

Fibre-Reinforced Generalized Thermoelastic Medium under Hydrostatic Initial Stress

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Abstract

The present problem is concerned with the deformation of an infinite fibre-reinforced generalized thermoelastic medium with hydrostatic initial stress under the influence of mechanical force. The normal mode analysis is used to obtain the analytical expressions of the displacement components, force stress and temperature distribution. The numerical results are given and presented graphically for Green-Lindsay [1] theory of thermoelasticity. Comparisons are made in the presence and absence of hydrostatic initial stress and anisotropy.

Keywords: Generalized Thermoelastic, Hydrostatic Initial Stress, Fibre-Reinforced, Temperature Distribution, Normal Mode, Anisotropy

1. Introduction

The classical theories of thermo-elasticity involving infinite speed of propagation of thermal signals, contradict physical facts. During the last three decades, non-classical theories involving finite speed of heat transportation in elastic solids have been developed to remove this paradox. In contrast to the conventional coupled thermo-elasticity theory which involves a parabolic-type heat transport equation, these generalized theories involving a hyperbolic-type heat transport equation are supported by experiments exhibiting the actual occurrence of wave-type heat transport in solids, called sound effect. The extended thermo-elasticity theory proposed by Lord and Shulman [2] incorporates a flux-rate term into Fourier's law of heat conduction, and formulates a generalized form that involves a hyperbolic-type heat equation admitting finite speed of thermal signals. Muller [3] in a review of the thermodynamics of a thermoelastic solid, proposed an entropy production inequality, with the help of which he considered restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws [4]. Green and Lindsay [4] developed temperature-type-dependent thermo-elasticity (TRDTE) theory by

introducing relaxation time factors that does not violate the classical Fourier's law of heat conduction and this theory also predicts a finite speed for heat propagation.

Barber [5] studied thermoelastic displacements and stresses due to a heat source moving over the surface of a half plane. Sherief [6] obtained components of stress and temperature distributions in a thermoelastic medium due to a continuous source. Dhaliwal *et al.* [7] investigated thermoelastic interactions caused by a continuous line heat source in a homogeneous isotropic unbounded solid. Chandrasekharaiah and Srinath [8] studied thermoelastic interactions due to a continuous point heat source in a homogeneous and isotropic unbounded body. Sharma *et al.* [9] investigated the disturbance due to a time-harmonic normal point load in a homogeneous isotropic thermoelastic half-space. Sharma and Chauhan [10] discussed mechanical and thermal sources in a generalized thermoelastic half-space. Sharma *et al.* [11] investigated the steady-state response of an applied load moving with constant speed for infinite long time over the top surface of a homogeneous thermoelastic layer lying over an infinite half-space. Deswal and Choudhary [12] studied a two-dimensional problem due to moving load in generalized thermoelastic solid with diffusion.

Fibre-reinforced composites are used in a variety of

structures due to their low weight and high strength. A continuum model is used to explain the mechanical properties of such materials. In the case of an elastic solid reinforced by a series of parallel fibres it is usual to assume transverse isotropy. In the linear case, the associated constitutive relations, relating infinitesimal stress and strain components, have five materials constants. The analysis of stress and deformation of fibre-reinforced composite materials has been an important subject of solid mechanics for last three decades. Pipkin [13] and Rogers [14,15] did pioneer works on the subject. Craig and Hart [16] studied the stress boundary-value problem for finite plane deformation of a fibre-reinforced material. Sengupta and Nath [17] discussed the problem of surface waves in a fibre-reinforced anisotropic elastic media.. Singh and Singh [18] discussed the reflection of plane waves at the free surface of a fibre-reinforced elastic half-space. Singh [19] discussed the wave propagation in an incompressible transversely isotropic fibre-reinforced elastic media. Singh [20] studied the effects of anisotropy on reflection coefficients of plane waves in fibre-reinforced thermoelastic solid. Kumar and Gupta [21] investigated a source problem in fibre-reinforced anisotropic generalized thermoelastic solid under acoustic fluid layer.

The development of initial stresses in the medium is due to many reasons, for example, resulting from differences of temperature, process of quenching, shot pinning and cold working, slow process of creep, differential external forces, gravity variations, etc. The earth is assumed to be under high initial stresses. It is, therefore, of much interest to study the influence of these stresses on the propagation of stress waves. Biot [22] showed the acoustic propagation under initial stress, which is fundamentally different from that under a stress-free state. He has obtained the velocities of longitudinal and transverse waves along the co-ordinates axis only.

The wave propagation in solids under initial stresses has been studied by many authors for various models. The study of reflection and refraction phenomena of plane waves in an unbounded medium under initial stresses is due to Chattopadhyay *et al.* [23], Sidhu and Singh [24] and Dey *et al.* [25]. Montanaro [26] investigated the isotropic linear thermoelasticity with hydrostatic initial stress. Singh *et al.* [27], Singh [28] and Othman and Song [29] studied the reflection of thermoelastic waves from a free surface under a hydrostatic initial stress in the context of different theories of generalized thermoelasticity. Ailawalia *et al.* [30] investigated deformation in a generalized thermoelastic medium with hydrostatic initial stress. Ailawalia [31] obtained the components of displacement, stresses, tem-

perature distribution of thermoelastic solid half-space under hydrostatic initial stress subjected to ramp-type heating and loading for G-N theory (type III).

The present paper is concerned with the investigations related to effect of hydrostatic initial stress in fibre-reinforced generalized thermoelastic medium. Effects of hydrostatic initial stress and anisotropy are shown graphically on normal displacement, normal force stress and temperature distribution for Green-Lindsay [1] theory of thermoelasticity.

