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De-Noising of ECG Signals by Design of an Optimized Wavelet

Vahid Makhdoomi Kaviri[1], Masoud Sabaghi[2*], Saeid Marjani[3]

[1]Department of Electrical Engineering, Islamshar Branch, Islamic Azad University, Tehran, Iran
[2]Laser and Optics Research School, Nuclear Science and Technology Research School, Atomic Energy Organization of Iran, Tehran, Iran
[3]Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
Email: *msabaghi@aeoi.org.ir

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Abstract

In this paper, a different method for de-noising of ECG signals using wavelets is presented. In this strategy, we will try to design the best wavelet for de-nosing. Genetic algorithm tests wide range of quadrature filter banks and the best of them will be chosen that minimize the Signal-to-Noise Ratio (SNR). Furthermore, the wavelet function and scaling function related to these filters are reported as the best wavelet for de-noising. Simulation results for de-noising of a noisy ECG signal show that using obtained wavelet by proposed method improves the SNR of about 2.5 dB.

Keywords

Wavelets, De-Noising, Genetic Algorithm, ECG Signals

1. Introduction

There are many methods to de-noise a signal. De-noising is so important in signal processing, particularly for biomedical signals. Discrete Wavelet Transform (DWT) is currently used in a wide variety of signal processing applications, such as audio and video compression, removal of noise in audio, and the simulation of wireless antenna distribution. The wavelet transforms have good properties such as the time-frequency localization, energy compaction and sub-band coding. Wavelet transform is one of the most powerful mathematical tools for digital signal processing [1], such as compression and de-noising.

Genetic algorithm (GA) is an optimization technique based on the "survival of the fittest" [2]. In this method, a wide range of inputs are checked and the best one of them is chosen. Genetic algorithm calculates the fitness of the solution according to fitness function by search approximately all of the possible inputs. Discrete wavelet transform is implemented by digital FIR filters.

When the voltage of the ECG waveform is at least larger than 75% of the peak value of the ECG in the comparator stage (digital-comparator) [3]-[5] of the peak detector, it allows oscillation generator to be fed into speaker for beeping. Digitizing ECG signals carry out using successive approximation ADC control system [6] [7] or delta-sigma analog to digital converter (decimation filter) [8]-[12]. In order to generate an error signal, a subtractor configures to subtract the filtered ECG signal that is generated by the adder [13] from the ECG signal input to the inputter. After proper amplification and filtration of the ECG signal, it is given to a voltage controlled oscillator (VCO) [14].

The ECG signal is a reference signal for pulse wave delay using photoplethysmographic signal and Laser-Doppler (LD) measurements. The basis of the registration is the selfmixing in the diode lasers cavities [15]-[25]. Also, the ECG signals are going to be transmitted into laser beam such as VCSELs [26]-[38] to stablish communication between the ECG and the medical center.

In this paper, the best coefficients of filters will be obtained using genetic algorithm. GA searches several coefficients and thresholds to reach the best output SNR. The paper is organized as follows: in Section 2, de-noising using wavelet implementation is presented. In Section 3, genetic algorithm is introduced and then the proposed method is developed. Simulation results are presented in Section 4. Finally conclusion is given in Section 5.

2. Wavelet Based De-Noising

The process of removing the noise, $e[n]$, from a signal, $\hat{x}[n] = x[n] + e[n]$ is called "de-nosing". For example, $e[n]$ may be a Gaussian white noise process, which is statistically independent of $x[n]$. A method for de-noising is the applying a nonlinear operation to a representation of $\hat{x}[n]$, like fourier transform or wavelet transform. In this work, wavelet transform is chosen. The de-noising procedure is as follows: First, the signal $\hat{x}[n]$ is decomposed using a filter bank, thus performing discrete wavelet transform. Then, the wavelet coefficients are manipulated in order to remove the noise component. Two approaches known as *hard* and *soft thresholding* have been proposed for this purpose. In hard thresholding, coefficients that their absolute values are smaller than a specific threshold, are replaced with zero. The idea of thresholding is that $x[n]$ can be represented via a number of wavelet coefficients, while the noise has wideband characteristics and spreads out on all coefficients. Thus, it provided the threshold $\varepsilon$is chosen appropriately, the signal constructed from the manipulated wavelet coefficients will contain much less noise than $\hat{x}[n]$ does [39] [40]. On the other hand, the waveform of used wavelet is so important for de-noising (or compression). In [41] [42], several de-noising methods using wavelets are compared.

3. Wavelet Design Using Genetic Algorithm

In this section after a brief summary of genetic algorithm, the concept of *multi-resolution* *analysis* and the efficient realization of the discrete wavelet transform based on multi-rate filter banks are presented. Then, the proposed method is discussed.

