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Forecast of Power Generation for Grid-Connected Photovoltaic System Based on Grey Theory and Verification Model

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ABSTRACT

Being photovoltaic power generation affected by radiation strength, wind speed, clouds cover and environment temperature, the generating in each moment is fluctuating. The operational characteristics of grid-connected PV systems are coincided with gray theory application conditions. A gray theory model has been applied in short-term forecast of grid-connected photovoltaic system. The verification model of the probability of small error will help to check the accuracy of the gray forecast results. The calculated result shows that the $GM(1,1)$ model accuracy has been greatly enhanced.

Keywords: Forecast of Power Generation; Grid-connected Photovoltaic System; Data Discretization; Greedy Algorithm; Continuous Attributes; Rough Sets

1. Introduction

The grid-connected PV power generation has become the trend of the development of solar photovoltaic applications. But its intermittent and randomness will cause the grid scheduling difficulties and bring some adverse effects of the grid. So the forecast of PV power generation will help the grid-connected PV capacity within the control range and minimize the adverse effects, as in [1,2].

The gray system theory is satisfied to the less data and uncertainty problem. It is consisted of a few basic part, they are gray system analysis, gray model, gray forecast, gray decisions, gray control and gray optimization techniques. The gray forecast establishes differential equations using gray numbers that is *GM* model (gray model) $GM(1,N)$ means that the first-order differential equations are composed of N variables. Commonly, used in load forecasting model, the essence is accumulated and generated by the original sequence in order to find some certain laws in generated sequences, to fit a typical curve and to establish the mathematical model, as in [3-5].

Known the photovoltaic power generation can be regarded as white elements and unknown generation as black elements. Therefore, a part of the information is clear, the other is unknown, and that is a typical gray system. Daily generation capacity of PV system shows normal distribution. Limited by maximum solar radiation values, the trend of generation is monotonous and graded. So this kind of incomplete information system is just suitable for the object of gray forecast, as in [6,7].

2. Gray Theory Model

With original non-negative sample sequence

$$x^{(0)} = \left\{ x^{(0)}(1), x^{(0)}(2), x^{(0)}(k) \right\},$$

the gray system theory uses a unique the data preprocessing way , in which one order sequence $x^{(0)}(k)$ is accumulated.

$$x^{(1)}(k) = \sum_{m=1}^{k} x^{(0)}(m), k(k = 1 \cdots n)$$

Then it can generate the series.

$$x^{(1)}(k) = \left\{ x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(k) \right\}$$

To establish the first-order linear differential equations about $x^{(1)}(k)$.

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u$$

Using the least-squares method solves the parameters

a, u.

$$\hat{\boldsymbol{A}} = \begin{bmatrix} a \\ u \end{bmatrix} = (\boldsymbol{B}^T \boldsymbol{B})^{-1}(\boldsymbol{B}^T \boldsymbol{Y_N})$$

$$\boldsymbol{B} = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n-1) + x^{(1)}(n)) & 1 \end{bmatrix}$$

$$\boldsymbol{Y_N} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

where, a is the development parameter for reflecting the development trends of sequence $x^{(1)}$, u is the coordination parameter for reflecting the change relation of data.

The gray forecast $GM(1,1)$ model about $x^{(1)}$ is

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{u}{a} \right] e^{-ak} + \frac{u}{a}$$
$$k = 0, 1, 2, \cdots\cdots, n-1$$

The actual forecast result $\hat{x}^{(0)}$ is

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$$
$$\hat{x}^{(0)}(1) = x^{(0)}(1)$$
$$k = 1, 2, \cdots, n-1$$

3. Verification Model

The generation forecast is based on the original input data to establish the forecast model, to identify the parameters of the model and to predict. Forecast error range is not only concerned about the forecast results, but also concerned about the forecast results. In the theory of gray forecast, the verification indicators of the model effect are the ratio of posterior error, the probability of small error and the degree of association.

$x^{(0)}(k)$, $k(k = 1 \cdots n)$ is the original sequence. The gray forecast data sequence is $\hat{x}^{(0)}(k), k = 1, 2, \cdots, n$. The process of verification model is shown as follow.