2. Basic Equations and Their Solutions

We consider a homogeneous thermally conducting transversely fibre-reinforced medium with hydrostatic initial stress of infinite extent with cartesian coordinates system (x, y, z) . To analyze the displacement components, stresses and temperature distribution at the interior of the medium, the continuum is divided into two half spaces defined by

- 1) half space I $|x| < \infty, -\infty < y \leq 0, |z| < \infty,$
- 2) half space II $|x| < \infty, 0 \leq y < \infty, |z| < \infty,$

if we restrict our analysis to the plane strain parallel to xy -plane with displacement vector $\mathbf{u} = (u_1, u_2, 0)$, then the field equations and constitutive relations for such a medium in the absence of body forces and heat sources are written as,

$$(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta) \frac{\partial^2 u_1}{\partial x^2} + \left(\lambda + \alpha + \mu_L + \frac{p}{2} \right) \frac{\partial^2 u_2}{\partial x \partial y} + \left(\mu_L - \frac{p}{2} \right) \frac{\partial^2 u_1}{\partial y^2} - \beta_1 \left(1 + \mathcal{G}_0 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial x} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{1}$$

$$(\lambda + 2\mu_T) \frac{\partial^2 u_2}{\partial y^2} + \left(\lambda + \alpha + \mu_L + \frac{p}{2} \right) \frac{\partial^2 u_1}{\partial x \partial y} + \left(\mu_L - \frac{p}{2} \right) \frac{\partial^2 u_2}{\partial x^2} - \beta_2 \left(1 + \mathcal{G}_0 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial y} = \rho \frac{\partial^2 u_2}{\partial t^2}, \tag{2}$$

$$K_1^* \left(n^* + t_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 T}{\partial x^2} + K_2^* \left(n^* + t_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 T}{\partial y^2} = \rho C^* \left(n_1 \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T + T_0 \left(n_1 \frac{\partial}{\partial t} + \tau_0 n_0 \frac{\partial^2}{\partial t^2} \right) \left(\beta_1 \frac{\partial u_1}{\partial x} + \beta_2 \frac{\partial u_2}{\partial y} \right), \tag{3}$$

$$t_{11} = -p + (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta) \frac{\partial u_1}{\partial x} + (\lambda + \alpha) \frac{\partial u_2}{\partial y} - \beta_1 \left(1 + \mathcal{G}_0 \frac{\partial}{\partial t} \right) T, \tag{4}$$

$$t_{12} = \left(\mu_L - \frac{p}{2} \right) \frac{\partial u_2}{\partial x} + \left(\mu_L + \frac{p}{2} \right) \frac{\partial u_1}{\partial y}, \quad (5)$$

$$t_{21} = \left(\mu_L + \frac{p}{2} \right) \frac{\partial u_2}{\partial x} + \left(\mu_L - \frac{p}{2} \right) \frac{\partial u_1}{\partial y}, \quad (6)$$

$$t_{22} = -p + (\lambda + \alpha) \frac{\partial u_1}{\partial x} + (\lambda + 2\mu_r) \frac{\partial u_2}{\partial y} - \beta_2 \left(1 + \mathcal{G}_0 \frac{\partial}{\partial t} \right) T. \quad (7)$$

where

$$\beta_1 = (2\lambda + 3\alpha + 4\mu_L - 2\mu_r + \beta) \alpha_1 + (\lambda + \alpha) \alpha_2,$$

$$\beta_2 = (2\lambda + \alpha) \alpha_1 + (\lambda + 2\mu_r) \alpha_2.$$

and $\lambda, \alpha, \beta, \mu_L, \mu_r$ are material constants, K_1^*, K_2^* are coefficients of thermal conductivity, α_1, α_2 are coefficients of linear expansion, τ_0, \mathcal{G}_0 are thermal relaxation times, u_1, u_2 are the components of displacement vector, ρ is the mass density, T is the temperature change of a material particle, T_0 is the reference uniform temperature of the body and C^* is the specific heat at constant strain.

For simplification, we shall use the following non-dimensional variables

$$\{x', y'\} = \frac{\omega^*}{c_1} \{x, y\}, \quad \{u'_1, u'_2\} = \frac{\rho c_1 \omega^*}{\beta_1 T_0} \{u_1, u_2\},$$

$$T' = \frac{T}{T_0}, \quad t'_{ij} = \frac{t_{ij}}{\beta_1 T_0},$$

$$t' = \omega^* t, \quad t'_1 = \omega^* t_1, \quad \tau'_0 = \omega^* \tau_0, \quad \mathcal{G}'_0 = \omega^* \mathcal{G}_0, \quad p' = \frac{p}{\beta_1 T_0}, \quad (8)$$

where

$$c_1^2 = \frac{(\lambda + 2\alpha + 4\mu_L - 2\mu_r + \beta)}{\rho}, \quad \omega^* = \frac{\rho C^* c_1^2}{K_1^*}.$$

Substituting non-dimensional variables into Equations (1)-(3), we obtain (after dropping the primes)

$$\frac{\partial^2 u_1}{\partial x^2} + a_1 \frac{\partial^2 u_2}{\partial x \partial y} + a_2 \frac{\partial^2 u_1}{\partial y^2} - \left(1 + \mathcal{G}_0 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial x} = \frac{\partial^2 u_1}{\partial t^2}, \quad (9)$$

$$a_3 \frac{\partial^2 u_2}{\partial y^2} + a_1 \frac{\partial^2 u_1}{\partial x \partial y} + a_2 \frac{\partial^2 u_2}{\partial x^2} - \bar{\beta} \left(1 + \mathcal{G}_0 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial y} = \frac{\partial^2 u_2}{\partial t^2}, \quad (10)$$

$$\begin{aligned} & \left(n^* + t_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 T}{\partial x^2} + \bar{K} \left(n^* + t_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 T}{\partial y^2} \\ & = \left(n_1 \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T \\ & + \left(n_1 \frac{\partial}{\partial t} + \tau_0 n_0 \frac{\partial^2}{\partial t^2} \right) \left(\varepsilon_1 \frac{\partial u_1}{\partial x} + \varepsilon_2 \frac{\partial u_2}{\partial y} \right). \end{aligned} \quad (11)$$