3.1. Genetic Algorithm (GA)

Genetic Algorithm is used to introduce computer-based problems solving systems, which use computational models of evolutionary processes. Different algorithms have been proposed in literature, such as: GAs, evolutionary programming, evolution strategies, classifier systems, and genetic programming. Via processes of selection, mutation and reproduction, these algorithms present a common conceptual base. The genetic algorithms are based on reproduction, fitness, crossover and mutation. The standard procedure of genetic algorithms is as follow:

1) Candidate solutions to a problem have been started with a randomly generated population of n 1-bit strings.

2) Fitness function $f(x)$ of each string in the population is calculated.

3) Until n new strings have been created, the following steps have to repeat:

a) From the current population, a pair of parent strings is selected where probability of selection being an increasing function of fitness.

b) In order to form two new strings, cross over the pair at a random point with the crossover probability.

c) With the mutation probability, the two new strings obtained from previous step mutate at each locus. Then they place the resulting strings in the new population.

4) The current population has to replace with the new population.

5) Go to step 2.

3.2. Multi-Resolution Analysis for Design of an Appropriate Wavelet for De-Noising

The main concept of wavelet transform based on *multi-resolution analysis* are presented in this section. This framework has been developed by Meyer, Mallat and Daubechies mainly, for the orthonormal cases [43] [44]. Design procedure is generally performed by designing a Quadrature Mirror filter Bank (QMFB) with Perfect Reconstruction (PR) conditions [40]. **Figure 1** shows the analysis and synthesis filters.

If the output signal be a delayed version of the input signal, perfect reconstruction is obtained. PR conditions for the filter bank that is shown in **Figure 1** are:

$$H_1(-z)F_1(z)+H_2(-z)F_2(z)=0 \tag{1}$$

$$H_1(z)F_1(z)+H_2(z)F_2(z)=2z^{-d} \tag{2}$$

Condition (1) says that the output signal contains no aliasing, but amplitude distortions may be occurred. If both (1) and (2) are satisfied, the amplitude distortions are also vanishing. There are many proper filter's coefficients to satisfy in (1), but condition (2) is only complied approximately. Proposed method is based on using Genetic Algorithm (GA) to reach the best filter coefficients. In other words, best wavelet function that results minimum SNR after de-noising. So, some parameters are arbitrary in design

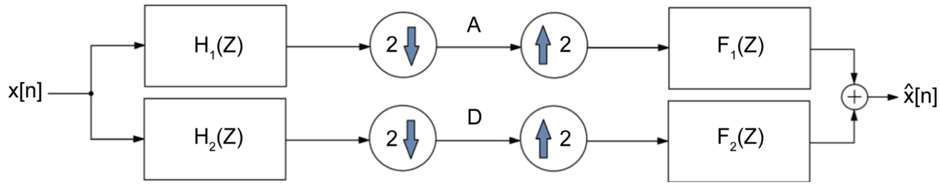

Figure 1. Quadrature mirror filter bank (QMFB).

procedure. A proper set of conditions used vastly in wavelet design are:

$$H_2(z) = -z^{-l} H_1(-z^{-1}) \tag{3}$$

$$F_1(z) = H_2(-z) \tag{4}$$

$$F_2(z) = -H_1(-z) \tag{5}$$

Note that the $H_1(z)$ is analysis low-pass filter. These conditions meet the Equation (1). Therefore, it is enough to find the filters that can satisfy Equation (2) exactly, or approximately. When the number of wavelet levels is not exceeded from 3 or 4, a near-PR filter bank is sufficient and gives good results. In this work, we use 3-level decomposition and reconstruction.

One chooses the coefficients of a PR two-channel filter bank in such a way that the wavelets and scaling functions associated with these filters have the desired properties. For constructing wavelets, we use two equations called two-scale relations [40]:

$$\phi(t) = \sum_n f_1[n]\sqrt{2}\phi(2t-n) \tag{6}$$

$$\psi(t) = \sum_n f_2[n]\sqrt{2}\phi(2t-n) \tag{7}$$

$\phi(t)$ and $\psi(t)$ are called "scaling function" and "waveletfunction", respectively. The starting point to constructing scaling functions is the first part of the two-scale relation. In this manner, first we must select coefficients of reconstruction low pass filter, $f_1[n]$, appropriately. To construct biorthogonal and orthonormal scaling functions and wavelets, the coefficients of PR two-channel filter banks are required. Since the scaling function, $\phi(t)$, is supposed to be a low-pass impulse response, generally, it introduces the normalization [40]:

$$\Phi(0) = \int_{-\infty}^{+\infty} \phi(t)\,\mathrm{d}t = 1 \tag{8}$$

where, $\Phi(\omega)$ is fourier transform of scaling function. By integrating from $\phi(t)$ and $\psi(t)$ in Equations (6) and (7), and this fact that $\int_{-\infty}^{+\infty} \psi(t) = 0$, we will obtain:

