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The Design of Heat Exchangers

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Abstract

A relation between heat transferred and energy loss, for turbulent flow. In different tube arrangements, is made. The conditions are determined which decide the dimensions and velocities for a heat exchanger. Also, a reference to the economic dimensioning of heat exchangers is presented. In this study, the conditions which a heat exchanger must satisfy represent the best balance between the amounts of material employed. The investigation is restricted to the case of turbulent flow.

Keywords: Heat Changers, Energy Loss, Economic Dimensioning, Counterflow, Crossflow and Parallelflow

1. Introduction

The heat exchanger is an equipment that allows heat transference between two fluids at different temperatures. Heat exchangers are extensively used in industry due to their wide variety of construction and applications in heat transference processes for producing conventional energy such as condensers, heaters, boilers or steam generators. They provide an adequate surface for heat transference to occur and their mechanical and thermal characteristics allow high pressure and high temperature processes.

Heat exchangers are important, their optimization rises the competitiveness and allows energy saving. The necessity of saving and recovering energy for different processes in industry makes essential the develop of new manufacturing technology for heat exchangers in order to cover a wide range of operation conditions. In recent years, new software for design heat exchangers has been focus in adapting the equipment to the required process and new solutions have been found that make the design time shorter.

2. Heat Transfer and Friction Work

Let us first seek a relation between the heat transfer and energy loss. The problem is to determine the energy E which has to be spent in order that a quantity of heat Q may be transferred to a surface F, the average temperature difference being Θ [1,2].

Symbols used:
Q = quantity of heat flowing per second.
G = weight of gas or liquid flowing per second.
Θ = heat transfer coefficient.
Cp = specific heat at constant pressure.
Δt = temperature variation of medium during flow.
w = velocity of flow.
ζ = coefficient in pressure drop formula
l = tube length.
d = tube diameter.
γ = specific weight.
μ = absolute viscosity.
λ = thermal conductivity.
System units: m, kg, sec.
The heat given up to the surface F is:

$$Q = \alpha \cdot \Theta \cdot F \tag{1}$$

And the gas loses a corresponding amount of heat

$$Q = G \cdot C_p \cdot \Delta t \tag{2}$$

whence

$$\frac{\Delta t}{\Theta} = \frac{\alpha F}{GC_p} \tag{3}$$

It is convenient to derive the expression of the energy loss first for longitudinal flow through the tubes, and applies it to the case of cross flow over a tube bank. The expression for the pressure drop in a tube is

$$\Delta p = \zeta \frac{1}{d} \cdot \frac{w^2}{2g} \gamma \tag{4}$$

Putting the free gas section equal to *f*, and remembering that $G = w\gamma f$ and further that $\frac{1}{d} = \frac{F}{4f}$ we obtain from Equation (3)

$$\frac{\Delta p}{\Delta t} = \zeta \frac{w^3 \gamma^2 Cp}{2g\alpha} \tag{5}$$

The ratio of the pressure drop in the heat transfer depends, therefore, of the velocity with which the heating surfaces are swept. If we group together those quantities which depend on the velocity, and introduce the Reynolds number $Re = \frac{w\gamma d}{\mu g}$ and the Nusselt number $Nu = \frac{\alpha d}{\lambda}$ we obtain the following relation between heat transfer and pressure drop:

$$\frac{\Delta p}{\Delta t/\Theta} = \frac{\mu^3 g^2 Cp}{8\gamma\lambda d^2} \cdot \zeta \cdot \frac{Re^3}{Nu}.$$

utting also:

$$\zeta \frac{Re^3}{Nu} = Z, \tag{6}$$

enables us to write the desired relation between the heat transfer and the energy loss in the case of longi tudinal flow through a tube, referred to 1 kg of medium flowing through the exchanger as

$$\frac{\varepsilon}{\Delta t/\Theta} = \frac{\mu^3 g^2 Cp}{8\gamma\lambda d^2} \cdot z \tag{7}$$

Similar expressions may be obtained for the case of a tube bank with cross flow, if ζ is taken as denoting the pressure drop coefficient per tube row, if the bank is z_1 the expression for the pressure drop becomes

$$\Delta p = \zeta \cdot z_1 \cdot \frac{w^2}{2g} \gamma \tag{4a}$$

where *w* is the velocity at the narrowest point between the tubes. If $s \cdot d$ denotes the pitch of the tubes across the flow, and z_q denotes the number of tubes per row also in the direction across the flow, then with $F = \pi d l z_q \cdot z_1$ we obtain

$$\frac{\Delta p}{\Delta t/\Theta} = \frac{w^3 \gamma^2 Cp}{8g\alpha} \cdot (s-1) \cdot \frac{4}{\pi} \zeta \tag{5a}$$

and introducing *Re* and *Nu* gives

$$\frac{\Delta p}{\Delta t/\Theta} = \frac{w^3 g^2 Cp}{8\gamma\lambda d^2} \cdot (s-1) \cdot \frac{4}{\pi} \cdot \zeta \cdot \frac{Re^3}{Nu}$$

Let us put

$$(s-1)\frac{4}{\pi} \cdot \zeta \cdot \frac{Re^3}{Nu} = Z_q \tag{6a}$$

and we obtain a new form of Equation (7), for the case of cross flow:

$$\frac{E}{\Delta t/\Theta} = \frac{\mu^3 g^3 Cp}{8\gamma^2 \lambda d^2} \cdot Z_q \tag{7a}$$

giving the relation between the energy loss and the heat transfer for cross flow.