3. Normal Mode Analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

$$[T, u_1, u_2, t_{ij}](x, y, t) = [T^*, u_1^*, u_2^*, t_{ij}^*](y) e^{(\omega t + i a x)}. \quad (12)$$

where ω is the complex time constant and a is the wave number in x -direction.

Using (12), Equations (9)-(11) take the form

$$[a_2 D^2 - a^2 - \omega^2] u_1^* + a_4 D u_2^* - a_5 T^* = 0, \quad (13)$$

$$a_4 D u_1^* + [a_3 D^2 - a_6 - \omega^2] u_2^* - a_7 D T^* = 0, \quad (14)$$

$$a_8 u_1^* + a_9 D u_2^* - [a_{10} D^2 - a^2 a_{11} - a_{12}] T^* = 0. \quad (15)$$

where

$$a_1 = \left(\lambda + \alpha + \mu_L + \frac{\beta_1 T_0 p}{2} \right) \frac{1}{\rho c_1^2},$$

$$a_2 = \left(\mu_L - \frac{\beta_1 T_0 p}{2} \right) \frac{1}{\rho c_1^2},$$

$$a_3 = (\lambda + 2\mu_r) \frac{1}{\rho c_1^2},$$

$$a_4 = a_1 i a, \quad a_5 = (1 + \mathcal{G}_0 \omega) i a,$$

$$a_6 = a_2 a^2, \quad a_7 = \bar{\beta} (1 + \mathcal{G}_0 \omega),$$

$$a_8 = \varepsilon_1 (n_1 \omega + \tau_0 n_0 \omega^2) i a,$$

$$a_9 = \varepsilon_2 (n_1 \omega + \tau_0 n_0 \omega^2),$$

$$a_{10} = \bar{K} (n^* + t_1 \omega), \quad a_{11} = (n^* + t_1 \omega),$$

$$a_{12} = (n_1 \omega + \tau_0 \omega^2), \quad a_{13} = \frac{(\lambda + \alpha)}{\rho c_1^2},$$

$$a_{14} = \left(\mu_L + \frac{\beta_1 T_0 p}{2} \right) \frac{1}{\rho c_1^2},$$

$$\bar{K} = \frac{K_2^*}{K_1^*}, \quad \bar{\beta} = \frac{\beta_2}{\beta_1}, \quad \varepsilon_1 = \frac{\beta_1^2 T_0}{\rho K_1^* \omega^*}, \quad \varepsilon_2 = \frac{\beta_1 \beta_2 T_0}{\rho K_1^* \omega^*}. \quad (16)$$

Eliminating u_2^* and T^* from Equations (13)-(15), we obtain

$$[\nabla^6 + \lambda_1 \nabla^4 + \lambda_2 \nabla^2 + \lambda_3] u_1^*(y) = 0, \quad (17)$$

where

$$\begin{aligned} \lambda_1 = & -\frac{1}{g_1} [a^2 a_3 (a_2 a_{11} + a_{10}) \\ & + \omega^2 a_{10} (a_2 + a_3) + a_2 g_2 + a_{10} g_3], \end{aligned} \quad (18)$$

$$\lambda_2 = \frac{1}{g_1} [a^2 a_{11} g_4 + a^2 g_5 + \omega^2 f_1 + a^2 \omega^2 f_2 + f_3], \quad (19)$$

$$\lambda_3 = \frac{1}{g_1} \left[a_5 a_6 a_8 - a^2 a_6 f_4 - a^2 \omega^2 a_{11} (\omega^2 + a_6) + \omega^2 f_5 - a^2 \omega^2 f_4 \right], \quad (20)$$

$$g_1 = a_2 a_3 a_{10}, \quad g_2 = a_3 a_{12} + a_7 a_9, \quad g_3 = a_2 a_6 + a_4^2, \\ g_4 = g_3 + a^2 a_3, \quad g_5 = g_2 + a_6 a_{10},$$

$$f_1 = g_5 + a_{10} \omega^2 + a_2 a_{12}, \quad f_2 = a_{11} (a_3 + a_2) + a_{10}, \\ f_3 = (a_4 (a_4 a_{12} + a_5 a_9 + a_7 a_8) + a_2 a_6 a_{12} - a_3 a_5 a_8),$$

$$f_4 = a_{11} a^2 + a_{12}, \quad f_5 = (a_5 a_8 - \omega^2 a_{12} - a_6 a_{12}), \quad D = \frac{d}{dy}. \quad (21)$$

The solution of Equation (17) has the form

$$u_1^*(y) = \sum_{n=1}^3 M_n(a, \omega) e^{-k_n y} + \sum_{n=1}^3 Q_n(a, \omega) e^{k_n y}, \quad (22)$$

$$u_2^*(y) = \sum_{n=1}^3 M_n'(a, \omega) e^{-k_n y} + \sum_{n=1}^3 Q_n'(a, \omega) e^{k_n y}, \quad (23)$$

$$T^*(y) = \sum_{n=1}^3 M_n''(a, \omega) e^{-k_n y} + \sum_{n=1}^3 Q_n''(a, \omega) e^{k_n y}. \quad (24)$$

where $M_n(a, \omega)$, $Q_n(a, \omega)$ are some parameters depending on a and ω . k_n^2 are the roots of the characteristic Equation(17).

