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An Adaptive Fuzzy Controller for Trajectory Tracking of Robot Manipulator

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Abstract

In this paper, an adaptive fuzzy control algorithm is proposed for trajectory tracking of an n-DOF robot manipulator subjected to parametric uncertainty and it is advantageous compared to the conventional nonlinear saturation controller. The asymptotic stability of the proposed controller has been derived based on Lyapunaov energy function. The design procedure is straightforward due to its simple fuzzy rules and control strategies. The simulation results show that the present control strategy effectively reduces the control effort with negligible chattering in control torque signals in comparison to the existing nonlinear saturation controller.

1. Introduction

The mathematical model of a dynamic robot manipulator involves highly nonlinearities, strong coupling and uncertain system dynamics [1-4]. To overcome this problem, various adaptive control strategies have been adopted for such uncertain systems including SISO and MIMO systems with guaranteed performance. The parametric uncertainty mainly occurs in robotic manipulator due to the variation in payloads and mass of links. Many efforts have been made in developing control scheme to achieve precise tracking controller of a robot manipulator in presence of parametric uncertainty. The adaptive control and nonlinear saturation control are the two major approaches in dealing the parametric uncertainty of a robot manipulator [5,6].

Fuzzy control has shown a great potential since it is able to handle uncertainty using the programming capability of human control behavior. Many results have been published in area of design and stability of fuzzy control systems [3,7,8]. Efforts have been made to improve tracking performance of fuzzy controllers with adaptive algorithms, which are well known as adaptive fuzzy approach [9]. A vast amount of research work on adaptive fuzzy control of nonlinear dynamic systems can be found in [10] and references cited therein.

In this paper, the possibility of replacing the additional control law given in [2] by an equivalent control law based on fuzzy-logic approach has been explored. Subsequently, an equivalent adaptive fuzzy controller is proposed to design a robust control law for trajectory tracking of a two-link robot manipulator with reduced control effort as well as reduced chattering in control force. Initially fuzzy controller is designed from a simple fuzzy IF-THEN rule and then an adaptive control law is adopted to update the parameters of the fuzzy controller during the adaptation procedure. Now, this adaptive fuzzy controller along with simple PD control term is used to construct the final control torque for tracking control of robot manipulators joints.

2. Dynamic Model and Robust Control Design

The dynamic equation of *n*-DOF robot manipulator is governed by [11]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \text{ with } q \in \mathfrak{R}^n, \ \tau \in \mathfrak{R}^n \quad (1)$$

where $M(q)$ is $n \times n$ inertia matrix, the vector $C(q,\dot{q})\dot{q}$ represents centrifugal and coriolis forces, and $g(q)$ is the vector of gravitational forces and the matrix $N(q,\dot{q}) = \dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric. The above dynamic Equation (1) is linearly parameterizable as

$$M(q)\ddot{q}+C(q,\dot{q})+g(q)=Y(q,\dot{q},\ddot{q})\theta \quad (2)$$

where θ is a constant p-dimensional parameter vector and Y is an $n \times p$ matrix of known functions of the generalized coordinates and their higher derivatives.

It is assumed that there exists an unknown bound on parametric uncertainty such that

$$\|\tilde{\theta}\|=\|\theta-\theta_0\|\le\rho \quad (3)$$

For any specific trajectory, $q_d, \dot{q}_d, \ddot{q}_d$ are the desired positions, velocities, accelerations whereas the position and velocity errors are defined as $\tilde{q}=q-q_d$, $\dot{\tilde{q}}=\dot{q}-\dot{q}_d$. The adaptive algorithm [1] is very useful for dealing the parametric uncertainties involve in a robot manipulator. The reference velocity and acceleration of the following quantities are defined as

$$\dot{q}_r=\dot{q}_d-\Lambda\tilde{q},\ \ddot{q}_r=\ddot{q}_d-\Lambda\dot{\tilde{q}} \quad (4)$$

and reference velocity error is given as

$$\sigma=\dot{q}-\dot{q}_r=\dot{\tilde{q}}+\Lambda\tilde{q} \quad (5)$$

The nominal control law is given as

$$\begin{aligned}\tau_0 &= M_0(q)\ddot{q}_r+C_0(q,\dot{q})\dot{q}_r+g_0(q)-K\sigma\\&=Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\theta_0-K\sigma\end{aligned} \quad (6)$$

where θ_0 represents fixed robot parameter, these parameters are not updated or changed with time. The gain matrices K and Λ are positive definite diagonal matrices.