$$\sum_n f_1[n] = \sqrt{2} \tag{9}$$

$$\sum_n f_2[n] = 0 \tag{10}$$

If $F_1(-1) = 0$, a low-pass behavior for $F_1(z)$ is achieved. Therefore:

$$\sum_n (-1)^n f_1[n] = 0 \tag{11}$$

From (3), (4), (5), $f_2[n] = -(-1)^n f_1[n]$. Then if $F_1(-1) = 0$, Equation (10) will be satisfied. A simple way to have zeros in $z = -1$ is to use a Type-II symmetric linear phase FIR filter for $f_1[n]$. An suitable set of coefficients can be realized when condition (2) is satisfied. $\phi(t)$ and $\psi(t)$ should possibly have been several continuous derivatives. A test that can check the regularity of the product is introduced by Daubechies. Assuming that $F_1(z)$ has $\boldsymbol{N}$ zeros in $z = -1$, $F_1(z)$ can be written as [40]:

$$F_1(z) = \sqrt{2}R(z)\frac{\left(1+z^{-1}\right)^N}{2^N} \tag{12}$$

In order to achieving smooth wavelets with continuous derivatives, three zeros in $z = -1$ are considered. Then, Equation (12) can be written as:

$$F_1(z) = \left(\frac{\sqrt{2}}{8}\right)R(z)\left(1+3z^{-1}+3z^{-2}+z^{-3}\right) \tag{13}$$

As seen from Equation (9), $R(1)=1$. Therefore, $R(z)$ can be considered as a polynomial of z^{-1}, with $\sum_n r[n]=1$. If $f_1[n]$ is a low-pass filter with 8 coefficients and three zeros in $z = -1$, $R(z)$ is a FIR system with five coefficient where its summation is equal to 1.

3.3. GA Strategy for Design Optimum Coefficients to Minimize the SNR

In this work, a 3-level wavelet decomposition is implemented. The FIR filters have 8-coefficients. Then, thresholding and reconstruction are performed on sub-bands to obtain the de-noised signal. The strategy is as follows: Fur coefficients of $r[n]$ is arbitrary chosen as first population for genetic algorithm. The 8 coefficients that constructing low-pass filter, $F_1(z)$, are made according to realization of the discrete wavelet transform based on multi-rate filter banks. Figure 2 shows a 3-level thresholding and reconstruction to attain the de-noised signal. In GA technique, thresholds also considered for all sub-bands to finding the best filter coefficients (used wavelet) and thresholds which minimize the SNR.

4. Simulation Results

We consider a smooth and noiseless ECG signal as a reference to calculate SNR. A Gaussian White Noise (GWN) is added to this pure signal to make a noisy ECG with a defined SNR. Then, the proposed algorithm is applied on this noisy signal and results are compared with other wavelets de-noising from a SNR point of view. Obtained filter coefficients define a wavelet function and a scaling function, which also will report. Simulation results are presented in **Table 1**: the wavelet that introduced by the proposed method improve the output SNR about 2.5 dB more than other wavelets implemented by means of filters which have 8 coefficients.

Figure 3 shows the noisy input signal and de-noised output signal using the proposed method for two situations. For an input SNR of 36.7 dB, the output SNR is 48.7 dB and for an input SNR of 45.2 dB, the output SNR is 59.2 dB. It's clear that if we use the filters that have more coefficients, the results will be improved. Finally, **Figure 4**

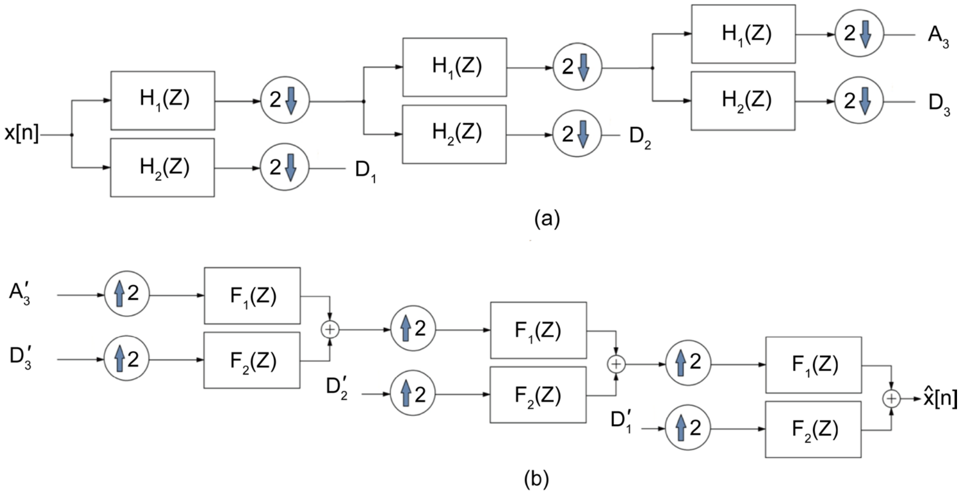

Figure 2. 3-level wavelet implementation. (a) Decomposition; (b) Reconstruction.