Substituting from Equations (22)-(24) into (13)-(15), we obtain the following relations

$$M_n' = H_{1n} M_n, \quad n = 1, 2, 3 \quad (25)$$

$$M_n'' = H_{2n} M_n, \quad n = 1, 2, 3 \quad (26)$$

$$Q_n' = -H_{1n} Q_n, \quad n = 1, 2, 3 \quad (27)$$

$$Q_n'' = H_{2n} Q_n, \quad n = 1, 2, 3 \quad (28)$$

where

$$H_{1n} = \frac{k_n [a_2 a_7 k_n^2 - a^2 a_7 - \omega^2 a_7 - a_4 a_5]}{[a_4 a_7 k_n^2 - a_3 a_5 k_n^2 + a_5 a_6 + \omega^2 a_5]},$$

$$H_{2n} = \frac{[a_2 k_n^2 - a^2 - \omega^2 - a_4 k_n H_{1n}]}{a_5},$$

$$H_{3n} = i a - a_{13} H_{1n} k_n - (1 + \mathcal{G}_0 \omega) H_{2n},$$

$$H_{4n} = i a a_2 H_{1n} - a_{14} k_n, \quad H_{5n} = i a a_{14} H_{1n} - a_2 k_n,$$

$$H_{6n} = i a a_{13} - a_3 k_n H_{1n} - \bar{\beta} (1 + \mathcal{G}_0 \omega) H_{2n}, \quad n = 1, 2, 3. \quad (29)$$

Thus the solution of Equations (1)-(3) are

$$u_1 = \sum_{n=1}^3 M_n(a, \omega) e^{-k_n y} e^{(\omega t + i a x)} + \sum_{n=1}^3 Q_n(a, \omega) e^{k_n y} e^{(\omega t + i a x)}, \quad (30)$$

$$u_2 = \sum_{n=1}^3 M_n'(a, \omega) e^{-k_n y} e^{(\omega t + i a x)} + \sum_{n=1}^3 Q_n'(a, \omega) e^{k_n y} e^{(\omega t + i a x)}, \quad (31)$$

$$T = \sum_{n=1}^3 M_n''(a, \omega) e^{-k_n y} e^{(\omega t + i a x)} + \sum_{n=1}^3 Q_n''(a, \omega) e^{k_n y} e^{(\omega t + i a x)}, \quad (32)$$

Normal mode analysis of the stress yields the following,

$$t_{22} = -p + \sum_{n=1}^3 H_{6n} M_n e^{-k_n y} e^{(\omega t + i a x)} + \sum_{n=1}^3 H_{6n} Q_n e^{k_n y} e^{(\omega t + i a x)}, \quad (33)$$

$$t_{21} = \sum_{n=1}^3 H_{5n} M_n e^{-k_n y} e^{(\omega t + i a x)} - \sum_{n=1}^3 H_{5n} Q_n e^{k_n y} e^{(\omega t + i a x)}, \quad (34)$$

$$t_{11} = -p + \sum_{n=1}^3 H_{3n} M_n e^{-k_n y} e^{(\omega t + i a x)} + \sum_{n=1}^3 H_{3n} Q_n e^{k_n y} e^{(\omega t + i a x)}, \quad (35)$$

$$t_{12} = \sum_{n=1}^3 H_{4n} M_n e^{-k_n y} e^{(\omega t + i a x)} + \sum_{n=1}^3 H_{4n} Q_n e^{k_n y} e^{(\omega t + i a x)}. \quad (36)$$

4. Boundary Conditions

In order to determine the parameters M_n ($n = 1, 2, 3$) and Q_n ($n = 1, 2, 3$), we consider the following boundary conditions at $y = 0$

- 1) $t_{22}(x, 0^+, t) - t_{22}(x, 0^-, t) = -P_1 e^{(\omega t + i a x)}$,
- 2) $t_{21}(x, 0^+) - t_{21}(x, 0^-) = 0$,
- 3) $u_1(x, 0^+) = u_1(x, 0^-)$,
- 4) $u_2(x, 0^+) = u_2(x, 0^-)$,
- 5) $T(x, 0^+) = T(x, 0^-)$,
- 6) $\frac{\partial T}{\partial y}(x, 0^+) = \frac{\partial T}{\partial y}(x, 0^-)$.

where P_1 is the magnitude of mechanical force.

Using Equations (30)-(34) in boundary condition (37), we get six equations with six unknown parameters M_n ($n = 1, 2, 3$) and Q_n ($n = 1, 2, 3$) as

$$\sum_{n=1}^3 H_{6n} M_n - \sum_{n=1}^3 H_{6n} Q_n = -P_1 + p e^{-(\omega t + i a x)}, \quad (38)$$

$$\sum_{n=1}^3 H_{5n} M_n + \sum_{n=1}^3 H_{5n} Q_n = 0, \quad (39)$$

$$\sum_{n=1}^3 M_n - \sum_{n=1}^3 Q_n = 0 \quad (40)$$

$$\sum_{n=1}^3 H_{1n} M_n + \sum_{n=1}^3 H_{1n} Q_n = 0, \quad (41)$$

$$\sum_{n=1}^3 H_{2n} M_n - \sum_{n=1}^3 H_{2n} Q_n = 0, \quad (42)$$

$$\sum_{n=1}^3 k_n H_{2n} M_n + \sum_{n=1}^3 k_n H_{2n} Q_n = 0. \quad (43)$$

Solving Equations (38)-(43), the parameters M_n ($n=1,2,3$) and Q_n ($n=1,2,3$) are derived as follows:

$$M_1 = \frac{\Delta_1}{\Delta}, M_2 = \frac{\Delta_2}{\Delta}, M_3 = \frac{\Delta_3}{\Delta},$$

$$Q_1 = \frac{\Delta_4}{\Delta}, Q_2 = \frac{\Delta_5}{\Delta}, Q_3 = \frac{\Delta_6}{\Delta}.$$

where $\Delta, \Delta_i, i=1,2,\dots,6$ are defined in the appendix.