Now, we define the control torque τ in terms of nominal control vector τ_0 as

$$\begin{aligned}\tau &= \tau_0-Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)u\\&=Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\theta_0-u)-K\sigma\end{aligned} \quad (7)$$

where u is an additional control input that will be designed to achieve robustness to parametric uncertainty represented by $\ddot{\theta}$. Substituting the control law (7) into (1) and after simplification we have,

$$M(q)\sigma+C(q,\dot{q})\sigma+K\sigma=Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\tilde{\theta}-u) \quad (8)$$

3. Simple Fuzzy Logic Control

In [2], the additional input is defined as

$$u=\begin{cases}-\rho\dfrac{Y^T\sigma}{\|Y^T\sigma\|} & \text{if } \|Y^T\sigma\|>\varepsilon\\[3mm]-\rho\dfrac{Y^T\sigma}{\varepsilon} & \text{if } \|Y^T\sigma\|\le\varepsilon\end{cases} \quad (9)$$

where $\varepsilon>0$ and the upper uncertainty bound is defined as

$$\|\theta-\theta_0\|\le\rho=\left(\sum_{i=1}^{p}\rho_i^2\right)^{1/2} \quad (10)$$

The parameter ρ indicates the measure of uncertainty that may leads to a higher controller gain. To overcome this problem different weights or gains can be assigned to each component of $u(t)$ and it is assumed there exists an upper bound on each parametric uncertainty as

$$|\tilde{\theta}_i|\le\rho_i,\ i=1,2,\cdots,p. \quad (11)$$

The control law (9) can be modified following the discussion in [2] as

$$u_i=\begin{cases}-\rho_i\dfrac{\xi_i}{|\xi_i|} & \text{if } |\xi_i|>\varepsilon_i\\[3mm]-\rho_i\dfrac{\xi_i}{\varepsilon_i} & \text{if } |\xi_i|\le\varepsilon_i\end{cases}\ ;\ i=1,2,\cdots,p. \quad (12)$$

where ξ_i denotes the ith element of the vector $Y^T\sigma$, ε_i is i^{th} element of ε and u_i is i^{th} element of u (see (9)).

It can be seen if ε_i is chosen very small then the nonlinear saturation control (12) will act as an ON-OFF controller. It is possible to avoid hard switching and also to achieve a better performance if we grade ξ_i in number of linguistic terms.

Therefore, we need to construct additional control input u with the help of "r" numbers of IF-THEN fuzzy rules to achieve robustness against the parametric uncertainty. The jth rule is given as:

Rule j:

$$\text{IF }\ \xi_i\ \text{ is }\ M_i^j\ \text{ THEN }\ u_i\ \text{ is }\ \psi_i^j \quad (13)$$

where, ξ_i and M_i^j, $i=1,2,\cdots,p$, $j=1,2,\cdots,r$ are the premise variables and the fuzzy terms, respectively; r is the number of IF-THEN rule; ψ_i^j is the singleton fuzzy inference. The crisp value of u_i can be obtained after weighted average deffuzzification,

$$u_i=\sum_{j=1}^{r}\psi_i^j w_i^j(\xi_i)=\Psi_i^T W_i(\xi_i) \quad (14)$$

and

$$w_i^j(\xi_i)=\frac{\mu_{M_i^j}(\xi_i)}{\displaystyle\sum_{k=1}^{r}\left(\mu_{M_i^k}(\xi_i)\right)}\ ;$$

$$\sum_{j=1}^{r}w_i^j(\xi_i)=1, w_i^j(\xi_i)\in[0\ 1]\ \text{ for all } j$$

$$W_i(\xi_i)=\left[w_i^1\ w_i^2,\cdots,w_i^r\right]^T,\ \Psi_i=\left[\psi_i^1\ \psi_i^2,\cdots,\psi_i^r\right]^T.$$

where $\mu_{M_i^j}$ is the grade of membership of ξ_i in the

fuzzy set M_i^j witht $\left|\psi_i^j\right| \leq \rho_i$.

4. Design of Adaptive Fuzzy Controller

In this section, we have proposed an adaptive fuzzy logic controller as an additional control input to achieve robustness against the parametric uncertainty. We have summarized the results of proposed work in following theorem.