Average of the original sequence is

$$\overline{x}^{(0)} = \frac{1}{n}\sum_{k=1}^{n} x^{(0)}(k)$$

Point wise residuals is

$$\varepsilon(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)$$
$$k = 1, 2, \cdots, n$$

Average residuals is

$$\overline{\varepsilon} = \frac{1}{n}\sum_{k=1}^{n} \left| \varepsilon(k) \right|$$

Average variance of original data is

$$s_1 = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left[x^{(0)}(k) - \overline{x}^{(0)} \right]^2} = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left[u(k) \right]^2}$$

Average variance of residuals is

$$s_2 = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left[\varepsilon(k) - \overline{\varepsilon} \right]^2}$$

Ratio of posterior error is

$$c = \frac{s_2}{s_1}$$

Probability of small error is

$$p = P\left\{ \left| \varepsilon(k) - \overline{\varepsilon} \right| < 0.6745 \cdot s_1 \right\}$$

Point wise association coefficient is

$$\xi(k) = \frac{\varepsilon_{\min} + \rho \cdot \varepsilon_{\max}}{\left| \varepsilon(k) \right| + \rho \cdot \varepsilon_{\max}},\ 0 < \rho < 1$$

Degree of association is

$$\xi = \frac{1}{n}\sum_{k=1}^{n} \xi(k)$$

The gray system error decision analysis is shown in **Table 1**.

The solution of gray forecast model $GM(1,1)$ is the exponential curve. The geometry of forecast value is a smooth curve, which is either monotone increasing or monotone decreasing. Although the daily power generation of PV systems trends the exponentially graded, the randomness of model data $GM(1,1)$ has been weakened in accumulating processing. But influenced by the forecast trends of original sequence,the randomness and volatility of data sequence fit ineffective.

Being photovoltaic power generation affected by radiation strength, wind speed, clouds cover and environment temperature, the generating in each moment is fluctuating. So only using the gray forecast model is difficult to achieve forecast accuracy. The forecast model based on advanced gray theory is necessary.

Table 1. Grade of forcats accuracy.

Grade	Probability of small error p	Ratio of posterior error c	Evaluate
1	$[0.95, 1.00]$	$[0.00, 0.35)$	excellent
2	$[0.80, 0.95)$	$[0.35, 0.50)$	good
3	$[0.70, 0.80)$	$[0.50, 0.65)$	pass
4	$[0.00, 0.70)$	$[0.65, 1.00]$	fail

Forecast steps of advanced gray model $GM(1,1)$ are given as follow.

Firstly, it is calculated relative deviation between the forecast value $\hat{x}^{(0)}(k)$ and actual value $x^{(0)}(k)$ during the former moment *n*.

$$\bar{\Delta}(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}, k = 1, 2, \cdots, n$$

Secondly, above relative error will be divided into *i* states., expressed as $s_1, s_2, \cdots, s_i$

$$s_i = [\theta_{1i}, \theta_{2i}], i = 1, 2 \cdots$$
$$\theta_{1i}, \theta_{2i} \in [\bar{\Delta}(k)_{\min}, \bar{\Delta}(k)_{\max}]$$

If $\hat{x}^{(0)}(n+1), \hat{x}^{(0)}(n+2), \cdots, \hat{x}^{(0)}(n+k)$ is the gray forecast value at moment $(n+1), (n+2), \cdots, (n+k)$.

$$\theta_{1i} = \frac{x_{1i} - \hat{x}^{(0)}(n)}{\hat{x}^{(0)}(n)}, \theta_{2i} = \frac{x_{2i} - \hat{x}^{(0)}(n)}{\hat{x}^{(0)}(n)}$$
$$i = 1, 2, \cdots, n = n+1, \cdots, n+k$$

$[x_{1i}, x_{2i}]$ is the gray forecast range. According to the biggest relative deviation state probability and the gray forecast range $[x_{1i}, x_{2i}]$, gray forecast results can be modified to increase the forecast accuracy.

4. Calculation Example

The actual operation data of one day is used as example in this paper, which is 15 minutes interval.

With photovoltaic generating presented synchronization trends with the sun radiation, two $GM(1,1)$ models are established respectively applying the data from 8:00 to 12:00 and 12:00 to 15:00.