5. Particular Cases

5.1. Isotropic Generalized Thermoelastic Medium with Hydrostatic Initial Stress

Substituting $\mu_L = \mu_T = \mu$, $K_1^* = K_2^* = K^*$, $\alpha_1 = \alpha_2 = \alpha_T$ and $\beta_1 = \beta_2 = (3\lambda + 2\mu)\alpha_T$, $\alpha = \beta = 0$ in Equations (30)-(36), we obtain the corresponding expressions of displacement, stress, and temperature distribution in isotropic generalized thermoelastic medium with hydrostatic initial stress.

5.2. Fibre-Reinforced Generalized Thermoelastic Medium

Letting $p \rightarrow 0$, the expressions (30)-(36) reduce to the case of fibre-reinforced generalized thermoelastic medium.

5.3. Isotropic Generalized Thermoelastic Medium

Substituting $\mu_L = \mu_T = \mu$, $K_1^* = K_2^* = K^*$, $\alpha_1 = \alpha_2 = \alpha_T$ and $\beta_1 = \beta_2 = (3\lambda + 2\mu)\alpha_T$, $\alpha = \beta = 0$ and letting $p \rightarrow 0$, the expressions (30)-(36) reduce to an isotropic generalized thermoelastic medium.

For all the cases discussed above the components of displacement, stresses and temperature distribution for the region $-\infty < y \leq 0$, are obtained by inserting

$M_1 = M_2 = M_3 = 0$ in Equations (30)-(36).

Similarly in the region $0 \leq y < \infty$, the components are obtained by inserting $Q_1 = Q_2 = Q_3 = 0$ in Equations (30)-(36).

6. Special Cases of Thermoelastic Theory

6.1. Equation of Coupled Thermoelasticity

The equations of the coupled thermoelasticity (C-T

theory) are obtained when

$$n^* = n_1 = 1, t_1 = \tau_0 = \mathcal{G}_0 = 0. \quad (44)$$

6.2. Lord-Shulman Theory

For the Lord-Shulman (L-S theory)

$$n^* = n_1 = n_0 = 1, t_1 = \mathcal{G}_0 = 0, \tau_0 > 0. \quad (45)$$

6.3. Green-Lindsay Theory

For Green-Lindsay (G-L theory),

$$n^* = n_1 = 1, n_0 = 0, t_1 = 0, \mathcal{G}_0 \geq \tau_0 > 0. \quad (46)$$

where \mathcal{G}_0, τ_0 are the two relaxation times.

6.4. Equations of Generalized Thermoelasticity

The equations of the generalized thermoelasticity without energy dissipation (the linearized G-N theory of type II) are obtained when

$$n^* > 0, n_1 = 0, n_0 = 1, t_1 = \mathcal{G}_0 = 0, \tau_0 = 1, \quad (47)$$

Equations (1) and (2) are the same and Equation (3) takes the form

$$K_1^* \frac{\partial^2 T}{\partial x^2} + K_2^* \frac{\partial^2 T}{\partial y^2} = \rho C^* \frac{\partial^2 T}{\partial t^2} + T_0 \frac{\partial^2}{\partial t^2} \left(\beta_1 \frac{\partial u_1}{\partial x} + \beta_2 \frac{\partial u_2}{\partial y} \right). \quad (48)$$

where n^* is constant with the dimension of $\left(\frac{1}{s}\right)$ and

K_1^*, K_2^* are characteristic constants of this theory.

7. Numerical Results

With a view to illustrating the analytical procedure presented earlier, we now consider a numerical example for which computational results are given. The results depict the variations of normal displacement, normal force stress and temperature distribution in the context of G-L theory. For this purpose, we take the following values of physical constants as Singh [20]

$$\rho = 2660 \text{ Kg/m}^3, \lambda = 5.65 \times 10^{10} \text{ Nm}^{-2},$$

$$\mu_T = 2.46 \times 10^{10} \text{ Nm}^{-2}, \mu_L = 5.66 \times 10^{10} \text{ Nm}^{-2},$$

$$C^* = 0.787 \times 10^3 \text{ JKg}^{-1} \text{ deg}^{-1},$$

$$\alpha = -1.28 \times 10^{10} \text{ Nm}^{-2}, \beta = 220.90 \times 10^{10} \text{ Nm}^{-2},$$

$$K_1^* = 0.0921 \times 10^3 \text{ Jm}^{-1} \text{ deg}^{-1} \text{ s}^{-1},$$

$$K_2^* = 0.0963 \times 10^3 \text{ Jm}^{-1} \text{ deg}^{-1} \text{ s}^{-1},$$

$$\alpha_1 = 0.017 \times 10^4 \text{ deg}^{-1},$$

$$\alpha_2 = 0.015 \times 10^4 \text{ deg}^{-1}, T_0 = 293^\circ \text{ K}.$$

The computations are carried out on the surface $y = 1.0$ at $t = 0.1$. The graphical results for normal displacement u_2 , normal force stress t_{22} and temperature distribution T are shown in **Figures 1-3** with $p = 1.5$, $\omega = \omega_0 + t\omega_1$, $\omega_0 = 2.3$, $\omega_1 = 0.1$, $a = 2.1$ for a

- 1) Fibre-reinforced generalized thermoelastic medium with hydrostatic initial stress (FRGTEHIS) by solid line.
- 2) Fibre-reinforced generalized thermoelastic medium without hydrostatic initial stress (FRGTEWHIS) by solid line with centered symbol (*).
- 3) Isotropic generalized thermoelastic medium with

hydrostatic initial stress (IGTEHIS) by dashed line.