It is seen that the first term of the Equations (7) and (7a) contains characteristic quantities of the medium and the tube diameter. This means that for a given tube diameter the heat exchanger is fully characterized by the number *Z*. Now the pressure drop coefficient ζ is function of *Re*, while *Nu* is a function of *Re* and of the Prandtl number, if we neglect the transition zone at the inlet, which is entirely permissible with cross flow exchangers many rows deep, or with longitudinal flow exchangers with relatively long tubes. But since the Prandtl number is purely a function of characteristic quantities of the medium, and we are interested only in a comparison of heat exchangers working with the same medium, and operating within the same temperature limits, *Nu* depends only on *Re*. This makes it possible to plot *Nu* as a function of *Z*. The *Nu*-*Z* diagram, therefore, gives a clear picture of the merit of a heat exchanger surface. Tube banks of different pitch are represented by different curves in the Nu-Z diagram. The higher a curve lies, the greater may be the heat transfer loading for a given energy expenditure, or, conversely, for a given surface and heat loading, the smaller the energy expenditure. **Figure** 1 which is drawn for gases contains curves relating to some of the most frequently used tube arrangements. In the case of cross flow, they are based on the values derived by Grimison from the tests of Pierson and Huge.

In order to make a comparison, curves for longitudinal flow have been inserted calculated with the aid of the formula given by Jung. It is seen that only at very high rates of heat transfer such as those which are achieved by the gas velocities attained in the Velox boiler, the longitudinal arrangement become more advantageous than the cross flow one. The curves giving the values of Z which are plotted in **Figure 1**, in a logarithmic scale is almost straight. It is, therefore, permissible when considering segments of these curves, and without introducing any appreciable error to assume the following relation

$$\log Z = B + m \log Nu.$$

or,

$$Z = BNu^m \tag{8}$$

where *B* and m are constants whose value depends on the position in the diagram of the segment under consideration.

The expression for power loss can be written using

Equation (7) and transforming Equation (3):

$$L = \frac{\mu^3 g^2}{8\gamma^2 d^3} \cdot F \cdot B \cdot Nu^{m+1} \quad (9)$$

1. Crossflow, staggered tubes, pitch 1.25 × 1.25.
2. Crossflow, tubes in line, pitch 1.25 × 1.25.
3. Crossflow, tubes in line, pitch 1.5 × 1.5.
4. Crossflow, tubes in line, pitch 2 × 2
5. Crosssflow, tubes in line, pitch 3 × 3.
6. Longitudinal flow in a tube.
7. Longitudinal flow between tubes, pitch 1.5 × 1 .5.
8. Longitudinal tiny- between tubes, pitch 2 × 2.
9. Single tube in crossflow.

The pitch is expressed as times the tubes diameter.

The diagram shows for some common tube arrangements, the relation between the heat transfer number *No* and characteristic number *Z* for the energy loss. For any given arrangement of the tubes, there is a definite value of *Z* for every value of *Nu* of heat transfer, with the help of which the pressure drop and the exchanger surface may be obtained from the Equation (7) or (7a) [3].

3. The Condition for the Right Exchanger

The merit of an exchanger can only be judged when it is known what quantity of heat is equivalent to the mechanical energy which has to be supplied in the form of compressor or pump work to overcome the resistance of the exchanger. If the exchanger is an air preheater forming part of a steam power unit or a gas turbine then the overall efficiency of the plant or the efficiency when the exchanger is in service, determines the amount of the

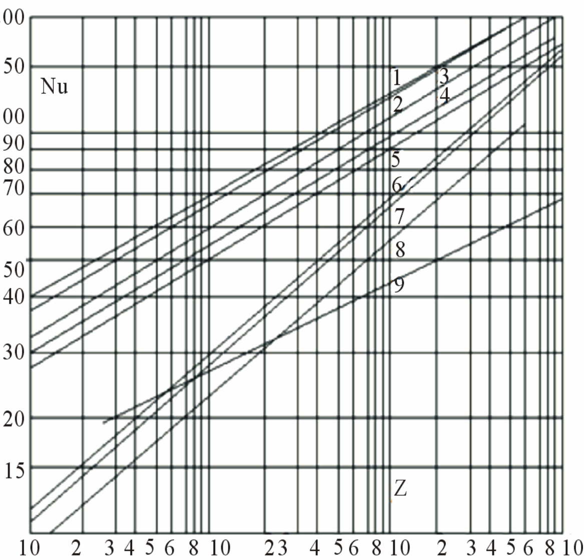

Figure 1. Relation between the energy loss and the heat transfer.

energy (expressed in kcal), to the heat consumption required to produce this energy [4,5].

On the other hand, if it is a case of plain heat transfer for instance, in blast heaters, furnaces, etc., where the energy absorbed in overcoming the resistance of the exchanger has to be supplied in the form of power purchased from an outside supply, then the cost of this power must be balanced against the cost of production of the more or less completely transferred heat in the exchanger. If η denotes the efficiency of the plant with which the heat energy of the fuel is converted into mechanical energy, then the economic performance of the exchanger is given by

$$Useful = Q - \frac{AL}{\eta}.$$

useful heat is, therefore, equal to the difference of the heat transferred and the heat required for the production of the mechanical work absorbed.

The right heat exchanger is, therefore, the exchanger which with a given surface and with a given diameter of tubes results in a maximum amount of useful heat. The condition for this is

$$\eta dQ - AdL = 0 \quad (10)$$

We shall now seek expressions for *dQ* and *dL* in terms of *Nu*.