Theorem: Let the *i*th element of the additional control input be defined as

$$u_i = \left(\tilde{\Psi}_i + \Psi_{oi}\right)^T W_i \qquad (15)$$

where, $\Psi_{oi}^T W_i = \left(\Psi_i - \tilde{\Psi}_i\right)^T W_i$ is adaptive and $\Psi_i^T W_i$ is fuzzy compensation (14) for $\tilde{\theta}_i$ with an adaptive law as

$$\dot{\tilde{\Psi}}_i = \zeta_i W_i \qquad (16)$$

and the gain matrix K in (8) is chosen such that

$$K > \gamma_{\max} I, \qquad (17)$$

where

$$\gamma_{\max} = \max\left(\gamma_i\right),$$

$$\gamma_i = \begin{cases} \dfrac{\left|\xi_i\right|\left|\rho_i - \Psi_{oi}^T W_i\right|}{\|\sigma\|^2} & \text{if } \|\sigma\| > \varepsilon_i \\[3mm] \dfrac{\left|\xi_i\right|\left|\rho_i - \Psi_{oi}^T W_i\right|}{\varepsilon_i} & \text{if } \|\sigma\| \leq \varepsilon_i \end{cases} \qquad (18)$$

Then for the trajectory tracking error of a robot manipulator is uniformly ultimately bounded along the trajectory *i.e.* the tracking errors $\tilde{q}$ and its derivatives $\dot{\tilde{q}}$ converge to zero with application of additional control law $u(t)$ (15) in presence of parametric uncertainties.

Proof: Let us assume that, the parameter vector Ψ_{oi} is selected in an adaptive way so that $\mu_{oi} = \Psi_{oi}^T W_i$ is the adaptive compensation for ρ_i.

Let us choose Lyapunov function as

$$V\left(\sigma, \tilde{\Psi}_i\right) = \frac{1}{2}\sigma^T M\left(q\right)\sigma + \frac{1}{2}\sum_{i=1}^{p}\tilde{\Psi}_i^T \tilde{\Psi}_i \qquad (19)$$

The time derivative of *V* along the trajectories is

$$\dot{V} = \sigma^T M\left(q\right)\dot{\sigma} + \frac{1}{2}\sigma^T \dot{M}\left(q\right)\sigma + \sum_{i=1}^{p}\tilde{\Psi}_i^T \dot{\tilde{\Psi}}_i \qquad (20)$$

Using the property $\sigma^T\left[\dot{M}\left(q\right) - 2C\left(q, \dot{q}\right)\right]\sigma = 0$, we have

$$\dot{V} = -\sigma^T K\sigma + \sigma^T Y\left(\tilde{\theta} - u\right) + \sum_{i=1}^{p}\tilde{\Psi}_i^T \dot{\tilde{\Psi}}_i \qquad (21)$$

Note that $\sigma^T Y = \left[\xi_1 \;\; \xi_2 \cdots \xi_p\right]$, $\tilde{\theta} = \left[\tilde{\theta}_1 \;\; \tilde{\theta}_2 \cdots \tilde{\theta}_p\right]$, (21) can be written as

$$\dot{V} = -\sigma^T K\sigma + \sum_{i=1}^{p}\xi_i\left(\tilde{\theta}_i - u_i\right) + \sum_{i=1}^{p}\tilde{\Psi}_i^T \dot{\tilde{\Psi}}_i \qquad (22)$$

We know that $\left|\tilde{\theta}_i\right| = \left|\theta_i - \theta_{0i}\right| \leq \rho_i$, and ξ_i can be negative also so (23) reduces to

$$\dot{V} \leq -\sigma^T K\sigma + \sum_{i=1}^{p}\left\{\left|\xi_i\right|\left(\rho_i - u_i\right) + \tilde{\Psi}_i^T \dot{\tilde{\Psi}}_i\right\} \qquad (23)$$

Substituting the adaptive fuzzy control from (15)

$$\dot{V} \leq -\sigma^T K\sigma + \sum_{i=1}^{p}\left\{\left|\xi_i\right|\left(\rho_i - \left[\tilde{\Psi}_i^T W_i + \Psi_{oi}^T W_i\right]\right) + \tilde{\Psi}_i^T \dot{\tilde{\Psi}}_i\right\} \qquad (24)$$

$$\dot{V} \leq -\sigma^T K\sigma + \sum_{i=1}^{p}\left\{\left|\xi_i\right|\left(\rho_i - \Psi_{oi}^T W_i\right) + \left(\tilde{\Psi}_i^T \dot{\tilde{\Psi}}_i - \tilde{\Psi}_i^T \zeta_i W_i\right)\right\} \qquad (25)$$