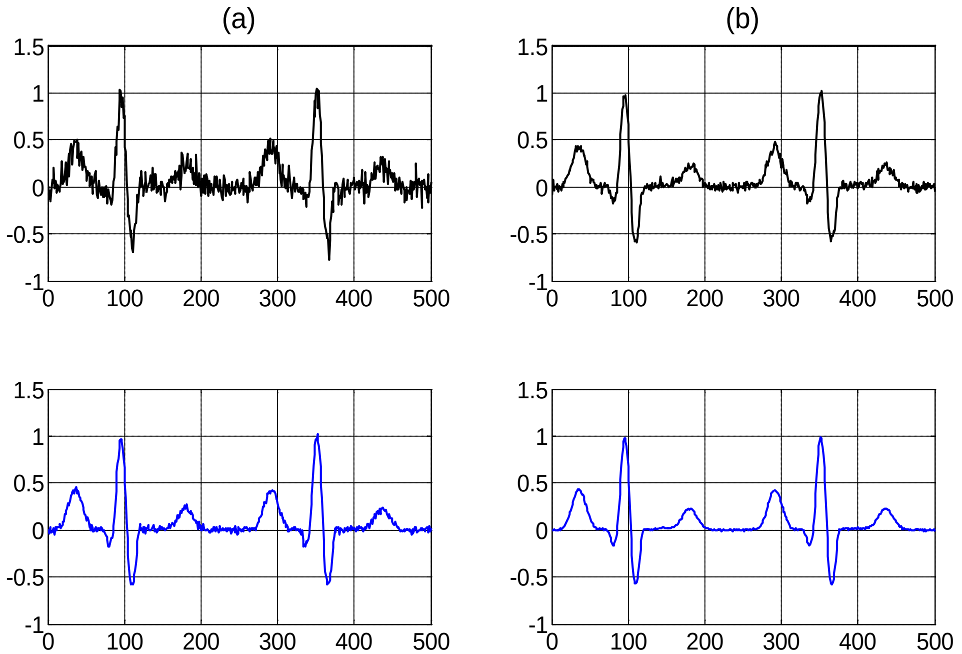

Figure 3. (a) The noisy and de-noised signal using proposed method (input SNR = 36.7 dB and output SNR = 48.7 dB) and (b) noisy and de-noised signal using proposed method (input SNR = 45.2 dB and output SNR = 59.2 dB).

Table 1. Output and input SNR for different wavelets.

Wavelet name	Input SNR (dB)	Output SNR (dB)	Improve (dB)
Bior3.3	36.7	46.57	9.87
Db. 4	36.7	46.19	9.49
Sym. 4	36.7	46.26	9.56
Proposed method	36.7	48.7	12

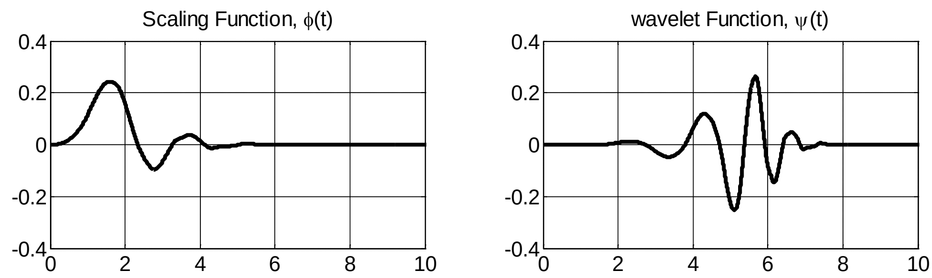

Figure 4. Optimized wavelet function and scaling function.

shows the wavelet function and scaling function constructed from 8-coefficcient filters found by Genetic Algorithm, respectively.

5. Conclusion

A new algorithm for de-noising of ECG signals is presented. The method is based on making a specific wavelet function for minimizing the SNR. By using genetic algorithms, the coefficients of wavelet filter bank alter smoothly until the best SNR for output signal achieved. We used this method and de-noised a noisy ECG signal with 3-level wavelet structure that used 8-coefficient-filters and the results were improved in comparison by typical wavelets such as Daubechies 4, Symlet4 and Bior 3.3.

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