Let the suffix 1 denote the hot medium, and the suffix 2 the cold one. The meaning of the symbols is made clear by the **Figure 2**. We may write for the temperature variation of the hot medium

$$t_1' - t_1'' = \varepsilon_1 \cdot \left(t_1' - t_2'\right).$$

and similarly, that of the colder one is

$$t_2' - t_2'' = \varepsilon_2 \cdot \left(t_1' - t_2'\right).$$

Further, let the mean temperature difference be given by

$$\Theta = a \cdot \left(t_1' - t_2'\right).$$

The factor a depends only on ε_1 and ε_2, and is plotted in **Figure 3** for counter flow, for cross flow and parallel flow, for the case $\varepsilon_1 = \varepsilon_2$. If we denote by *k* the overall heat transfer coefficient, the heat transferred is given by

$$Q = a \cdot k \cdot F\left(t_1' - t_2'\right) \quad (11)$$

For a small change in the rate of heat transfer we have.

$$dQ = \left(a + kF\frac{da}{dkF}\right) \cdot dkF \cdot \left(t_1' - t_2'\right).$$

Since *kF* may be treated as a single quantity.

We put

$$dQ = b \cdot dkF \cdot \left(t_1' - t_2'\right) \quad (12)$$

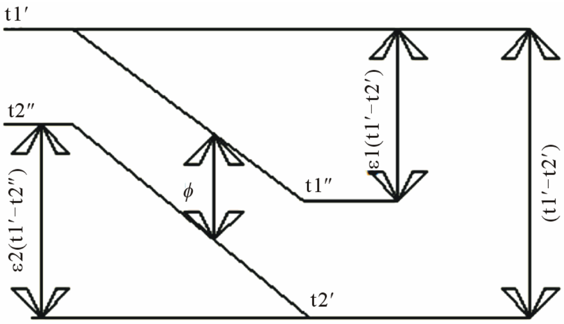

Figure 2. Diagram of temperature variation in an exchanger.

where b depends only on ε_1 and ε_2. From Equations (11) and (12), and from the relation $Q=\varepsilon GCp\left(t_1'-t_2'\right)$, it is found that

$$b=\frac{a}{1-\frac{\varepsilon}{a}\frac{da}{d\varepsilon}} \quad (13)$$

Curves for b are given in **Figure 3**. In the case of heat transfer in metal exchangers the thermal resistance of the exchanger wall may be neglected without introducing any appreciable error. Hence, it is permissible to write

$$\frac{1}{kF}=\frac{1}{\alpha_1F_1}+\frac{1}{\alpha_2F_2} \quad (14)$$

Differentiating and introducing the Nusselt number in place of the quantities $d\alpha_1$ and $d\alpha_2$ gives

$$dkF=\left(\frac{kF}{\alpha_1F_1}\right)^2\cdot F_1\cdot\frac{\lambda_1}{d_1}\cdot dNu_1+\left(\frac{kF}{\alpha_2F_2}\right)^2\cdot F_2\cdot\frac{\lambda_2}{d_2}\cdot dNu_2 \quad (15)$$

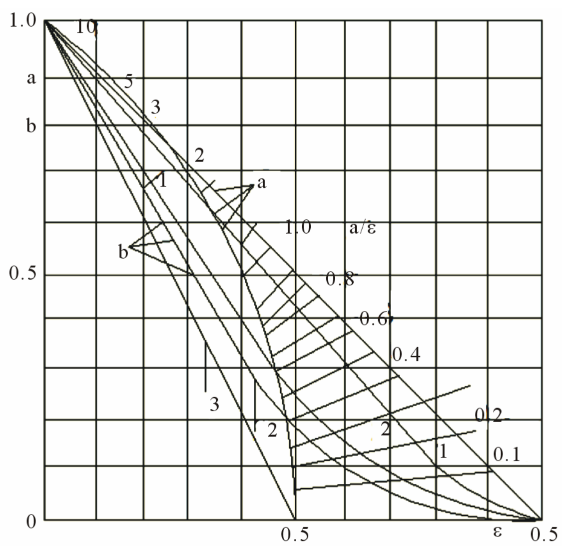

Figure 3. Characteristic numbers *a* and *b* of Equations (11) and (12) for $\varepsilon_1=\varepsilon_2$ in the case of counterflow, crossflow and parallelflow.

The total power loss is the sum of the losses for the hot and cold mediums. We use Equation (9) and put

$$F\frac{\mu^3g^2B}{8\gamma^2d^3}=P \quad (16)$$

and obtain for the total energy loss

$$L=P_1Nu_1^{m_1+1}+P_2Nu_2^{m_2+1}.$$

The change in loss with change of velocity is then given by

$$dL=P_1\left(m_1+1\right)_1Nu_1^{m_1}dNu_1+P_2\left(m_2+1\right)Nu_2^{m_2}dNu_2 \quad (17)$$

The change in heat quantity transferred with change of specific loading kF is given by Equation (12). Inserting Equations (12) and (17) in Equation (10) and taking into account Equation (15) gives

$$\frac{\eta b}{A}\left(t_1'-t_2'\right)\left[\left(\frac{kF}{\alpha_1F_1}\right)^2\cdot F_1\cdot\frac{\lambda_1}{d_1}\cdot dNu_1+\left(\frac{kF}{\alpha_2F_2}\right)^2\cdot F_2\cdot\frac{\lambda_2}{d_2}\cdot dNu_3\right]-P_1\left(m_1+1\right)Nu_1^{m_1}dNu_1-P_2\left(m_2+1\right)Nu_2^{m_2}dNu_2=0 \quad (18)$$

