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Finite Element Analysis of Von-Mises Stress Distribution in a Spherical Shell of Liquified Natural Gas (LNG) Pressure Vessels

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Abstract

This research work investigated the modeling of Von Mises stress in LNG Spherical Carbon Steel Storage tank using assumed displacement Finite Element analysis based on shallow shell triangular elements. Using equations of elasticity, constant thickness carbon steel spherical storage tanks were subjected to different loading conditions. This paper stresses the need for proper definition of shallow element using sector angles to obtain the shallowness. The shallow spherical triangular element has five degrees of freedom at each of its corner node, which are the essential external degrees of freedom. The assumed displacement fields of these shallow triangular elements satisfied the exact requirement of rigid body modes of motion. The FORTRAN 90 programming language was used for the programme coding to solve finite element equations resulting from the model while Von Mises stresses distribution within the spherical storage tank shell subjected to different internal pressures were determined. The results showed that the use of non-shallow elements due to improper sector angles resulted in unreliable results while real shallow elements produced results that tallied with ASME Section VIII Div 1, Part UG values.

Keywords: Von Mises Stress, FE Modeling, LNG, Spherical Storage Tank, ASME, ASCE

1. Introduction

Finite element modeling of pressure vessels response to stress analyses is a much inviting option than performing physical model analyses demonstrated by the amount of published work in the area. A large above-ground full containment LNG storage tank was modeled to check against local earthquake loading properties [1] while Korea Gas Corporation (KOGAS) developed the world's largest above-ground full containment LNG storage tank with a gross capacity of 200,000 m^3 [2]. The stability of cylindrical above-ground steel tanks under imposed support settlements and wind pressures have also been done [3]. In another study, the FEM code, MARC, was used to simulate the hydrobulging process of a single-curvature polyhedron, including loading and offloading conditions [4]. Pascal Pourcel and others worked on the seismic post elastic behavior of an existing equipment with a volume of 1000 m^3 containing 85% of LPG [5]. In another study, the seismic response of a cylindrical steel liquid storage tank was examined by using a coupling method that combined the finite elements and the boundary elements [6]. However, in all cases of modeling, it is of utmost importance to select elements that would aptly describe the shape of the vessel for the element meshing and interpolation functions. Inapt defining of the element shape could be very disastrous in critical equipment modeling and design for applications. This paper looks at the critical area of using shallow shell element for finite element modeling of LNG spherical tanks.

2. Finite Element Theory

2.1. General

Assumed displacement method was employed in this research work to develop a shallow triangular spherical shell element without an in-plane rotation as a sixth degree of freedom.

Assumptions

- Uniform Pressure is assumed for the LNG storage tank.

- External wind load, seismic loads, tremors or earthquakes are not considered here.

2.2. Displacement Functions

The accuracy which may be obtained by the finite element method depends directly on the accuracy with which the deformation patterns are selected. The assumed deformation patterns should reproduce the distortions actually developed within the element. If deformation patterns are not properly chosen, the deformations will not necessarily converge to correct values when the mesh size is decreased. On the other hand, very good results may be obtained with a very coarse mesh if the element deformation patterns selected closely correspond to the actual patterns. Thus, the most critical factor in the entire finite element analysis is the proper selection of the element displacement field. To fulfill the conditions of the principle of minimum potential energy, the interpolation functions must be such that the displacements along the inter-element boundaries are compatible.

The assumed displacement relationships for the triangular shallow shell (**Figure 1**) are expressed in curvilinear coordinates. Polynomial displacement function is assumed; the polynomial being of the highest order that will permit evaluation of the coefficients. Since displacements *u* and *v* are known at three points, nodes 1, 2 and 3, the highest-order expressions which can be assumed for *u* and *v* are

$$u(x,y)=a_1+a_2x+a_3y \quad (1)$$

$$v(x,y)=a_4+a_5x+a_6y \quad (2)$$

The displacement w with its derivates, θ_x, θ_y has nine known values; hence, it may be assumed that

$$\begin{aligned} w(x,y)&=a_7+a_8x+a_9y+a_{10}x^2+a_{11}xy+a_{12}y^2\\ &+a_{13}x^3+a_{14}xy^2+a_{15}y^3 \end{aligned} \quad (3)$$

from which it follows that

$$\theta_x(x,y)=\frac{\partial w}{\partial y}=a_9+a_{11}x+2a_{12}y+2a_{14}xy+3a_{15}y^2 \quad (4)$$