Using the adaptive law (16)

$$\dot{V} \leq -\sigma^T K\sigma + \sum_{i=1}^{p}\left\{\left|\xi_i\right|\left(\rho_i - \Psi_{oi}^T W_i\right) + \left(\tilde{\Psi}_i^T \dot{\tilde{\Psi}}_i - \tilde{\Psi}_i^T \dot{\tilde{\Psi}}_i\right)\right\} \qquad (26)$$

$$\dot{V} \leq -\sigma^T K\sigma + \sum_{i=1}^{p}\left|\xi_i\right|\left(\rho_i - \Psi_{oi}^T W_i\right) \qquad (27)$$

There exists a small positive scalar $\beta_i > 0$ satisfying following inequality [12],

$$\left|\rho_i - \Psi_{oi}^T W_i\right| \leq \beta_i \quad i = 1, 2, \cdots, p. \qquad (28)$$

Similarly, with $\gamma_i > 0$ we can write,

$$\left|\rho_i - \Psi_{oi}^T W_i\right| \leq \beta_i \leq \gamma_i \frac{\|\sigma\|^2}{\left|\xi_i\right|} \quad i = 1, 2, \cdots, p. \qquad (29)$$

Substituting (29) in (27),

$$\dot{V} \leq -\sigma^T K\sigma + \sum_{i=1}^{p}\gamma_i\|\sigma\|^2 \qquad (30)$$

Using $\gamma_{\max}$ for γ_i with $\gamma_{\max} = \max\left(\gamma_i\right), i = 1, 2, \cdots, p$ the above Equation (30) can be written as

$$\dot{V} \leq -\sigma^T K\sigma + \sigma^T\left(\gamma_{\max} I\right)\sigma = -\sigma^T\left(K - \gamma_{\max} I\right)\sigma$$

From (17), $K - \lambda_{\max} I > 0$, hence $\dot{V} \leq 0$ and the closed-loop system is asymptotically stable and the position and velocity tracking errors $\tilde{q}$ and $\dot{\tilde{q}}$ converge to zero. Moreover, from (29) one can get (18). □

The design procedure of adaptive fuzzy controller for robot manipulator can be summarized in following steps:

Step 1: Obtain the parameter vector Ψ_i, $i = 1, 2, \cdots, 6$, according to fuzzy IF-THEN Rules.

Step 2: Using the adaptive law (16) obtain $\tilde{\Psi}_i$, $i = 1, 2, \cdots, 6$ to compute optimal parameter vector

$$\Psi_{0i} = \Psi_i - \tilde{\Psi}_i, \quad i = 1, 2, \cdots, 6.$$

Step 3: Compute the additional control input (15),

$$u_i = \left(\tilde{\Psi}_i + \Psi_{oi} \right)^T W_i, \quad i = 1, 2, \cdots, 6.$$

Step 4: Obtain γ_i from (18) and get $\gamma_{max} = \max(\gamma_i)$ for $i = 1, 2, \cdots, p$.

Step 5: Choose the gain matrix K such that $K > \gamma_{max} I$.

Step 6: Obtain the final control torque according to (7)
$$\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)(\theta_0 - u) - K\sigma.$$

5. Description of Two-Ink Robot Manipulator

In order to show effectiveness of the proposed controller, computer simulations have been performed with the two-link planar manipulator model [11]. **Figure 1** shows this two link planar manipulator with the following parameterizations are considered as

$$\begin{aligned}
\theta_1 &= m_1 l_{c1}^2 + m_2 l_1^2 + I_1 & \theta_2 &= m_2 l_{c2}^2 + I_2 \\
\theta_3 &= m_2 l_1 l_{c2} & \theta_4 &= m_1 l_{c1} \\
\theta_5 &= m_2 l_1 & \theta_6 &= m_2 l_{c2}
\end{aligned} \tag{31}$$

where m_1, m_2 the masses of links 1 and 2 respectively, l_1, l_2 are the lengths of link 1 and 2 respectively and l_{c1}, l_{c2} are the lengths where the masses of link 1 and 2 are assumed to be concentrated respectively. Using the above parameters, the matrices $M(q)$, $C(q, \dot{q})$ and the vectors $g(q)$, τ in (1) are given as

$$M(q) = \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3 \cos(q_2) & \theta_2 + \theta_3 \cos(q_2) \\ \theta_2 + \theta_3 \cos(q_2) & \theta_2 \end{bmatrix} \tag{32}$$