The coefficients of dNu_1 and dNu_2 must be equal 0. This gives two new equations namely,

$$\frac{\eta b}{A}\left(t_1'-t_2'\right)\left[\left(\frac{kF}{\alpha_1F_1}\right)^2\cdot F_1\cdot\frac{\lambda_1}{d_1}\right]=P_1\left(m_1+1\right)Nu_1^{m_1} \quad (19a)$$

$$\frac{\eta b}{A}\left(t_1'-t_2'\right)\left[\left(\frac{kF}{\alpha_2F_2}\right)^2\cdot F_2\cdot\frac{\lambda_2}{d_2}\right]=P_2\left(m_2+1\right)Nu_2^{m_2} \quad (19b)$$

Dividing Equation 19 (b) by Equation 19(a), inserting for P_1 and P_2 the values given by Equation (16) and substituting Nu for α_1 and α_2 gives

$$\frac{Nu_2^{m_2+2}}{Nu_1^{m_1+2}}=\frac{\left(m_1+1\right)\cdot B_1\cdot F_1^2\mu_1^3\gamma_2^2\lambda_1d_2^4}{\left(m_2+1\right)\cdot B_2\cdot F_2^2\mu_2^3\gamma_1^2\lambda_2d_1^4} \quad (20)$$

This equation determines the ratio of the velocities in the right heat exchanger; it does not, however, say anything about the absolute value of the velocities. It means that heat exchangers in which this ratio of the velocities is observed have for a given surface and a given heat quantity the lowest friction loss.

If we take the roots of Equations 19(a) and 19(b), and remembering that $\frac{kF}{\alpha_1F_1}+\frac{kF}{\alpha_2F_2}=1$ we obtain as the second condition for the right heat exchanger:

$$\sqrt{\frac{\eta}{A}b\left(t_1'-t_2'\right)}=\sqrt{R_1Nu_1^{m_1}}+\sqrt{R_2Nu_2^{m_2}} \quad (21)$$

The factor R contains only constants.

$$R=\left(m+1\right)\frac{\mu^3 g^2 B}{8\gamma^2 d^2 \lambda}.$$

Two Equations (20) and (21) completely determine the most favourable heat exchanger. The resulting transcend-dental equation must be solved by trial. The surfaces are found with the aid of the numbers Nu, and Nu, and since Nu is a function of the Reynolds number, the velocities w_1 and w_2 are also determinate. The dimensions of the heat exchanger are, therefore, fixed.

4. The Economic Dimensioning of a Heat Exchanger

It was seen in the first part that there is a function Z which serves as a criterion of the merit of tube arrange-ments in heat exchangers. In the second part the condi-tions fix the dimensions of the surface and sections of the right heat exchanger. The exchanger should, however, like every other apparatus be correctly dimensioned from the economic point of view, that is the total sum of the capital charges and of the running costs should be a minimum.

If P denotes the capital cost, n the interest and depre-cition rate, then the capital chargers are

$$K_1 = nP.$$

and if L is the power absorbed, η the efficiency, h the operating hours in one year and p the price per kilo-watt-hour, then the power costs are

$$K_2 = Lh\frac{1}{\eta}p.$$

and the total costs in one operational year

$$K_1+K_2 = nP + h\frac{1}{\eta}pL.$$

there should be a minimum hence,

$$nP + h\frac{1}{\eta}pL = 0 \tag{22}$$

The capital costs will increase approximately in pro-portion to the exchanger surface, and for a given tube diameter, inversely proportionally to the heat transfer number, or

$$F = Nu^{-1},\ P = Nu^{-1}.$$

But according to Equation (9), the energy proportional to $F\,Nu^{m+1}$; hence substituting for $F = Nu^{-1}$

$$L = CNu^m \text{ or } P \sim L^{-\frac{1}{m}}.$$

differentiating and dividing by P

$$\frac{dP}{P}+\frac{1}{m}\frac{dL}{L}=0 \tag{23}$$

Dividing Equation (22) by Equation (23)

$$nP + h\frac{1}{\eta}pmL = 0 \tag{24}$$

The total costs in an operational year are a minimum when the capital charges amount to m-times the power costs. Within the range of practical application, that is, for Nu = 40 to 120 the curve which is over this range can be seen upon as a straight line, for instance curve 3 (cross flow heat exchanger with a tube pitch 1.5 × 1.5) gives an exponent m = 3.84 and a constant B = 166; for curve 7 (longitudinal flow with a tube pitch 1.5 d) the figures are m = 2.67 and B = 99 500.

The starting point for this study has been the as sump-tion of a fixed tube diameter and tube pitch. These and the choice between staggered or straight arrangement of the tubes are determined by dirt deposit and cleaning considerations. How closely these assumptions and the results of the calculation of the right heat exchanger may be adhered to in practice depends on manufacturing con-ditions, but in any case the above exposition serves as a guide to show in what direction and to what extent modi-fications are desirable.

5. Example

Designing a compact heat exchanger for heating wings continuous 20,000 kg/h of air from an inlet temperature (T_{iair}) 10˚C to an out temperature (air $T_{o\ air}$) of 55˚C heating fluid used water volumetric flow $\dot{V}_w$ (10 m^3/h at temperature T of 95˚C, the input date are shown in **Table 1** [6,7].

The density and heat capacity of both fluids are ob-tained from tables to standard atmosphere conditions. The characteristic of tube and of the wins are shown in **Table 2.**

Diagram inlets from primary and secondary of the compact heat exchanger are show in **Figure 4**.

Unit thermal water consumption

$$q_{\text{t agua}} = \dot{m}_{\text{agua}} cp_{\text{agua}} = \dot{V}_{\text{agua}}\rho_{\text{agua}} cp_{\text{agua}}$$

$$q_{t\ agua} = 10\frac{\text{m}^3}{\text{h}}\times\frac{1\text{h}}{3600\text{s}}\times 962.036\frac{\text{kg}}{\text{m}^3}\times 4217\frac{J}{\text{kg}C}$$
$$=11269{,}18\frac{W}{0_C} = 11.269\frac{kW}{0_C}$$

Unit thermal air consumption

$$q_{\text{t aire}} = \dot{m}_{\text{aire}} cp_{\text{aire}}$$

Table 1. Input data of the compact heat exchanger.