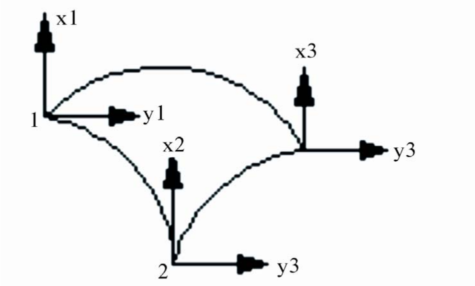

Figure 1. Shallow triangular element.

$$\theta_y(x,y)=-\frac{\partial w}{\partial x}=-a_8-2a_{10}x-a_{11}y-3a_{13}x^2+2a_{14}y^2 \quad (5)$$

to determine constants as, known displacements at nodes are substituted and the equations become

$$[a]=\left[A^{-1}\right][\delta] \quad (6)$$

where [δ] is the nodal degrees of freedom, $[A^{-1}]$ is inverse of transformation matrix and [a] is vector of independent constants.

2.3. Strain Displacement Relationship

The strain-displacement relationships for thin shells as given by Reissner [7] are simplified for the shallow shell and expressed as follows in curvilinear coordinates.

$$\begin{aligned} &\varepsilon_x=\frac{\partial u}{\partial x}+\frac{w}{r},\varepsilon_y=\frac{\partial v}{\partial y}+\frac{w}{r},\varepsilon_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x},\\ &k_x=-\frac{\partial^2 w}{\partial x^2},k_x=-\frac{\partial^2 w}{\partial y^2} \end{aligned} \quad (7)$$

The strain equations can be written in matrix form after necessary substitutions of *u*, *v* and *w* into the above strain equations.

2.4. Stress in a Triangular Element

Stress varies from point to point along the shell profile and also through the thickness of the shell. It is thus in reality an unknown function of two variables, which leads us to the equations below:

$$\sigma_b=\frac{6M}{t^2},\quad \sigma_m=\frac{N}{t} \quad (8)$$

where: *M* is the moment per unit length, *M* and σ_b is the bending stress at the surface.

N is to be force per unit length and σ_m which is membrane stress.

2.5. Strain Energy

The strain energy of an isotropic linear shell is given by Langhaar [8] as;

$$U=\iint_A\int_{-\frac{t}{2}}^{\frac{t}{2}}\frac{E}{2\left(1-v^2\right)}\left[\varepsilon_x^2+\varepsilon_y^2+2v\varepsilon_x\varepsilon_y+\frac{1}{2}(1-v)\gamma_{xy}^2\right]\mathrm{d}\varsigma\mathrm{d}x\mathrm{d}y \quad (9)$$

where, *t* = thickness of the shell, *v* = Poisson's ratio and *E* = Modulus of elasticity.

After substitution for strains in the above expression and integration with respect to ς, the strain energy can be separated into the membrane energy U_m and the bending energy U_b.

$$U = U_m + U_b \tag{10}$$

$$U_m = \frac{Et}{2(1-v^2)} \iint_A \left[e_x^2 + e_y^2 + 2ve_x e_y + \frac{1}{2}(1-v) e_{xy}^2 \right] \mathrm{d}x\mathrm{d}y \tag{11}$$

$$U_b = \frac{Et^3}{24(1-v^2)} \iint_A \left[k_x^2 + k_y^2 + 2vk_x k_y + \frac{1}{2}(1-v) k_{xy}^2 \right] \mathrm{d}x\mathrm{d}y \tag{12}$$

The potential energy is then written as:

$$\Phi = U - W$$

where W represents the work done by the external load on the system. In the finite element method, the potential energy of a shell is expressed as:

$$\Phi = \sum_{k=1}^{n} \phi_k \tag{13}$$

where ϕ_k is the potential energy of the k^{th} element.

2.6. Stiffness Matrix

By writing strain energy equations in terms of displacements, element stiffness matrix can be determined in the usual manner,

$$k_m = t\left[A^{-1}\right]^T \iint_A B_m^T D_m B \mathrm{d}x\mathrm{d}y \left[A^{-1}\right] \tag{14}$$

$$k_b = t\left[A^{-1}\right]^T \iint_A B_b^T D_b B \mathrm{d}x\mathrm{d}y \left[A^{-1}\right] \tag{15}$$

k_m and k_b are element stiffness matrices due to membrane and bending stresses respectively.