$$C(q, \dot{q}) = \begin{bmatrix} -\theta_3 \sin(q_2)\dot{q}_2 & -\theta_3 \sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ \theta_3 \sin(q_2)\dot{q}_1 & 0 \end{bmatrix} \tag{33}$$
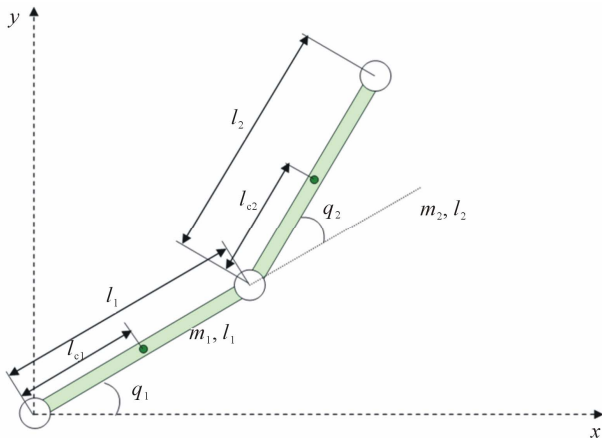

Figure 1. Two-link planar robot.

$$g(q) = \begin{bmatrix} g(\theta_4 + \theta_5)\cos(q_1) + g\theta_6 \cos(q_1 + q_2) \\ g\theta_6 \cos(q_1 + q_2) \end{bmatrix} \tag{34}$$

$$\tau = Y(q, \dot{q}, \ddot{q})\theta; \theta \in \mathfrak{R}^6 \tag{35}$$

where the component y_{ij} of $Y(q, \dot{q}, \ddot{q})$ are given as

$$\begin{aligned}
y_{11} &= \ddot{q}_1 & y_{12} &= \ddot{q}_1 + \ddot{q}_2 \\
y_{13} &= \cos(q_2)(2\ddot{q}_1 + \ddot{q}_2) \\
&\quad - \sin(q_2)(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2) & y_{14} &= g\cos(q_1) \\
y_{15} &= g\cos(q_1) & y_{16} &= g\cos(q_1 + q_2) \\
y_{21} &= 0 & y_{22} &= \ddot{q}_1 + \ddot{q}_2 \\
y_{23} &= \cos(q_2)\ddot{q}_1 + \sin(q_2)\dot{q}_1^2 & y_{24} &= 0 \\
y_{25} &= 0 & y_{26} &= g\cos(q_1 + q_2)
\end{aligned} \tag{36}$$

The each component of $Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$ in (7) is given as

$$\begin{aligned}
y_{11} &= \ddot{q}_{r1} & y_{12} &= \ddot{q}_{r1} + \ddot{q}_{r2} \\
y_{13} &= \cos(q_2)(2\ddot{q}_{r1} + \ddot{q}_{r2}) \\
&\quad - \sin(q_2)(\dot{q}_2\dot{q}_{r2} + 2\dot{q}_1\dot{q}_{r2}) & y_{14} &= g\cos(q_1) \\
y_{15} &= g\cos(q_1) & y_{16} &= g\cos(q_1 + q_2) \\
y_{21} &= 0 & y_{22} &= \ddot{q}_{r1} + \ddot{q}_{r2} \\
y_{23} &= \cos(q_2)\ddot{q}_{r1} + \sin(q_2)(\dot{q}_1\dot{q}_{r1}) & y_{24} &= 0 \\
y_{25} &= 0 & y_{26} &= g\cos(q_1 + q_2)
\end{aligned} \tag{37}$$

The unloaded manipulator parameters used in simulation are given in **Table 1**. Using the values from **Table 1** in (31) we can get θ_i as shown in **Table 2**.

We assume that due to unknown load carried by the robot as part of second link, the parameters m_2, l_{c2} and I_2 will change to $m_2 + \Delta m_2$, $l_{c2} + \Delta l_{c2}$ and $I_2 + \Delta I_2$, respectively. We will design an adaptive controller that will ensure the asymptotic stability in presence of parameter uncertainties in the intervals

$$0 \le \Delta m_2 \le 10; \ 0 \le \Delta l_{c2} \le 0.5; \ 0 \le \Delta I_2 \le \frac{15}{12}. \tag{38}$$

The nominal parameter vector θ_0 is shown in **Table 3**, is obtained after choosing the mean value for the range of possible θ_i. The uncertainty bound for each parameter ρ_i is shown in **Table 4**. The design steps of the proposed adaptive fuzzy controller are discussed below:

5.1. Controller Design

Step 1: The parameter vector Ψ_i, $i = 1, 2, \cdots, 6$, is the fuzzy controller consequent can be obtained according to fuzzy IF-THEN rules (13). Each premises variable ξ_i is described with help of five fuzzy terms NB, NS, Z, PS,

Table 1. Parameters of the unloaded arm.

m_1	m_2	l_1	l_2	l_{c1}	l_{c2}	I_1	I_2
10	5	1	1	0.5	0.5	10/12	5/12

Table 2. θ_i for the unloaded arm.