4) Isotropic generalized thermoelastic medium without hydrostatic initial stress (IGTEWHIS) by dashed line with centered symbol (*).

These graphical results represent the solutions obtained by using the generalized theory with two relaxation times (G-L theory) by taking $\tau_0 = 0.02$, $\vartheta_0 = 0.03$.

8. Discussions

The values of normal displacement for the case of FRGTEHIS increase sharply in the range and then

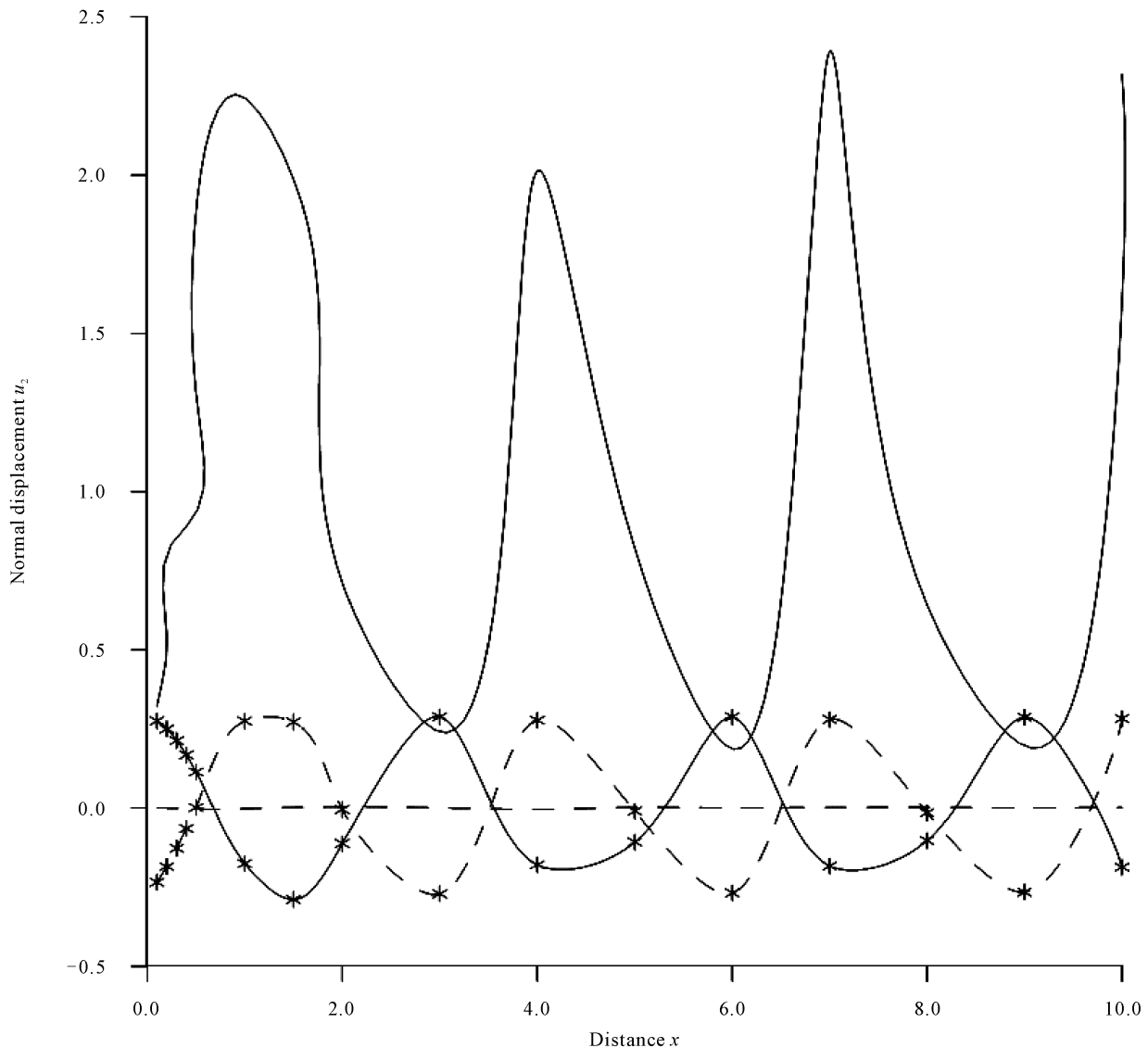


Figure 1. Variations of normal displacement u_2 with horizontal distance x for mechanical force: (—) FRGTEHIS – Fibre-reinforced generalized thermoelastic medium with hydrostatic initial stress, (-----) FRGTEWHIS- Fibre-reinforced generalized thermoelastic medium without a hydrostatic initial stress, (*-*) IGTEHIS – isotropic generalized thermoelastic medium with a hydrostatic initial stress, (*-*) IGTEWHIS – isotropic generalized thermoelastic medium without hydrostatic initial stress.