The original non-negative sample sequence form 8:00 to 11:15 is $x_1^{(0)}$

$$x_1^{(0)} = \{0.2305, 0.3197, 0.3308, 0.3714, 0.4443, 0.4868, 0.5383, 0.5310, 0.6025, 0.6528, 0.6828, 0.7055, 0.6704, 0.7140\}$$

The original non-negative sample sequence form 12: 15 to 14:15 is $x_2^{(0)}$

$$x_2^{(0)} = \{0.7338, 0.7333, 0.7079, 0.6938, 0.6463, 0.5901, 0.5518, 0.5358, 0.4840\}$$

The sequences of acculturational data is

$$x_1^{(1)} = \{0.2305, 0.5502, 0.8810, 1.2524, 1.6966, 2.1833, 2.7216, 3.2526, 3.8551, 4.5079, 5.1906, 5.8961, 6.5665, 7.2805\}$$

$$x_2^{(1)} = \{0.7338, 1.4670, 2.1749, 2.8687, 3.5149, 4.1050, 4.6567, 5.1925, 5.6765\}$$

Using the least-squares method to solve parameters a, u, there are

$$\hat{\boldsymbol{A}}_1 = \begin{bmatrix} a_1 \\ u_1 \end{bmatrix} = (\boldsymbol{B}_1^T \boldsymbol{B}_1)^{-1} (\boldsymbol{B}_1^T \boldsymbol{Y}_{N1}) = \begin{bmatrix} -0.0630 \\ 0.3374 \end{bmatrix}$$

$$\hat{\boldsymbol{A}}_2 = \begin{bmatrix} a_2 \\ u_2 \end{bmatrix} = (\boldsymbol{B}_2^T \boldsymbol{B}_2)^{-1} (\boldsymbol{B}_2^T \boldsymbol{Y}_{N2}) = \begin{bmatrix} 0.0589 \\ 0.8180 \end{bmatrix}$$

Two gray forecast $GM(1,1)$ models are

$$\text{model 1}: \frac{dx_1^{(1)}}{dt} - 0.0630 x_1^{(1)} = 0.3374$$

$$\text{model 2}: \frac{dx_2^{(1)}}{dt} + 0.0589 x_2^{(1)} = 0.8180$$

As the actual situation, the relative deviation range of model 1 and model 2 are divided into four states and two states respectively, as shown following

$$\text{model 1:} \quad \begin{aligned} s_1 &= [-20\%, -5\%] \\ s_2 &= (-5\%, 0\%] \\ s_3 &= (0\%, 5\%] \\ s_4 &= (5\%, 10\%] \end{aligned}$$

$$\text{model 2:} \quad \begin{aligned} s_1 &= [-10\%, 0\%] \\ s_2 &= (0\%, 10\%] \end{aligned}$$

Table 2 and **Table 3** are $GM(1,1)$ forecast results.

Table 2. Frecast result fpom 8:00 to 12:00.

Time	Actual Value(kWh)	Forecast value(kWh)	Relative deviation $\bar{\Delta}$(%)	State
8:00	0.2305	0.2305	0.0000	S_2
8:15	0.3197	0.3633	-13.6234	S_1
8:30	0.3308	0.3869	-16.9634	S_1
8:45	0.3714	0.4120	-10.9295	S_1
9:00	0.4443	0.4388	1.2355	S_3
9:15	0.4868	0.4673	4.0022	S_3
9:30	0.5383	0.4976	7.5467	S_4
9:45	0.5310	0.5300	0.1952	S_3
10:00	0.6025	0.5644	6.3241	S_4
10:15	0.6528	0.6011	7.9175	S_4
10:30	0.6828	0.6401	6.2434	S_4
10:45	0.7055	0.6817	3.3714	S_3
11:00	0.6704	0.7260	-8.2948	S_1
11:15	0.7140	0.7732	-8.2886	S_1
11:30	0.7343	0.8234	-12.1440	
11:45	0.7634	0.8769	-14.8702	
12:00	0.7885	0.9339	-18.4396	

Table 3. Frecast result from 12:15 to 15:00.