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
8.33	1.67	2.5	5	5	2.5

Table 3. Nominal parameter vector θ_0.

θ_{01}	θ_{02}	θ_{03}	θ_{04}	θ_{05}	θ_{06}
13.33	8.96	8.75	5	10	8.75

Table 4. Uncertainty bounds ρ_i.

ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
5	6.25	6.25	0	5	6.25

and PB. Therefore, it requires five rules to describe for each u_i, $i = 1, 2, \cdots, 6$. These fuzzy IF-THEN rules are as follows

1) IF ξ_i is NB THEN u_i is NB;
2) IF ξ_i is NS THEN u_i is NS;
3) IF ξ_i is Z THEN u_i is Z;
4) IF ξ_i is PS THEN u_i is PS;
5) IF ξ_i is PB THEN u_i is PB.

These fuzzy rules are generated using the control structure (12) of [2]. The membership functions for input and output variables are shown in **Figure 2**.

Table 5 shows the parameters of membership functions for input variables and **Table 6** shows the parameters of membership functions for output variables. Note that there is no uncertainty in parameter θ_4, *i.e.* $\rho_4 = 0$. Therefore, the component u_4 of u is zero.

Step 2: $\tilde{\Psi}_i$, $i = 1, 2, \cdots, 6$ are computed using the adaptive law (16). Now we can get the optimal parameter vector as $\Psi_{0i} = \Psi_i - \tilde{\Psi}_i$, $i = 1, 2, \cdots, 6$.

Step 3: The additional control input is obtained based on adaptive fuzzy control law (15), $u_i = \left(\tilde{\Psi}_i + \Psi_{oi} \right)^T W_i$, $i = 1, 2, \cdots, 6$.

Step 4: Now we will compute γ_i using (18) and maximum value of it chosen as $\gamma_{max} = \max(\gamma_i)$.

Step 5: The gain matrix K is now updated so as to meet the stability condition (17) $K > \lambda_{max} I$.

Step 6: The final control torque is obtained according to (6) $\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)(\theta_0 - u) - K\sigma$. Note that the proposed control algorithm updates the gain K of PD term unlike the previous algorithm presented in [2].
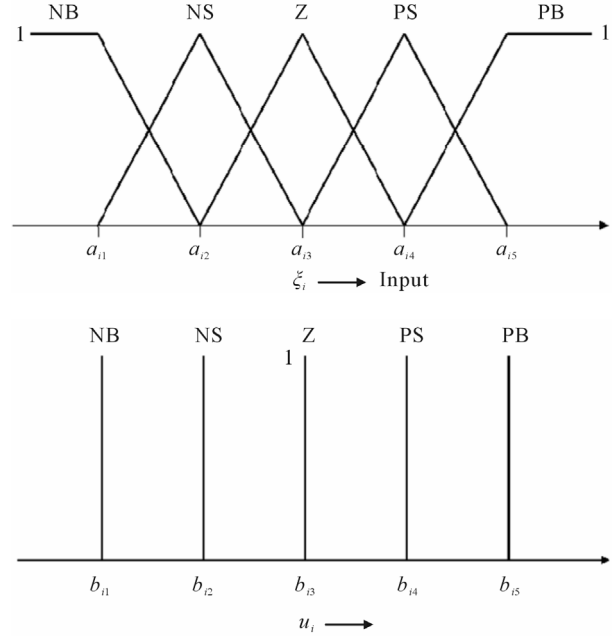
Figure 2. Membership functions for input and output variables.

Table 5. Parameters of membership functions for input variable.

	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}
ξ_1	−0.040	−0.020	0.000	0.020	0.040
ξ_2	−0.200	−0.100	0.000	0.100	0.200
ξ_3	−0.150	−0.075	0.000	0.075	0.150
ξ_5	−0.200	−0.100	0.000	0.100	0.200
ξ_6	−0.400	−0.200	0.000	0.200	0.400

Table 6. Parameters of membership functions for output variable.

	b_{i1}	b_{i2}	b_{i3}	b_{i4}	b_{i5}
u_1	−5.000	−2.500	0.000	2.500	5.000
u_2	−7.290	−3.645	0.000	3.645	7.290
u_3	−6.250	−3.125	0.000	3.125	6.250
u_5	−5.000	−2.500	0.000	2.500	5.000
u_6	−6.250	−3.125	0.000	3.125	6.250