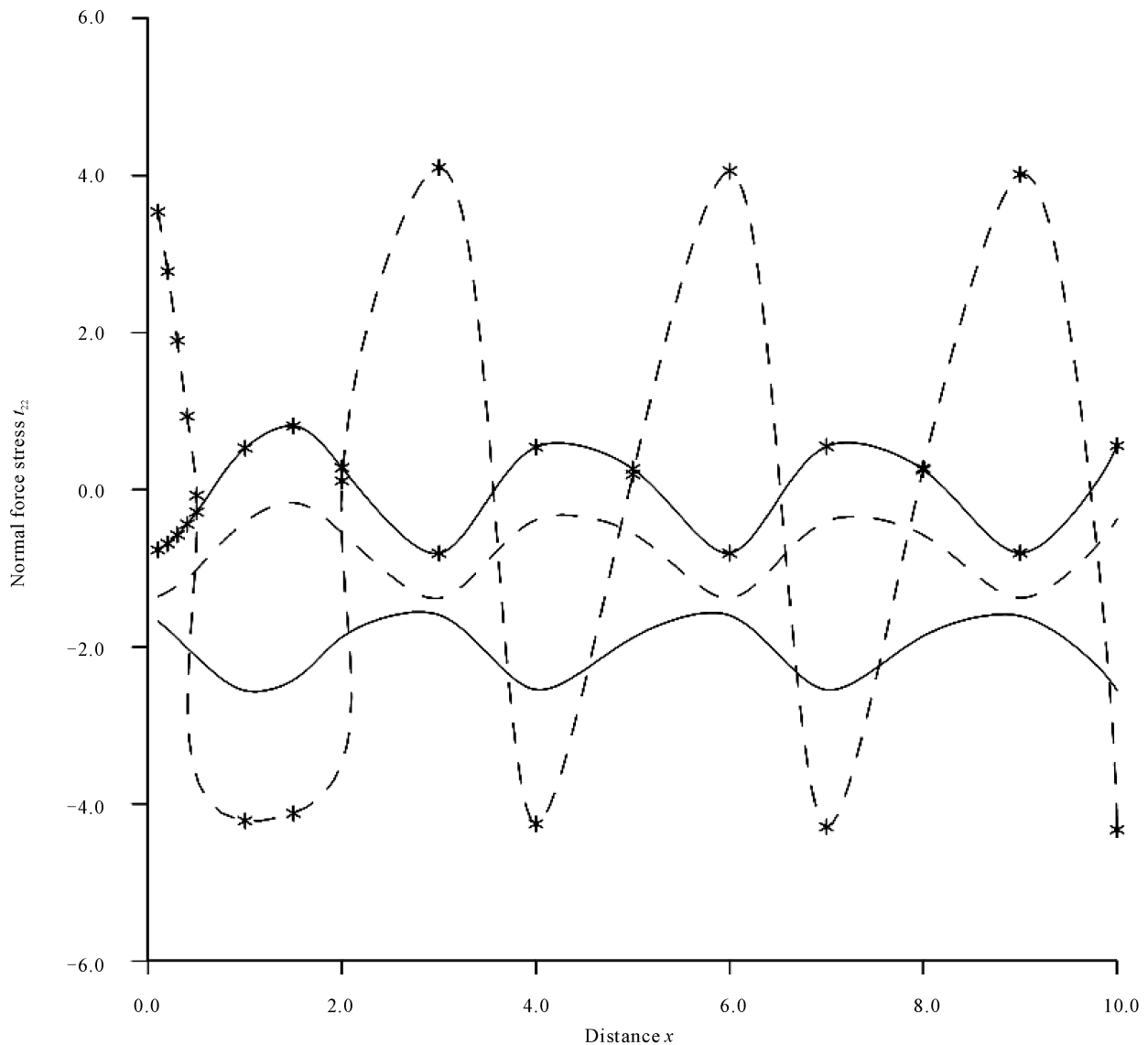


Figure 2. Variations of normal force stress t_{22} with horizontal distance x for mechanical force: (—)FRGTEHIS, (-----)FRGTEWHIS, (-*-*)IGTEHIS, (*-*)IGTEWHIS. [see Figure 1 for explanation of symbols].

oscillate with distance. In case of IGTEHIS, the variations of normal displacement are very less in magnitude and for FRGTEWHIS, IGTEWHIS, these variations are quite uniform and at a particular point these variations are opposite in nature *i.e.* when one is on the zenith and the other one is on the lowest point. These variations are shown in **Figure 1**.

It is observed from **Figure 2** that the variations of normal force stress for FRGTEHIS, FRGTEWHIS, IGTEHIS are similar in oscillating manner. The variation of normal force stress for IGTEWHIS is highly oscillating in nature in comparison to the variations obtained for FRGTEHIS, FRGTEWHIS, IGTEHIS.

The deformation of the body effects the change in

temperature to a large extent as compared to normal displacement and normal force stress which is evident from **Figure 3**. Among all the mediums, the variation of temperature distribution is least oscillating for IGTEHIS.

9. Conclusions

- 1) The affects of anisotropy and hydrostatic initial stress are observed on all the quantities.
- 2) The variations of the temperature distribution are more oscillatory in nature than those of of normal force stress and normal displacement .
- 3) The variations for L-S and G-L theory of thermoelasticity are close, although the authors have de-

