious design method is proposed as follows: just the integrator is taken as $\dot{\sigma}^T = \left(\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma\right)^{-1}\partial V_x(x)/\partial x$, which is obtained by differentiating the function $\varphi(\sigma)$ and using the partial derivative of Lyapunov function $\partial V_x(x)/\partial x$ as the indispensable element of integrator, and then we get $\dot{\varphi}^T(\sigma) = \partial V_x(x)/\partial x$. Thus, we not only obtain a bounded integral control action $K_\sigma\varphi(\sigma)$ but also cancel the terms on $\varphi(\sigma)-\varphi(\sigma_0)$ in the time derivative of Lyapunov function, and then Theorem 1 can be established only by some bounded information. Consequently, the justification of general concave integral control is verified. Moreover, this resulted in a class of new integrator with a concave function gain $\left(\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma\right)^{-1}$, see **Figure 2**. This is why the control law (10) is called general concave integral control.

Remark 2: From the control law (10), it is obvious that the highlight point of this integral control strategy is that the integrator output could tend to infinity but the integral control action is finite, which is the same as the one proposed by [19]. This means that this kind of integral control can devote its mind to counteract the unknown constant uncertainties or disturbances and filter out the other action, and then the stability analysis is easy to be achieved in theory and actuator saturation is easy to be eliminated in practice.

Remark 3: From the statement above, it is easy to see that: for achieving the integral control, we only need to find a control input $u_x(x)$ and a Lyapunov function $V_x(x)$ such that $x=0$ is an exponentially stable equilibrium point of the system (5). Especially, when the
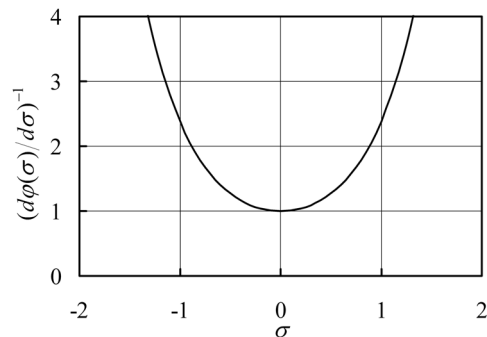

Figure 2. The concave function gain curve.

function $g(x, w)$ is equal to a constant, the dilemma condition (14) can be removed, that is, the stable conditions on the closed-loop system (11), except for the condition (13), is the same as the one of the system (5). This not only results in a class of general strategy to transform ordinary control into general integral control but also the guess [17], that is, many control laws can easily be transformed into general integral control laws, is verified partly. Moreover, there is great freedom in the choice of $u_x(x)$ and $V_x(x)$ such that the control engineers can choose the most appropriate control input $u_x(x)$ in hand to design their own general integral controller.

Based on these statements above, it is not hard to know that all of them constitute a simple and ingenious method to design general integral control together.

4. Simulation

Consider the pendulum system [21] described by,

$$\ddot{\theta} = -a\sin\theta - b\dot{\theta} + cT$$

where $a = g/l > 0$, $b = k/m > 0$, $c = 1/ml^2 > 0$, θ is the angle subtended by the rod and the vertical axis, and T is the torque applied to the pendulum. View T as the control input and suppose we want to regulate θ to δ. Taking $x_1 = \theta - \delta$, $x_2 = \dot{\theta}$ and $u = T$, the pendulum system can be written as,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -a\sin(x_1 + \delta) - bx_2 + cu \end{cases} \quad (19)$$

It is easily to know that the desired equilibrium point is $x_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ and $u_0 = a\sin(\delta)/c$ is the steady-state control that is needed to maintain equilibrium at x_0. Thus, the control law in Assumption 3 can be taken as, $u_x(x) = -k_1 x_1 - k_2 x_2$, where k_1 and k_2 are all positive constants.