5.2. Simulation Results

Desired trajectories for both the joints are given as

$$q_d = 1 - \cos(at), \quad \dot{q}_d = a\sin(at) \tag{39}$$

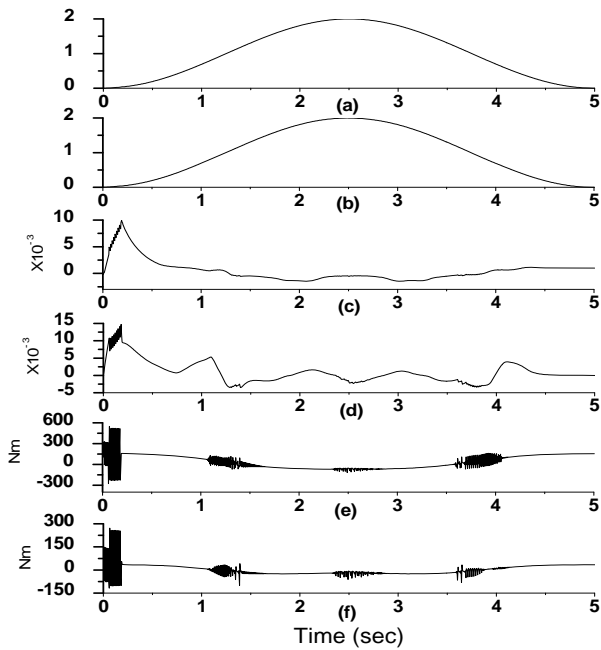where $a = 2\pi/5$ and simulation results are obtained with a sampling time T = 0.005 sec. **Figures 3-6** show

Figure 3. System response based on [2] using controller (12) with *ε* = 0.1.

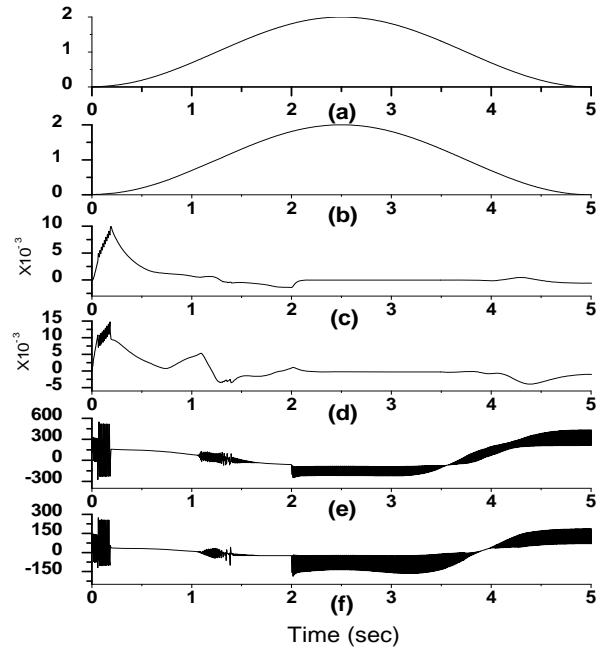**Figure 5. System response based on [2] using controller (12) with *ε* = 0.1 (Manipulator is loaded at time 2 sec).**
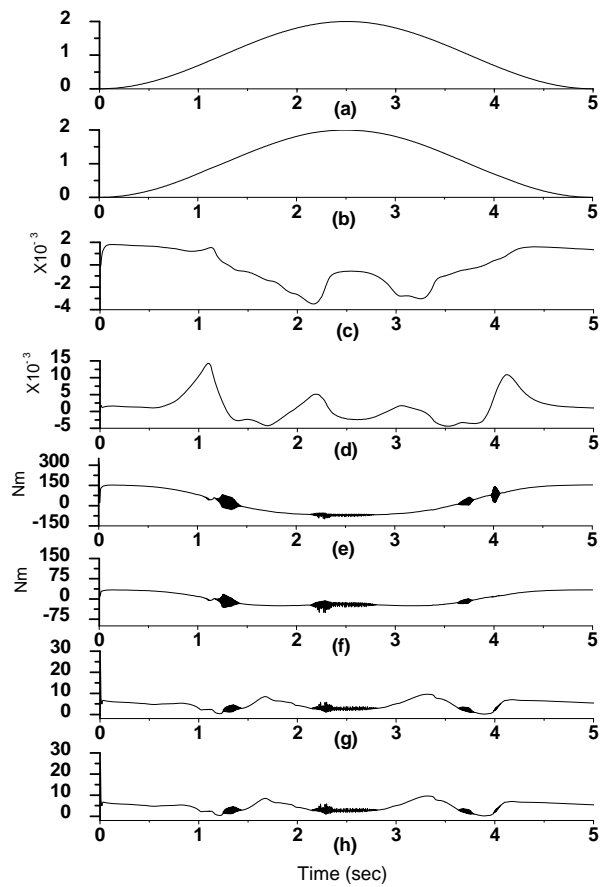