D_m and D_b are elasticity matrices for membrane and bending stresses respectively.

B_m and B_b are strain matrices for membrane and bending stresses respectively.

Therefore, element total stiffness matrix is

$$k = k_b + k_m \tag{16}$$

Element stiffness matrix is then combined to give system stiffness matrix.

2.7. Consistent Load Vector

The simplest method to establish an equivalent set of nodal forces is the lumping process. An alternative and more accurate approach for dealing with distributed loads is the use of a consistent load vector which is derived by equating the work done by the distributed load through the displacement of the element to the work done by the nodal generalized loads through the nodal displacements. If a triangular shell element is acted upon by a distributed load q per unit area in the direction of w, the work done by this load is derived as follows:

$$P_1 = \int_A qw \mathrm{d}x\mathrm{d}y \tag{17}$$

If w is taken to be represented by:

$$\{w\} = \left[C^T\right]\{a\} = \left[C^T\right]\left[A^{-1}\right]\{\delta\} \tag{18}$$

where $[C^T]$ for the present element is given by

$$C^T = \left[0\ 0\ 0\ 0\ 0\ 0\ 1\ x\ y\ x^2\ xy\ y^2\ x^3\ xy^2\ y^3\right] \tag{19}$$

The work done by the consistent nodal generalized force through the nodal displacements $\{\delta\}$ is given by:

$$P_2 = \left\{F^T\right\}\{\delta\} \tag{20}$$

Hence, from Equations (18)-(20), the nodal forces are obtained

$$F = \left[A^{-1}\right]\int\left[C^T\right][q]\mathrm{d}x\mathrm{d}y \tag{21}$$

Equation (21) gives the nodal forces for a single element; and the nodal forces for the whole structure is obtained by assembling the elements' nodal forces.

2.8. Boundary Conditions

In order to reduce computing time, symmetrical nature of the storage tank was considered. Shown below is the typical shallow spherical triangular shell mesh for finite element modeling of LNG spherical storage tanks. This mesh in the **Figure 2** below has six shallow spherical triangular shell elements with eight nodes.

Each node has five degree of freedoms; therefore the mesh in **Figure 2** has forty degrees of freedom. In considering known displacements, all displacements at given node were given zero values with the exception of radial displacements, w.

2.9. Sector Angles

It can be taken that there are many lines of symmetry in

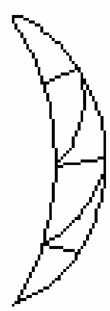

Figure 2. Typical spherical shell mesh.

the spherical tank with uniform internal/external pressure loading. For the purpose of the finite element analysis, it is better to take advantage of this symmetry by considering portions of the spherical tank instead of the whole tank. Thus, if we used a quarter of the spherical tank to analyze the entire system, to know the size of this quarter of the spherical tank in terms of angle size, we take 1/4 of 360˚—which is equal to 90˚. The calculation of angle size of symmetrical portion of the spherical tank used for finite element analysis with respect to the whole tank is what we termed "sector angle".

3. Problem Considered

The direct stresses, N_x, N_y and the bending moment, M_x and M_y were computed along x and y axes respectively. Maximum Von Mises stresses are determined. The spherecal storage tank considered has the following properties:

Diameter = 40 in
Thickness = 1 in
Young Modulus of Elasticity = $30*10^6$ Psi
Poisson Ratio = 0.3
Shell Material = A516M Grade 70
Shell Material Minimum Yield Stress = $38*10^3$ psi

Results and Discussions

The maximum Von-Mises stresses variations with internal pressures are given in **Figure 3**. Here, the shallow shell element used in modeling was well described at a sector angle of 90 degrees. Thus, the Finite element values showed close tally with ASME Section VIII Div 1, Part UG values. Equivalent maximum Von mises stresses were well within the limits of shell material minimum yield stress for internal pressure between 500 psi and 3800 psi. For internal pressure above 3800 Psi, the equivalent maximum Von mises yield stresses exceeded the shell material minimum yield stress.

To prevent failure of LNG storage tank at pressures above 3800 psi, one of the following has to be done.