Substituting $u_x(x)$ into (19) and deleting the constant term $-a\cos(\delta)$, and then Linearization of the system about the origin, we obtain,

$$\dot{x} = Ax \quad (20)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -(a\cos(\delta) + ck_1) & -(b + ck_2) \end{bmatrix}$$

Now, using the linear system theory, the choice of $k_1 > -a\cos(\delta)/c$ and $k_2 > 0$ ensures that the matrix A is Hurwitz for all the parameter perturbations on $a > 0$, $b > 0, c > 0$, and all $\delta \in [-\pi, \pi]$, and then $x = 0$ is an exponentially stable equilibrium point of the system (20). Therefore, for any given positive define symmetric matrix Q there exists a unique positive define symmetric matrix P that satisfied Lyapunov equation $PA + A^T P = -Q$, and then the Lyapunov function in Assumption 3 can be taken as $V(x) = x^T Px$. Thus, taking $\varphi(\sigma) = \tanh(\sigma)$,

$\alpha = \tanh(1)$, $\beta = 1$ and choosing k_σ, such that $ck_\sigma \varphi(\alpha\beta) > a\sin(\delta)$ holds for all $a > 0$, $c > 0$ and $\delta \in [-\pi, \pi]$, and then a globally exponentially stable controller can be given as,

$$\begin{cases} u(x, \sigma) = -k_1 x_1 - k_2 x_2 - k_\sigma \tanh(\sigma) \\ \dot{\sigma} = \cosh^2(\sigma)(p_{12} x_1 + (p_{21} + p_{22})x_2) \end{cases}$$

By taking $k_1 = 8$, $k_2 = 3$, $k_\sigma = 10$, $a = 10$, $b = 1$, and $c = 10$, and then solving the Lyapunov equation $PA + A^T P = -Q$, we obtain,

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 38 & 1 \\ 1 & 0.1 \end{bmatrix}$$

where

$$Q = \begin{bmatrix} 140 & 0 \\ 0 & 4.2 \end{bmatrix} \quad \text{and} \quad A = -\begin{bmatrix} 0 & -1 \\ 70 & 31 \end{bmatrix}$$

In simulation, the normal parameters are $a = c = 10$ and $b = 1$. In the perturbed case, b and c are reduced to 0.5 and 5, respectively, corresponding to doubling of the mass. Moreover, we consider an additive impulse-like disturbance $d(t)$ of magnitude 60 acting on the system input between 18 s and 19 s.

Figure 3 showed the simulation results under normal (solid line) and perturbed (dashed line) cases. The following observations can be made: under the normal and perturbed cases, the optimum response in the whole domain of interest can all be achieved by a set of the same control gains, even under the case that the payload is changed abruptly. This demonstrates that general concave integral control has strong robustness, fast convergence and good flexibility and can effectively deal with unknown exogenous disturbances, nonlinearity and uncertainties of dynamics.

5. Conclusions

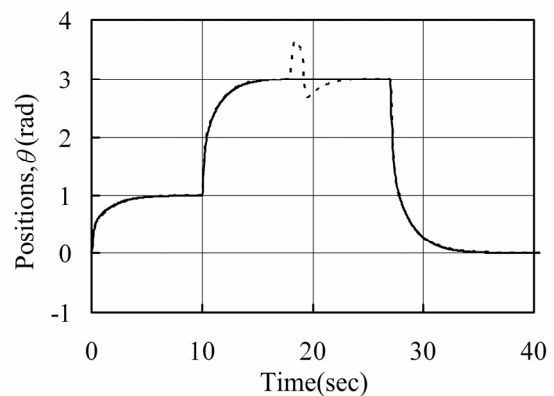A class of fire-new general integral control named gen-

Figure 3. System output under normal (solid line) and perturbed case (dashed line).

eral concave integral control was proposed in this paper. The main contributions are as follows: 1) the partial derivative of a class of general Lyapunov function is firstly introduced into the integrator design; 2) the bounded integral control action and concave function gain integrator are normalized; 3) a general strategy to transform ordinary control into general integral control is proposed; 4) by using Lyapunov method and LaSalle's invariance principle, the theorem to ensure regionally as well as semi-globally asymptotic stability is established only by some bounded information. Moreover, the highlight point of this integral control strategy is that the integrator output could tend to infinity but the integral control action is finite. Therefore, a simple and ingenious method to design general integral control is founded.

In this note, only a class of general integral control was presented. It is clear that we can not expect one particular procedure to apply to all system. Therefore, new design techniques for general integral control are needed to solve the wider theoretical and practical problems.

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