Input data		
T_{inlet} of Secondary fluid (Tfi)	10.00	˚C
T_{out} of Secondary fluid (Tfo)	55.00	˚C
T_{inlet} of Primary fluid (Tci)	95.00	˚C
velocity primary fluid (Wp)	1.00	m/s
velocity secondary fluid (Ws)	3.00	m/s
Mass flow of primary fluid (mp)	2.77	kg/s
Mass flow of secondary fluid (ms)	5.55	kg/s
heat capacity primary fluid (Cpp)	4217	J/kg-˚C
heat capacity secondary fluid (Cps)	1005.00	J/kg-˚C
density primary fluid (ρp)	962.04	kg/m^3
density secondary fluid (ρs)	1.15	kg/m^3

Table 2. Characteristic of tube and of the wing.

Characteristic of tube		
Outside diameter of tube (do)	0.012700	m
Inside diameter of tube (di)	0.011280	m
Wheelbase of the tubes in the direction of the tubes (l)	0.080	m
Wheelbase of the tubes in the direction of height (h)	0.040	m
Area inside of the tube (At-int)	9.993E - 505	m^2
Characteristics of the wing		
Thickness of the wing (e)	3.00E - 04	m
Thermal conductivity (Aluminum , k)	203.52	W/m˚C

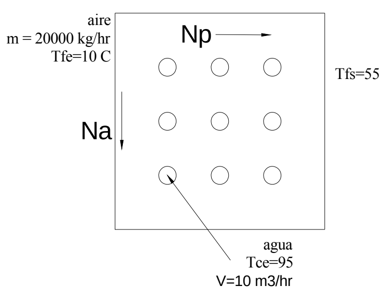

Figure 4. Diagram inlets from primary and secondary fluids.

$$q_{t\,aire} = 20000\frac{kg}{h}\times\frac{1h}{3600s}\times 1.005\frac{kJ}{kgK} = 5.577\frac{kW}{C}$$

$$T_{m\,aire} = \frac{55+10}{2} = 32.5°C$$

The density of air to Tm is show in **Table 3**.

Table 3. Density of air to Tm.

T(°C)	$\rho\left(\frac{kg}{m^3}\right)$
30	1.165
32.5	$\rho_{air} = 1.15\left(\frac{kg}{m^3}\right)$
40	1.128

Themal flow

$$\dot{Q} = q_{t\,air}\left(T_{s\,air} - T_{e\,air}\right) = \dot{m}_{air}cp_{air}\left(T_{s\,air} - T_{e\,air}\right)$$

$$\dot{Q} = 5577{,}75\frac{W}{°C}$$

$$\left(55°C - 10°C\right) = 250.99KW = 250998{,}75W$$

Temperature of exit of the water

$$T_{s\,water} = T_{e\,water} - \frac{\dot{Q}}{q_{t\,water}} = 95°C - \frac{250998{,}75W}{11269{,}18\,W/°C} = 73.51°C$$

Velocities are: 2.5 m/s and 3.5 m/s

$$W_{water} = 1\ m/s$$

$$W_{air} = 3\ m/s$$

Logarithmic mean temperature

$$\Delta T_{ml} = \frac{\Delta\theta_1 - \Delta\theta_2}{\ln\frac{\Delta\theta_1}{\Delta\theta_2}} = \frac{(95-55)-(73.51-10)}{\ln\frac{40}{63.51}}$$

$$\Delta T_{ml} = 50.85°C$$

Surface airflow

$$S_p = \frac{\dot{V}_{air}}{W_{air}}$$

$$\dot{V}_{air} = \frac{W_{air}}{\rho_{air}}$$

Table gives the density of dry air at a temperature of 58.38˚C

$$\rho_{air} = 1.15\frac{kg}{m^3}$$

Volumetric air flow

$$\dot{V}_{air} = \frac{20000\frac{kg}{h}\times\frac{1h}{3600s}}{1.15\frac{kg}{m^3}} = 4.83\frac{m^3}{s}$$

$$S_p = \frac{4.83\frac{m^3}{s}}{3\frac{m}{s}} = 1.61m^2$$

$$\frac{d_{\text{int}}}{d_{out}} = \frac{11.66mm}{15.88mm} \qquad h = 40mm$$

Surface water flow

$$S_T = \frac{\pi}{4} d_{\text{int}}{}^2 N_A; \qquad W_{\text{water}} = 1 \text{ m/s}$$

$$S_T = \frac{\dot{V}_{\text{water}}}{\text{W}_{\text{water}}} = \frac{10\frac{m^3}{h} \times \frac{1h}{3600s}}{1m/s} = 2.88 \times 10^{-3} \text{ m}^2$$

Number of tubes in the direction of height (H) heat exchanger

$$N_A = \frac{S_T}{A_{\text{tube,inlet.}}} = \frac{4}{\pi} \frac{S_T}{d_{\text{inlet}}{}^2}$$
$$= \frac{4 \times 2.78 \times 10^{-3} \text{m}^2}{\pi (0.01128) \text{m}^2} = 29 \text{tubes}$$

Height of heat exchanger

$$H = h \cdot N_A = 0.040\ m \cdot 29 = 1.15\ m$$

Winged tube length

$$S_p = H \cdot La \therefore La = \frac{S_p}{H} = \frac{1.61m^2}{1.15m} = 1.40m$$

In **Table 4** are shown calculated values of Na, H and L to the compact heat exchanger.