- The shell thickness has to be increased accordingly so that equivalent maximum Von mises stress will be lower than the shell material minimum specified yield stress.
- Select shell material with higher minimum yield value so that the equivalent maximum Von mises stress calculated will be less than the shell material minimum specified yield stress.
- Decrease the working internal pressure of the spherical storage tank so that the equivalent maximum Von mises stress will fall below the shell material minimum specified yield stress.

Figure 4 shows the result when an element does not describe well the structure being modeled. Using a sector angle of 25 degrees, the shell element was not shallow but curved. This showed a deviation from ASME, s value by about 20 percent. The figure also shows that increasing the number of elements (*i.e.* making the element smaller) will not solve the problem of a bad choice of element configuration. The values kept diverging away from ASME standard values.

Figure 5 shows the full divergence of the modeled values from standard ASME results using improper element. The implication is that the designer will use thicker materials for designs that ought not to be so bulky. On other occasions when the divergence is negative (*i.e.* lower values than ASME values), then the designer could cause a catastrophic failure of the storage tank when operated at pressures than it can withstand (see **Figure 6**).

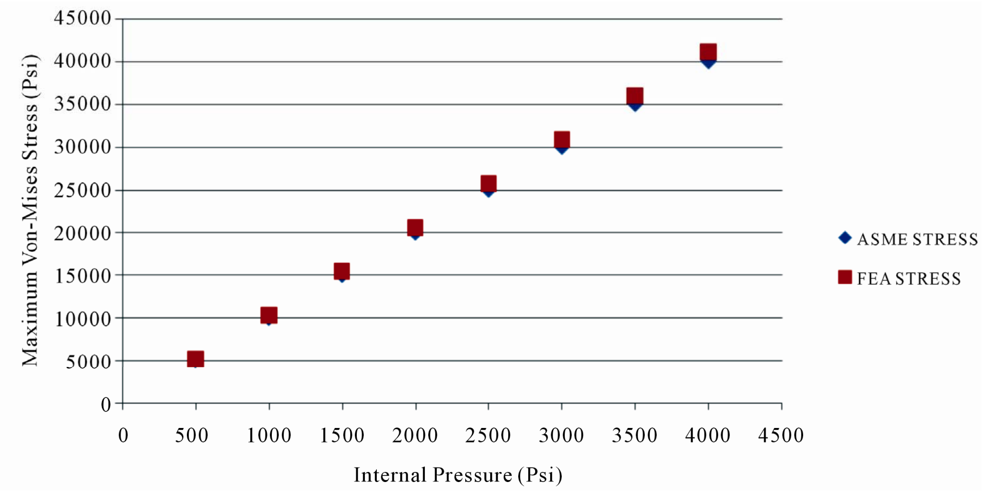

Figure 3. Graph of Internal Pressure (Psi) against Maximum Von-Mises Stress (Psi) for Spherical vessel Dia. of 40 inches and Sector Angle of 90 Degrees producing a shallow shell element.