Time	Actual Value(kWh)	Forecast value(kWh)	Relative deviation $\overline{\Delta}$(%)	State
12:15	0.7338	0.7338	0.0000	S_1
12:30	0.7333	0.7524	-2.6160	S_1
12:45	0.7079	0.7094	-0.2095	S_1
13:00	0.6938	0.6688	3.5969	S_2
13:15	0.6463	0.6305	2.4321	S_2
13:30	0.5901	0.5945	-0.7385	S_1
13:45	0.5518	0.5604	-1.5763	S_1
14:00	0.5358	0.5284	1.3752	S_2
14:15	0.4840	0.4982	-2.9239	S_1
14:30	0.4460	0.4697	-5.3029	
14:45	0.4186	0.4428	-5.7766	
15:00	0.3914	0.4174	-6.6551	

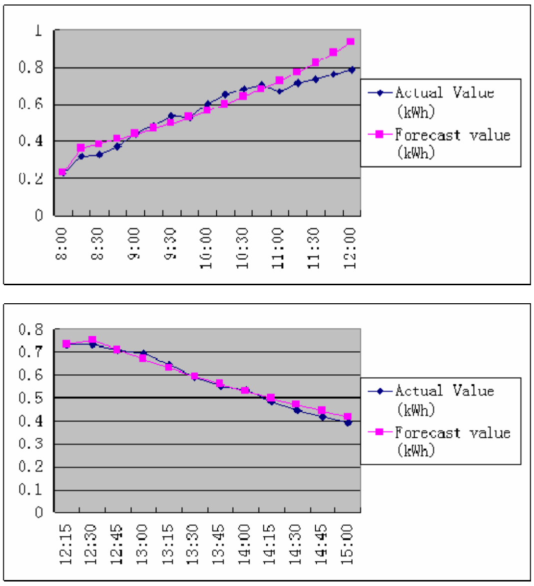

Figure 1. *GM*(1,1) forecast results.

Figure 1 is $GM(1,1)$ forecast results.

According to the verification model, the average of the original sequence is

$$\overline{x}^{(0)} = \frac{1}{3}(0.7343 + 0.7634 + 0.7885) = 0.7621$$

$$\varepsilon(1) = 0.8234 - 0.7343 = 0.0891$$

$$\varepsilon(2) = 0.8769 - 0.7634 = 0.1135$$

$$\varepsilon(3) = 0.9339 - 0.7885 = 0.1454$$

$$\overline{\varepsilon} = \frac{1}{3}(0.0891 + 0.1135 + 0.1454) = 0.1160$$

$$s_1 = \sqrt{\frac{1}{3}\sum_{k=1}^{3}\left[x^{(0)}(k) - \overline{x}^{(0)}\right]^2}$$

$$= \sqrt{\frac{1}{3}\left[\begin{array}{l}(0.7343 - 0.7621)^2 + (0.7634 - 0.7621)^2 \\ +(0.7785 - 0.7621)^2\end{array}\right]}$$

$$= 0.0153$$

$$s_2 = \sqrt{\frac{1}{3}\sum_{k=1}^{3}\left[\varepsilon(k) - \overline{\varepsilon}\right]^2}$$

$$= \sqrt{\frac{1}{3}\left[\begin{array}{l}(0.0891 - 0.1160)^2 + (0.1135 - 0.1160)^2 \\ +(0.1454 - 0.1160)^2\end{array}\right]}$$

$$= 0.0231$$

$$c = \frac{s_2}{s_1} = 0.0231/0.0153 = 1.5098$$

Probability of small error is

$$\left|\varepsilon(1) - \overline{\varepsilon}\right| = \left|0.0891 - 0.1160\right|$$
$$= 0.0269 > 0.6745 \times 0.0153 = 0.0103$$

So the error can be concluded that the model does not meet the requirements, that is to say, the other method must be used in amend the gray model. Otherwise, the gray forecast model is correct.

5. Conclusions

Photovoltaic (PV) power generation is a kind of green renewable energy. As distributed generation, it can not only complement energy to power system, but also enhance the reliability of power supply. But the randomness of photovoltaic power generation impacts the stability of power system and the actual load demand directly. Therefore, the accurate forecast of power generation in grid-connected photovoltaic system is extremely important, and it will be useful to the planning and operation of whole system, so do well to load demand management.

Operational characteristics of grid-connected PV systems are coincided with gray theory application conditions. In accordance with the basic principles of gray theory, the greater of the data and the more comprehensive of information, the more accurate is forecast results.

After leading into the verification model, the probability of small error can suggest the accuracy of the gray forecast results. Once it exceeds the precision of allowed values, the gray forecast model must combined with other advanced method, that is the combination model to ensure the forecast result, as in [8-11].

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