Wing thickness, this value is assumed by the types of materials available in the industry

$$e = 0.3mm$$

Calculation of Reynolds number (Re), Nusselt (Nu) and the convective coefficient (hi) of primary fluid
Whereas, *dh* = *di*; and the Prandtl Number (*Prp*), dynamic viscosity (μp) and conductivity of the primary fluid are determined from tables to the average temperature of primary fluid (*Tmp*). These values are shown in **Table 5**.

$$Rep = \frac{Wp \cdot dh}{\mu p}$$

$$Nu = 0.023 \cdot Rep^{0.8} \cdot Prp^{0.3}$$

$$hi = \frac{Nu \cdot kp}{dh}$$

The values of Reynolds number, Nusselt number and convection coefficient inside of primary fluid are shown in **Table 6**.

Calculation of the mass velocity and maximum mass velocity of the secondary fluid.

$$s = \frac{h}{de}$$

$$Wms = Ws \cdot \rho s$$

$$Wms' = Wms\left(\frac{s}{s-1}\right)$$

The values of mass velocity, maximum mass velocity and relationship s are shown in **Table 7**

Calculation of hydraulic diameter (*dh*′), Reynolds (*Re*) and convection coefficient (*I*) as a function the secondary fluid pitch between wing (*p*) Tables obtained forthe Prandtl number (*Pr*), the dynamic viscosity (μs) and thermal conductivity (*ks*) of secondary fluid at film temperature (*Tmp*). These values are shown in **Table 8.**

Table 4. Values of *Na*, *H* and *L* to the compact heat exchanger.

Number of tubes in direction of height (*Na*)	29	Tubos
Exchanger height (*H*)	1.15	m
Overall longitude of the tube (*L*)	1.40	m

Table 5. Values of *Prp*, *v* and *kp*.

Prandtl number of primary fluid (*Prp*)	2.11	
Viscosity of primary fluid (*v*)	3.50E - 07	m^2/s
Conductivity of primary fluid (*kp*)	0.68	w/m ˚C

Table 6. Values of Rep, Nup and hi to the compact heat exchanger.

Reynolds number of primary fluid (*Rep*)	32228.57	
Nusselt number of fluid primary (*Nup*)	116.31	
Convection coefficient inside (*hi*)	6972.23	W/m^2-K

Table 7. Values of *s*, *Wms* and *Wms* of the secondary fluid.

Relationship between h and the outer diameter (*s*)	2.36	
Mass velocity of secondary fluid (*Wms*)	3.45	kg/s-m^2
Maximus velocity of secondary fluid (*Wms*')	5.98	kg/s-m^2

Table 8. Values of *Tp*, Pr, ρs and *ks* of secondary fluid.

Film Temperature (*Tp*)	58.33	˚C
Prantl Number of secondary fluid (*Prs*)	0.6963	
Dynamic Viscosity of secondary fluid (ρs)	0.0781	kg/m-s
Conductividad del flujo secundario (*ks*)	0.0287	w/m ˚C

$$dh' = 2p\left(1 - \frac{\pi \cdot de^2}{4h \cdot l} - \frac{e}{p}\right)$$

$$\mathrm{Re} = \frac{Wms' \cdot dh'}{\mu s}$$

$$Nu = 0.26 \cdot Re^{0.6} \cdot Prp^{1/3}$$

$$he = \frac{Nu \cdot ks}{dh'}$$

Table 9. show the values of *m*, *dh*', *Re*, *Nu* and *he* of the secondary fluid.

Calculation of equivalent diameter (dea) and efficiencies of wings. (**Table 10**)

In **Table 11** show characteristics of wing, Aluminum considering.

$$dea = \frac{2 \cdot h \cdot l}{h + l}$$

$$\mu = \frac{dea}{de}$$

$$\beta = \sqrt{\frac{2 \cdot he}{ks \cdot e}}$$

$\left[de \cdot \beta \cdot \frac{\mu - 1}{2}\right]$, Relationship for the efficiencies.

Calculation of the ratio number of wings with borders with the total number of wings.

Considering that $N = Na\ x\ Np$ where Na is the number of tubes in direction of height and *Np* is the number of tubes in the direction of secondary flow.

Number of wings without borders

$$*\ N0 = Na \cdot Np - 2(Na + Np) + 4$$

Number of wings with a border

$$*\ N1 = 2(Na + Np) - 8$$

Number of wings with two borders

$$*\ N2 = 4$$

*Note: This means that the equations are modified as a function on *Np*. (**Table 12**)

$$\psi 0 = \frac{N0}{N} \quad \psi 1 = \frac{N1}{N} \quad \psi 2 = \frac{N2}{N}$$

Calculation of correction coefficients for the wings. (**Table 13**)

$$C_0 = 1$$

$$C_1 = 1 + 0.5\left[\frac{h_e}{k_{al}}\left(\frac{l}{h} + \frac{h}{l}\right)\right]^{1/4}$$

$$C_2 = 1 + \left[\frac{h_e}{k_{al}}\left(\frac{l + h}{h}\right)\right]^{1/3}$$

Table 9. Values of *m*, *dh*′, *Re*, *Nu* and *he* of the secondary fluid.

