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A Note on Economic Order Quantity Model

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Abstract

Economic Order Quantity (*EOQ*) model has been extended. We formulate *EOQ* model as a calculus of variations. This new extended problem is a simple optimal control problem with an unknown initial state. By solving this problem we generalize *EOQ* formula [1].

Keywords

EOQ, Cost Minimization, Inventory Management

1. Introduction

The economic order quantity (*EOQ*) formula plays an important role in inventory management. *EOQ* model has had on a century of researchers and practitioners in the fields of operations management and operations research. *EOQ* model appeared in Harris (1913) describes a very simple deterministic inventory planning model with a tradeoff between fixed ordering cost and inventory carrying cost [1]. *EOQ* lays the foundation for all kinds of extensions and real world management applications [2] [3] [4] [5]. Both the deterministic and the stochastic *EOQ* models were developed in [1] [5] [6] [7] [8]. The Economic Order Quantity (*EOQ*) is the number of units that a company should add to inventory with each order to minimize the total costs of inventory—such as holding costs, and shortage costs. The *EOQ* is used as part of a continuous review inventory system in which the level of inventory monitored at all times and a fixed quantity is ordered each time the inventory level reaches a specific reorder point. The *EOQ* provides a model for calculating the appropriate reorder point and the optimal reorder quantity to ensure the instantaneous replenishment of inventory with no shortages. It can be a valuable tool for small business owners who need to make decisions about how much inventory to keep on hand, how many items to order each time, and how often to reorder to incur the lowest possible costs.

The *EOQ* model assumes that demand is constant, and that inventory is depleted at a fixed rate until it reaches zero. At that point, a specific number of items arrive to return the inventory to its beginning level. Since the model assumes instantaneous replenishment, there are no inventory shortages or associated costs. Therefore, the cost of inventory under the *EOQ* model involves a tradeoff between inventory holding costs (the cost of shortage, as well the cost of tying up capital in inventory rather than investing or using it other purposes) and order costs (any fees associated with placing orders, such as delivery charges). Ordering a large amount at one time will increase a small business's holding costs, while making more frequent orders of fewer items will reduce holding costs but increase order costs. The *EOQ* model finds the quantity that minimizes the sum of these costs.

The *EOQ* model computed by the following formula $EOQ = \sqrt{\frac{2Dc}{c_h}}$, where D is the demand per time unit, c is the ordering cost, c_h is the holding cost a per unit and time unit.

Research on the *EOQ* can be primarily classified into three areas of interest:

1) The performance of the *EOQ* against other lot-sizing rules: Lot size refers to the quantity of an item ordered for delivery on a specific date or manufactured in a single production run. Choi *et al.* [9] examined the *EOQ* versus eight other rules in multi-echelon MRP systems using FORTRAN, with the *EOQ* performing in the lower third. Evan L. Porteus [10] has introduced a model that shows a significant relationship between quality and lot size. Melnyk and Piper [11] examined the effects of lead time errors on different lot sizing rules.

2) Extensions of the *EOQ* model: *EOQ* extensions include an application to retail cycle stock inventories [12], the addition of cost changes under a finite or infinite time horizon [13], the inclusion of storage size considerations [6], and the addition of damage costs [5] [14] provided an exhaustive summary of the research on *EOQ* models that handle partial backordering, and some even more current models with partial or full backordering include those by [7] [15] [16] and [17].

3) The role of the *EOQ* in logistics: Research that addressed the use of *EOQ* models in transportation and logistics first appeared in the 1980s. Eppen [18] was the first to discuss the "impact of inventories" on locations by exploiting risk pooling effects. Tanchoco *et al.* [19] considered the impact of material handling and the transportation of unit loads on lot sizing. Ng *et al.* [20] studied a type of *EOQ* problem where the (maximum) warehouse capacity is a decision variable. Furthermore, they assumed that the warehouse cost dominates all the other inventory holding costs. Keskin *et al.* [21] study generalized vendor selection models aimed at optimizing the total logistical costs including not only the vendor-specific fixed management and purchasing costs considered in traditional models, but also the transportation, inventory replenishment, and holding costs.

In their models, the authors mainly used convex optimization as a mathematical tool. However, dynamic models for *EOQ* have been less considered. To fulfill this gape, we formulate in this paper *EOQ* model as a calculus of variations. The new problem is a simple optimal control which can be solved by Euler-Lagrange equation as optimality conditions. In particular, we obtain the well-known *EOQ* formula [1].