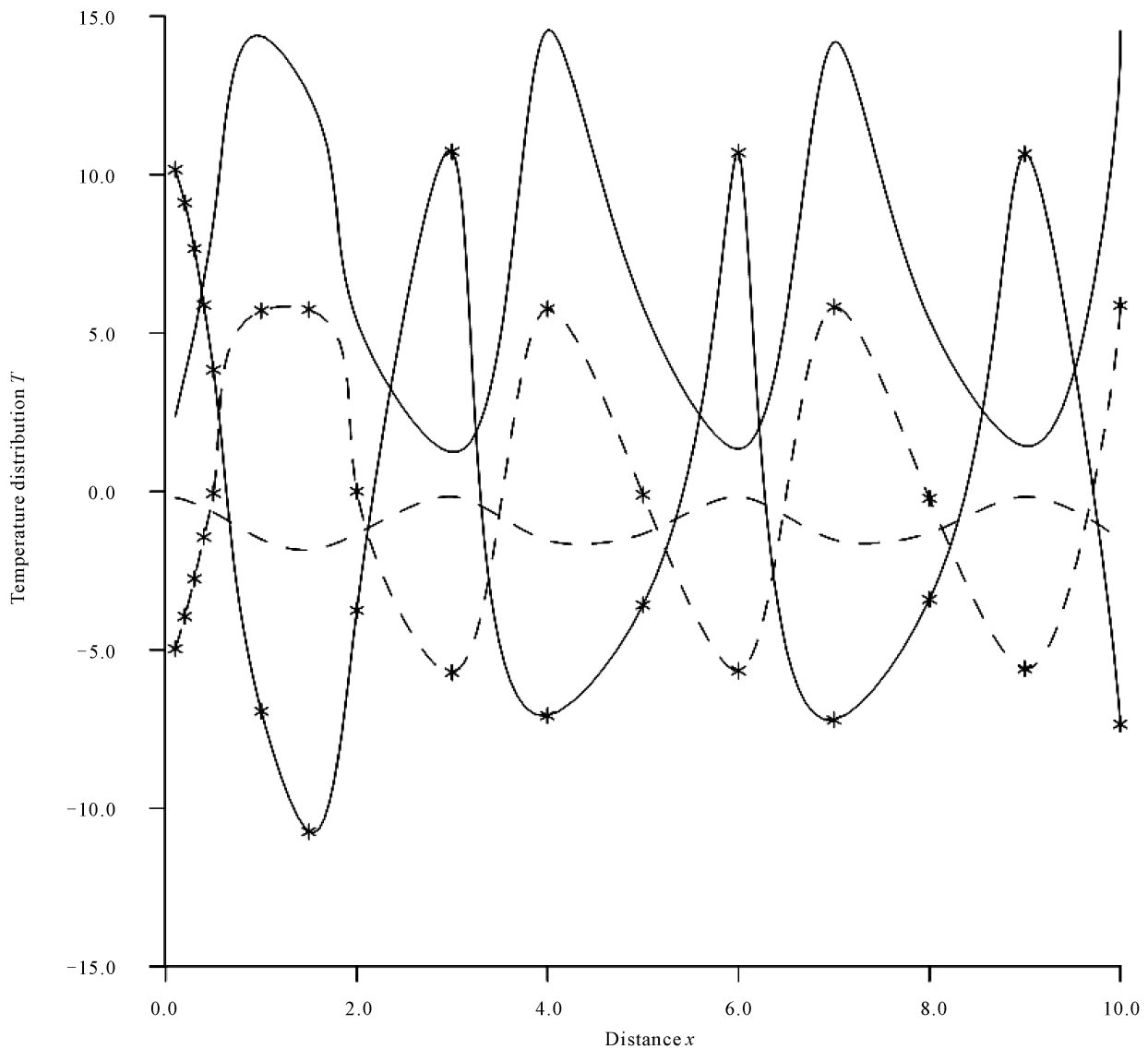


Figure 3. Variations of temperature distribution T with horizontal distance x for mechanical force: (—)FRGTEHIS, (-----)FRGTEWHIS, (*—*—*) IGTEHIS, (*-----*)IGTEWHIS. [see Figure 1 for explanation of symbols].

picted the graphical results only for G-L theory.

10. References

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Appendix

$$\begin{aligned} \Delta &= 8 \left[H_{61} (H_{22} - H_{23}) + H_{62} (H_{23} - H_{21}) + H_{63} (H_{21} - H_{22}) \right] \\ &\quad \cdot \left[H_{51} (k_3 H_{12} H_{23} - k_2 H_{13} H_{22}) + H_{52} (k_1 H_{13} H_{21} - k_3 H_{11} H_{23}) + H_{53} (k_2 H_{11} H_{22} - k_1 H_{12} H_{21}) \right], \\ \Delta_1, \Delta_4 &= \pm \left(-P_1 + p e^{-(\omega t + i a x)} \right) (H_{22} - H_{23}) \\ &\quad \cdot \left[H_{51} (k_3 H_{12} H_{23} - k_2 H_{13} H_{22}) + H_{52} (k_1 H_{13} H_{21} - k_3 H_{11} H_{23}) + H_{53} (k_2 H_{11} H_{22} - k_1 H_{12} H_{21}) \right], \\ \Delta_2, \Delta_5 &= \pm \left(-P_1 + p e^{-(\omega t + i a x)} \right) (H_{23} - H_{21}) \\ &\quad \cdot \left[H_{51} (k_3 H_{12} H_{23} - k_2 H_{13} H_{22}) + H_{52} (k_1 H_{13} H_{21} - k_3 H_{11} H_{23}) + H_{53} (k_2 H_{11} H_{22} - k_1 H_{12} H_{21}) \right], \\ \Delta_3, \Delta_6 &= \pm \left(-P_1 + p e^{-(\omega t + i a x)} \right) (H_{21} - H_{22}) \\ &\quad \cdot \left[H_{51} (k_3 H_{12} H_{23} - k_2 H_{13} H_{22}) + H_{52} (k_1 H_{13} H_{21} - k_3 H_{11} H_{23}) + H_{53} (k_2 H_{11} H_{22} - k_1 H_{12} H_{21}) \right]. \end{aligned}$$