Pitch (*m*)	Hydraulic Diameter (*dh*′)	Reynolds Number secondary (*Re*)	Nusselt Number secondary (*Nu*)	Convective Coefficient secondary (*he*)
0.002	3.08E - 03	718.4305	11.923624	110.9873
0.003	4.92E - 03	1147.548	15.792099	92.027767
0.004	6.77E - 03	1576.665	19.108255	81.04597
0.005	8.61E - 03	2005.782	22.077386	73.606079
0.006	1.04E - 02	2434.899	24.800817	68.113749
0.007	1.23E - 02	2864.017	27.337746	63.831783
0.008	1.41E - 02	3293.134	29.726447	60.364755

Table 10. Values of equivalent diameter and efficiencies of wings.

Pitch (*m*)	Convective Coefficient secondary (*he*)	Equivalence Factor wings (*β*)	*dea*** = (*de***β*)*(*μ*-1/2)	Efficiency obtained from the graphic *ηf*
0.002	110.9873	60.295115	0.823028315	0.9
0.003	92.027767	54.904119	0.749441229	0.92
0.004	81.04597	51.524193	0.703305238	0.93
0.005	73.606079	49.102356	0.670247159	0.95
0.006	68.113749	47.234887	0.644756207	0.96
0.007	63.831783	45.726081	0.624161001	0.97
0.008	60.364755	44.466936	0.606973681	0.98

Table 11. Characteristics of Aluminum.

Thermal conductivity of aluminum (k)	203.525	w/mC
Wing thickness (e)	3.00E - 04	m

Table 12. Relations with number of wings on the total number of wings.

Relations with number of wings on the total number of wings						
ψ	(Np) = 1	(Np) = 2	(Np) = 3	(Np) = 4	(Np) = 5	(Np) = 6
$\tilde{\psi}_0$	0	0	0.3101952	0.4652928	0.5583514	0.620390394
ψ_1	0	0.930586	0.6435285	0.5	0.4138829	0.35647147
ψ_2	0.930585591	0.069414	0.0462763	0.0347072	0.0277658	0.023138136

Table 13. Correction factors for each type of wing.

Correction factors for each type of wing				
Pitch (m)	h_e	c_0	c_1	c_2
0.002	110.987	1	1.510965	2.029347
0.003	92.0278	1	1.487588	1.967038
0.004	81.046	1	1.472341	1.926932
0.005	73.6061	1	1.461107	1.897653
0.006	68.1137	1	1.452253	1.874747
0.007	63.8318	1	1.444972	1.856018
0.008	60.3648	1	1.438802	1.840231

Calculation of the overall efficiency of the equivalent circular wing. (**Table 14**)

$$\eta g = \eta f \left[\frac{\psi_0}{C_0} + \frac{\psi_1}{C_1} + \frac{\psi_2}{C_2} \right]$$

Calculation the overall surface heat Exchange where, $X = Np \cdot l$. (**Table 15**)

$$Se = \left(La \cdot X \cdot H \right) \left(\left(\frac{\pi \cdot de}{h \cdot l} \right) + \frac{2}{p} - \frac{\pi \cdot de^2}{2 \cdot p \cdot h \cdot l} \right)$$

Calculating the overall coefficient of heat transfer function and pitch Np

$$U = \frac{1}{\frac{Se}{Si}\left(\frac{1}{hi} + \frac{et}{kt} \right) + \frac{1}{he}\left[1 - \frac{Ss}{Se}\left(1 - \eta g \right) \right]}$$

where,

$$Si = \pi \cdot dik \cdot La \cdot Na \cdot Np$$

$$Ss = 2 \cdot \frac{La}{p} \left(X \cdot H - Np \cdot Na \cdot \left[\frac{\pi \cdot de^2}{4} \right] \right)$$

*kt = conductivity of tube, e_t = Thickness of the wall of tube.

The value $\frac{et}{kt}$ is neglected as being small compared to $\frac{1}{hi}$. (Table 16**)

Calculation Heat flow based on Np and pitch (p)

$$Q = U \cdot A \cdot \left(\Delta T \right).$$

Table 14. Overall efficiency (ηg) as a function of Np and pitch (p).

Overall efficiency (ηg) as a function of Np and pitch (p).						
ηf	(Np) = 1	(Np) = 2	(Np) = 3	(Np) = 4	(Np) = 5	(Np) = 6
0.9	0.412708	0.585084	0.683014	0.731979	0.761358	0.780944
0.92	0.435243	0.607987	0.705014	0.753528	0.782636	0.802041
0.93	0.449131	0.621303	0.717299	0.765297	0.794095	0.813294
0.95	0.465868	0.639809	0.736269	0.784499	0.813437	0.832729
0.96	0.476524	0.650701	0.746883	0.794974	0.823829	0.843066
0.97	0.486346	0.660974	0.757071	0.80512	0.833949	0.853168
0.98	0.495576	0.670808	0.766957	0.815031	0.843876	0.863105

Table 15. Total surface heat exchange.

TOTAL SURFACE HEAT EXCHANGE						
$P(m)$	(Np) = 1	(Np) = 2	(Np) = 3	(Np) = 4	(Np) = 5	(Np) = 6
0,002	60,85782	121,7156	182,5734	243,4313	304,2891	304,2891
0,003	41,10674	82,21349	123,3202	164,427	205,5337	205,5337
0,004	31,23121	62,46242	93,69363	124,9248	156,156	156,156
0,005	25,30589	50,61178	75,91766	101,2236	126,5294	126,5294
0,006	21,35567	42,71135	64,06702	85,42269	106,7784	106,7784
0,007	18,53409	37,06818	55,60228	74,13637	92,67046	92,67046
0,008	16,41791	32,83581	49,25372	65,67162	82,08953	82,08953

Table 16. Overall rate of heat transfer (W/m^2 C).