2. Calculus Variations Approach to *EOQ*

Assume that holding cost per unit depends on its rate and quantity of items.

In other words,

$$C = C(q, q', t), \tag{1}$$

where $C: R \times R \times R \to R, q: R \to R, q = q(t)$, C is the cost function at moment t per each inventory. And q is the quantity inventory.

Then total cost is

$$TC = \int_0^T C(q, q', t) q(t) \mathrm{d}t.$$

Economic order quantity problem can be formulated as a problem of calculus of variations with a free left side constraint q_0.

$$\begin{cases} \min \int_0^T F(q, q', t) \mathrm{d}t \\ q(0) = q_0 \\ q(T) = 0 \end{cases} \tag{2}$$

where, $q(t)$: order quantity function, q_0: economic order quantity, T: final time, $F(q, q', t) = C(q, q', t) q(t)$.

In order to solve problem (2), we need to solve Euler-Lagrange equation [22] as an optimality condition for the calculus of variations.

$$\frac{\partial F(q, q', t)}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial F(q, q', t)}{\partial q'}\right) = 0 \tag{3}$$

Let $q^* = q^*(c_1, c_2, t)$ be a solution to problem (3). The constants c_1 and c_2 can be found from the boundary conditions [15] solving a system of nonlinear equations.

$$\begin{cases} \dfrac{\partial F(q^*, q^{*'}, 0)}{\partial q^*} = 0 \\ q^*(T) = 0 \end{cases} \tag{4}$$

For example, if we have $F(q, q', t) = 12qt + qq' + q'^2, q(0) = q_0$ and $q(1) = 0$, then Euler-Lagrange equation is:

$$12t + q' - q' - 2q'' = 0.$$

The solution is $q(t, c_1, c_2) = t^3 + c_1 t + c_2$.

Clearly, $q(0) = c_2 = q_0$, and the system (4) gives

$$\begin{cases} \dfrac{\partial F}{\partial q'} = q + 2q' = \left(t^3 + c_1 t + c_2\right) + 2\left(3t^2 + c_1\right)\Big|_{t=0} = c_2 + 2c_1 \\ q(T) = T^3 + c_1 T + c_2 \end{cases}$$

Now we have condition (4) as

$$\begin{cases} c_2 + 2c_1 = 0 \\ T^3 + c_1 T + c_2 = 0 \end{cases}$$

Hence, we have $c_1 = \dfrac{T^3}{2-T}$ and $c_2 = \dfrac{2T^3}{T-2}$, consequently, $q_0 = \dfrac{2T^3}{T-2}$. Then optimal solution is

$$q^*(t) = t^3 + \frac{T^3}{2-T} t + \frac{2T^3}{T-2}:$$

In practice, usually type of order quantity function is given. For instance, $q(T) = bT + d$, and holding cost per unit is constant c_h. Then total cost minimum problem is:

$$\begin{cases} \min TC = \dfrac{Dc}{q_0} + \displaystyle\int_0^T (bt + d) c_h \mathrm{d}t \\ q(0) = d = q_0' \\ q(t) = bT + d = 0, \end{cases}$$

where, D is demand for the period T, and c ordering cost. b can be expressed as

$$b = -\frac{d}{T} = -\frac{q_0}{T}.$$

Then the cost function TC has the form.

$$\min TC = \frac{DC}{q_0} + \int_0^T \left(-\frac{q_0}{T} t + q_0\right) c_h \mathrm{d}t$$

or equivalently,

$$TC = \frac{Dc}{q_0} - \frac{q_0 c_h T}{2} + q_0 c_h T.$$

Taking derivative of TC, we have

$$\frac{\mathrm{d}TC}{\mathrm{d}q_0} = -\frac{DC}{q_0^2} - \frac{c_h T}{2} + q c_h T = 0.$$

Now we obtain

$$\frac{Dc}{q_0^2} = \frac{c_h T}{2}.$$

We find optimal order quantity q_0 as

$$q_0 = \sqrt{\frac{2Dc}{c_h T}}.$$

If we take $T = 1$, then we have well known economic order quantity formula (EOQ) in the literature [1].

$$q_0 = \sqrt{\frac{2Dc}{c_h}}.$$

3. Conclusion

An extended *EOQ* model has been considered for the first time as a simple optimal control problem with an unknown initial state. In particular, from the proposed model we obtain the well-known *EOQ* formula. The proposed approach allows to examine the order quantity function from a view point of dynamic system. That is why the stock can be controlled at each action time. The model was tested on a numerical example.

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