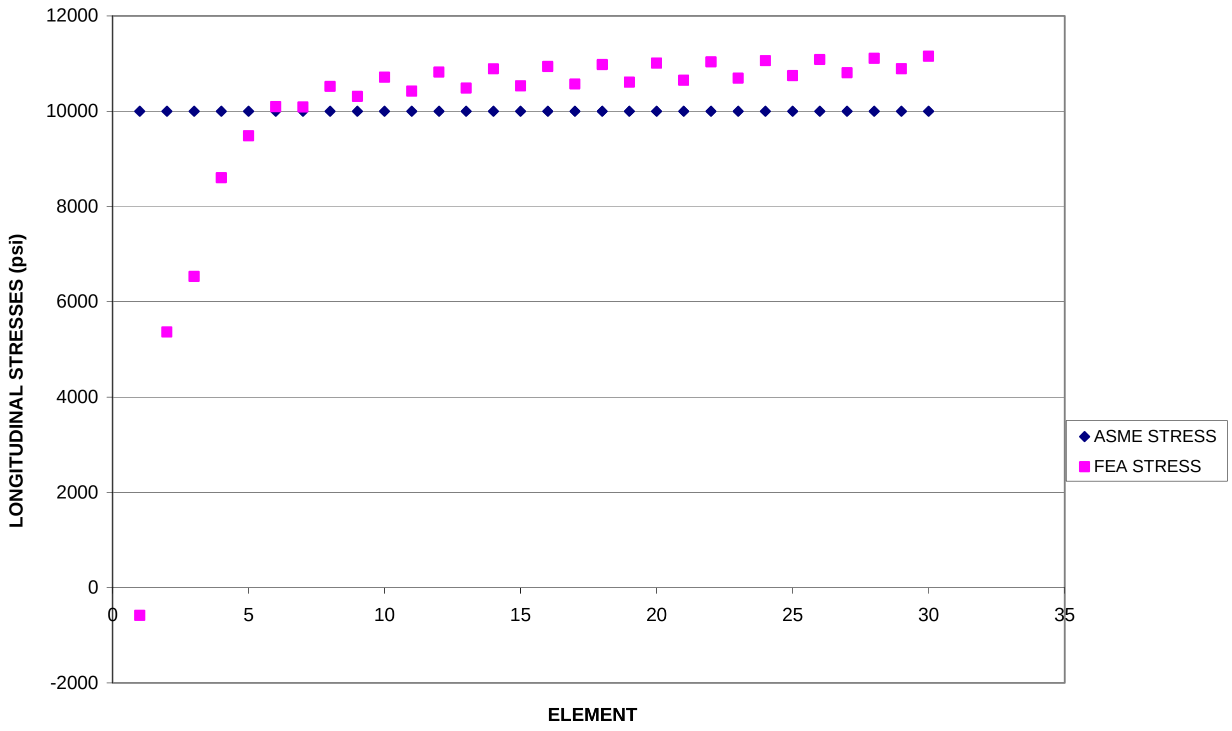

Figure 4. Plot of number of mesh elements against longitudinal stresses (Psi) using a sector angle of 25 for a 40 in Dia. Vessel and internal pressure of 1000 Psi (sector angle here produced a curved element and not a shallow shell element).