OVERALL RATE OF HEAT TRANSFER (W/m^2 C)						
$P(m)$	(Np) = 1	(Np) = 2	(Np) = 3	(Np) = 4	(Np) = 5	(Np) = 6
0.002	100.1748	86.99647	80.94672	78.22676	76.68079	84.68974
0.003	109.8194	91.66171	83.87258	80.4542	78.53373	84.50856
0.004	110.6942	90.50486	82.55818	78.52662	76.50163	81.06726
0.005	107.1119	86.58511	65.14468	74.68027	72.68163	76.02262
0.006	102.8529	82.72923	62.38494	71.19159	69.25976	71.86373
0.007	98.2215	78.86459	72.69445	67.83032	65.9839	68.03117
0.008	93.64498	75.20035	69.27388	64.71031	62.95396	64.56843

where

$A = Se$ = Overall surface heat exchange

$\Delta T = Tml$ = Difference logarithmic mean temperature. (**Table 17**)

Calculation $U \cdot Np \cdot St_1$ for comparison of the relationship ($Q/°C$) as a function of *Np* and the pitch between wings.

$$Q = U \cdot Np \cdot St_1 \cdot (\Delta\theta_{ml})$$

$$\therefore U \cdot Np \cdot St_1 = \frac{Q}{\Delta\theta_{ml}} = \frac{250998,75}{50.85} = 4935,724 \ W/°C$$

Where:

St = Overall surface heat exchange with respect to pitch

Np = Pitch number of tubes in the direction of secondary flow. (**Table 18**)

Selection based on the results obtained in the above table $U \cdot Np \cdot St_1 = \frac{Q}{\Delta\theta_{ml}}$ can be compared to select the exchange with their respective sizes, data are compared

$$4935,724059 \approx 5300,392$$

This variation is important for any factor that our team needs to transfer more heat.

Table 17. Values of heat flow (*Q*).

Heat flow (*Q*)						
P(*m*)	(*Np*) = 1	(*Np*) = 2	(*Np*) = 3	(*Np*) = 4	(*Np*) = 5	(*Np*) = 6
0.002	310024.2	538478.9	751549.4	968394.7	1186571	1310502
0.003	229568.8	383223.1	525987	672732.6	820842.8	883292.4
0.004	175806.2	287482.5	393360.6	498868.8	607505.5	643761.5
0.005	137841.4	222851.5	251502.6	384421.9	467667.2	489164.7
0.006	111699.4	179689.6	203252.1	309259.2	376084.1	390223.8
0.007	92576.03	148663.4	205548.6	255726.6	310956.7	320604.8
0.008	78184.91	125570.7	173511.9	216108.5	262803.7	269543.4

Table 18. Values of $U \cdot Np \cdot St_1$ (W/˚C).

$U \cdot Np \cdot St_1$ (W/˚C)						
P(*m*)	(*Np*) = 1	(*Np*) = 2	(*Np*) = 3	(*Np*) = 4	(*Np*) = 5	(*Np*) = 6
0,002	6096,42	10588,83	14778,72	19042,84	23333,13	25770,16
0,003	4514,317	7535,829	10343,19	13228,84	16141,33	17369,36
0,004	3457,113	5653,152	7735,176	9809,925	11946,19	12659,14
0,005	2710,561	4382,226	4945,632	7559,402	9196,366	9619,099
0,006	2196,494	3533,477	3996,817	6081,377	7395,444	7673,492
0,007	1820,446	2923,367	4041,977	5028,693	6114,758	6304,48
0,008	1537,454	2469,264	3411,996	4249,631	5167,861	5300,392

The selection of compact heat exchanger required to have 6 columns of tubes (*Np* = 6) in the direction of flow and pitch inlet wings of 8 mm, so *X* will be worth

$$X = 1xNp = (0.008)(6) = 0.18\text{m}$$

The dimensions of the compact heat exchanger will (*L*) *x*(*H*) *x*(*X*), (1.40 × 1.15 × 0.18) m.

With piping HWG 22 *de* = 12.7 mm y *di* = 11.28 mm The thick aluminum wings *e* = 0.3 mm and *K* = 203.52 W/m C. In **Table 19** are shown characteristics of the exchanger.

6. Conclusions

This paper identifies the advantages of having the appropriate exchanger with working conditions, environmental conditions and economic aspects, it is also necessary to mention the following regarding the general utility of this work.

- In addition to the thermal design, mechanical design of heat exchangers is also a part of it. The mechanical design is done under the ASME Section VIII, which is entitled “Pressure Vessel Design”
- Although the subject of this work is the design of heat exchangers, which, as noted was achieved successfully, the utility of it is wider and there are several methods for the design of heat exchangers.

Table 19. Characteristics of the exchange.

CHARACTERISTICS OF THE EXCHANGE		
de:	Outside diameter of the tubes	12.7 mm
di:	Internal diameter of the tubes.	11.28 mm
La:	Wings tube length	1.40 m
L:	Overall length of the tube.	1.40 m
N_a:	Number of vertical tubes in the exchanger.	29 tubos
N_p:	Number of tubes in the direction of secondary flow.	6 tubos
N:	Overall number of tubes ($N = Na \cdot Np$).	174 tubos
h:	Wheelbase of the tubes in the vertical direction.	0.04 m
l:	Wheelbase of the tubes in the direction of secondary flow.	0.08m
n:	Number of wings $(n = La/p)$.	175 wings
p:	Wings pitch (from center to center of wings)	0.008 m
e:	Wing thickness.	0.3 mm
H:	Height of heat exchanger (continuous wing)	1.15 m
X:	Heat exchanger width (within the meaning of secondary flow).	0.18 m

- While this paper addresses just one example of a heat exchanger, this could vary, the same results in the thermal design, these variations are affected primarily for economic reasons or space.

7. References

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