Figure 4. System response based on proposed adaptive fuzzy controller (15) (manipulator is unloaded).
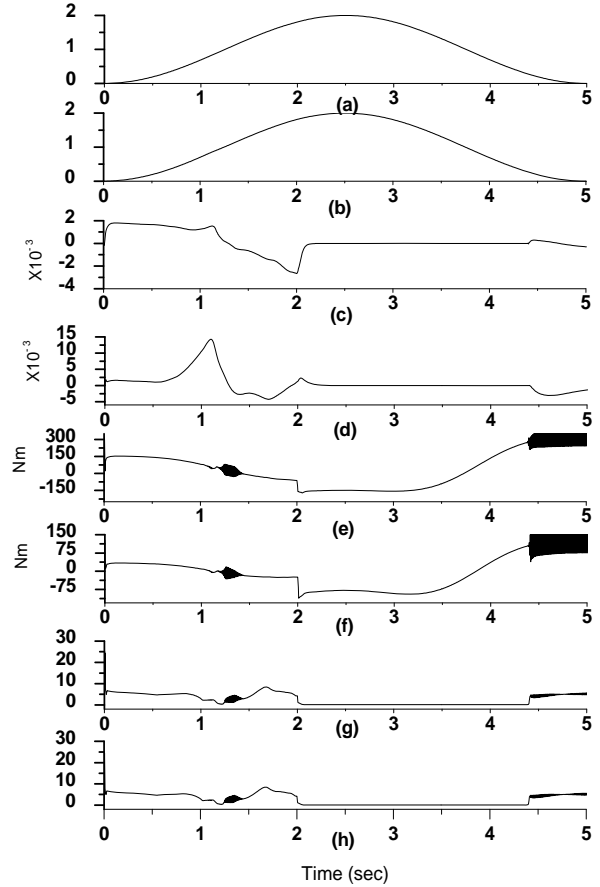

Figure 6. System response based on proposed adaptive fuzzy controller (15) (manipulator is loaded at time 2 sec).

the system response of a two-link robot manipulator based on nonlinear saturation controller [2] and proposed adaptive fuzzy controller (15) for two cases while the robot manipulator is (*i*) unloaded (ii) loaded. Results are presented in the following sequences in the figures: joint positions (joint-1 (q_1), joint-2 (q_2)), tracking errors (q_1 and q_2), and input torques (τ_1 and τ_2) respectively. It may be mentioned that the robot manipulator is loaded at time 2 sec. *i.e.* the second link lifts some mass due to which the parameter of the robot changes as given in (38).

The results of the proposed adaptive fuzzy controller are compared with that of [2] (see (12)) where fixed PD controller gains are considered as $K = \mathrm{diag}(75\ 50)$ and $\Lambda = \mathrm{diag}(40\ 15)$. In both the methods, it is observed that the tracking error responses remain almost the same order and insensitive irrespective of the payload variation. Further it has been observed through simulation studies that the proposed technique effectively alleviates or reduces the chattering effect in control signals. A significant chattering in the control signal is noticed while a robot arm takes a sharp turn under loaded condition. Simulation result shows that the proposed controller effectively reduces the magnitude of input torque or in other words effectively reduces the control effort compared to the method discussed in [2]. It may be further observed from the figures ((see **Figures 4(g)-(h)** and **Figures 6(g)-(h)**) how the diagonal elements of gain matrix K (K_{11}, K_{22}) of PD controller are updated adaptively.

6. Conclusions

In this paper, an adaptive fuzzy control law is proposed for trajectory tracking of robot manipulator with a view to reduce the chattering effect in torque control signal. The advantages of fuzzy and adaptive control strategies are combined and subsequently the stability condition of robot manipulator is derived based on Lyapunov theorem. The implementation of proposed controller is very straightforward due to the use of simple fuzzy rules and control strategies. The gain K of PD term is updated with time and hence proposed adaptive control scheme removes the disadvantage of using fixed large gain values in [2]. Simulation results show that the present con-

trol scheme is effective in reducing chattering in torque control signal and simultaneously control effort is less in comparison to the results in [2].

7. References

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