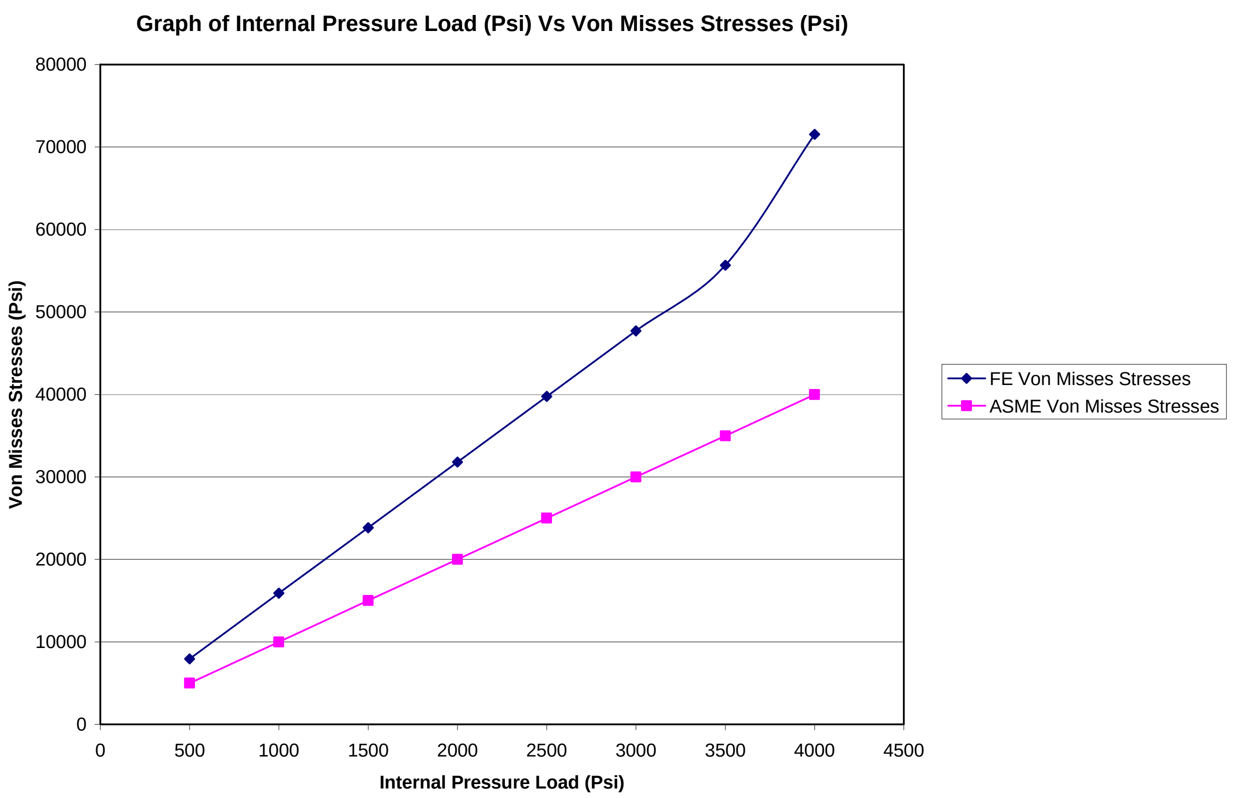

Figure 5. Internal Pressure Loading against Von Mises Stresses using a sector angle of 25 degrees and hence curved shell element.

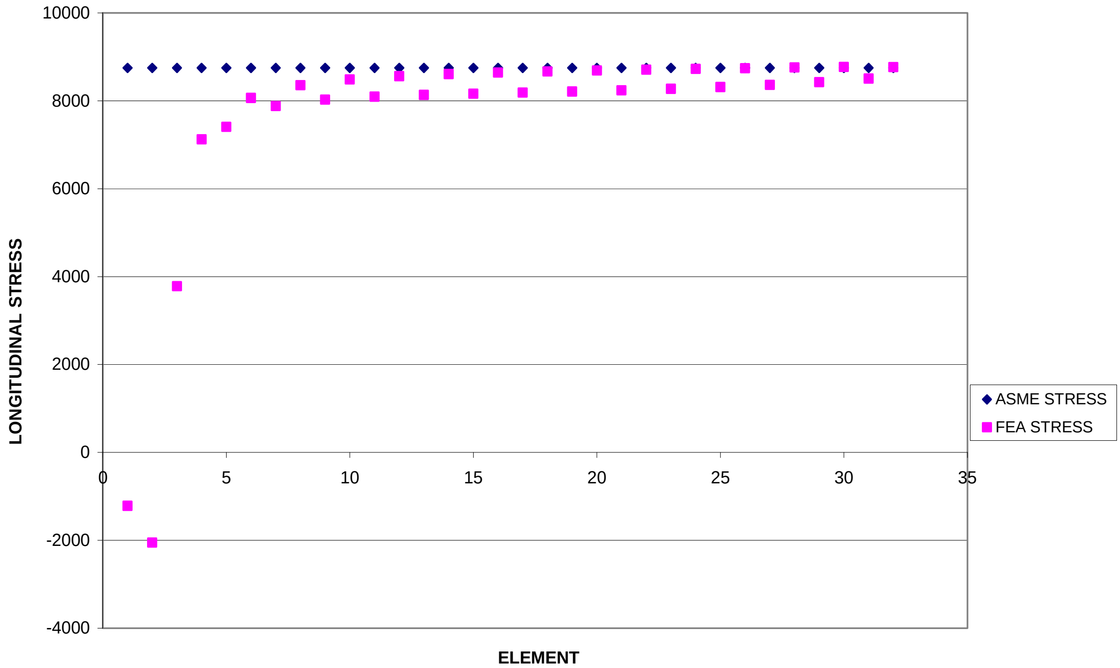

Figure 6. Plot of number of elements against longitudinal stresses for a 35 in Dia vessel modeled with a 30 degrees sector angle shell element showing negative deviation from ASME values.

The onus is thus on the design engineer to take responsibility for proper and painstaking modeling for design whichever modeling method he/she chooses.

4. Conclusions

The usefulness of finite element modeling of spherical steel shells for LNG pressure storage vessels using shallow shell elements has been indicated in this work, the usefulness of ASME standards as guides cannot be over emphasized. Strict adherence to adaptation of physical parameters to modeling parameters is indicated as seen in the use of shallow shells which aptly describe the tank structure as opposed to curved elements. A wrong choice of mesh element configuration will not make results converge to correct results no matter the discretization size. Design Engineers must carry out their design and modeling base on storage tank capacities and maximum loads that may act on the spherical storage tank during its design life.

5. References

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