

Bipolar Valued Fuzzy α -Ideal of BF-Algebra

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Abstract

In the mathematical applications, ideal concepts are involved. They have been studied and analyzed in various ways. Already ideal and α -ideal concepts were discussed in BF-algebras. In this paper the idea of bipolar valued fuzzy α -ideal of BF algebra is proposed. The relationship between bipolar valued fuzzy ideal and bipolar valued fuzzy α -ideal is studied. Some interesting results are also discussed.

Keywords

BF-Algebra, Ideal, Bipolar Valued Fuzzy Ideal, Bipolar Valued Fuzzy α -Ideal

1. Introduction

After the concept of fuzzy sets of Zadeh [1], Lee [2] proposed an extension of fuzzy sets namely Bipolar Valued Fuzzy Sets (BVFS). Their range of membership degree has been extended from the interval $[0, 1]$ to $[-1, 1]$ and in [3]. He made a comparison with other fuzzy settings. These Bipolar valued fuzzy sets possess degrees of membership that denote the degree of satisfaction to the property corresponding to a fuzzy set and its counter-property in a bipolar valued fuzzy set. The membership degree 0 refers that the elements are irrelevant to the corresponding property. Further, the membership degrees $(0, 1]$ show that the elements somewhat satisfy the property, and the membership degrees $[-1, 0)$ denote that the elements somewhat satisfy the implicit counter property.

There are two kinds of representations in the definition of bipolar valued fuzzy sets. They are canonical representation and reduced representation. In this work, the canonical representation of bipolar valued fuzzy sets is utilized.

In 2011, Bipolar valued fuzzy K-subalgebras are discussed by Farhat Nisar [4]. The authors of [5] studied the

concepts of Intuitionistic L-fuzzy p-ideals of BF-algebras and their related results.

Inspired by the concepts recently, the concept of Filters of BCH-Algebras Based on Bipolar-Valued Fuzzy Sets [6] has been discussed. In this paper, these concepts are intended to α -ideal of BF-algebras and bipolar valued fuzzy α -ideal of a BF-algebra is proposed. The nature of the homomorphic images of bipolar valued fuzzy α -ideal of a BF-algebra is also analyzed.

The paper is organized as follows: Section 2 provides the preliminaries. In Section 3, Bipolar valued fuzzy α -ideal is discussed and in Section 4, homomorphism on Bipolar valued fuzzy α -ideal is studied. Section 5 gives the conclusion.

2. Preliminaries

In this section, some basic definitions and results that are required in the sequel are recalled. The notations $\min(a, b) = a \wedge b$ and $\max(a, b) = a \vee b$ are used.

2.1. Basic Results on BF-Algebras

Definition 2.1. [7] A BF algebra is a non-empty set X with a constant 0 and a single binary operation $*$ which satisfies the following axioms:

- 1. $x * x = 0$
- 2. $x * 0 = x$
- 3. $0 * (x * y) = y * x$ for all $x, y \in X$.

Example 2.2. Let $X = \{0, 1, 2, 3, 4\}$ be a set which comprises the following table.

*	0	1	2	3	4
0	0	4	3	2	1
1	1	0	4	3	2
2	2	1	0	4	3
3	3	2	1	0	4
4	4	3	2	1	0

Then $(X, *, 0)$ is BF-algebra.

Definition 2.3. [7] A BG-algebra is a non-empty set X with a constant 0 and a single binary operation $*$ satisfying the following axioms:

- $x * x = 0$
- $x * 0 = x$
- $(x * y) * (0 * y) = x$ for all $x, y \in X$.

A binary relation \leq in a BF-algebra X can be defined as $x \leq y$, if and only if $x * y = 0$.

A subset S of a BF-algebra X is called a subalgebra of X , if $x * y \in S$ for all $x, y \in S$.

An ideal of a BF-algebra X is a subset I of X consisting 0 such that, if $x * y \in I$ and $y \in I$, then $x \in I$.

An ideal I of a BF-algebra X is called closed, if $0 * x \in I \forall x \in I$.

A non-empty subset I of a BF-algebra X is α -ideal, if for all $x, y, z \in X$, $(x * z) * (y * z) \in I$ and $y \in I \Rightarrow x \in I$.

An α -ideal I of X is called closed, if $0 * x \in I \forall x \in X$.

A fuzzy set μ in a BF-algebra X can be called as a fuzzy subalgebra of X , if it satisfies:

$$\mu(x * y) \geq \mu(x) \wedge \mu(y) \text{ for all } x, y \in X \quad (1)$$

A fuzzy set μ in a BF-algebra X can be called as a fuzzy ideal of X , if it satisfies:

$$\mu(0) \geq \mu(x) \text{ for all } x \in X \quad (2)$$

$$\mu(x) \geq \mu(x * y) \wedge \mu(y) \text{ for all } x, y \in X \quad (3)$$

A fuzzy set μ in a BF-algebra X can be called as a fuzzy α -ideal of X , if it satisfies:

$$\mu(0) \geq \mu(0)(x) \text{ for all } x \in X \tag{4}$$

$$\mu(x) \geq \mu((x * z) * (y * z)) \wedge \mu(y) \text{ for all } x, y, z \in X \tag{5}$$

Definition 2.4. [6] A function $f : X \rightarrow Y$ of BF-algebras is considered to be homomorphism of X , if

$$f(x * y) = f(x) * f(y) \quad \forall x, y \in X.$$

Remark 2.5. If $f : X \rightarrow Y$ is a homomorphism on BF-algebras, $f(0_x) = 0_y$.

Definition 2.6. [6] A function $f : X \rightarrow Y$ of BF-algebras is said to be anti-homomorphism of X if

$$f(x * y) = f(y) * f(x) \quad \forall x, y \in X.$$

2.2. Basic Results on Bipolar Valued Fuzzy Set

Fuzzy sets are generally useful mathematical structures which represent a collection of objects whose boundary is vague. Several kinds of fuzzy set extensions are there in the fuzzy set theory. The examples are intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, etc. This section starts with the definition of Bipolar Valued Fuzzy Set.

Definition 2.7. Let X be a non empty set. A Bipolar Valued Fuzzy Set (BVFS) B in X is an object with the form

$$B = \{(x; \mu^+(x), \nu^-(x)) \mid x \in X\}$$

where $\mu^+ : X \rightarrow [0, 1]$ and $\nu^- : X \rightarrow [-1, 0]$ are mappings.

The positive membership degree $\mu^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set $B = \{(x; \mu^+(x), \nu^-(x)) \mid x \in X\}$ and the negative membership degree $\nu^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set $B = \{(x; \mu^+(x), \nu^-(x)) \mid x \in X\}$. If $\mu^+(x) \neq 0$ and $\nu^-(x) = 0$, x is regarded as possessing only positive satisfaction for $B = \{(x; \mu^+(x), \nu^-(x)) \mid x \in X\}$. If $\mu^+(x) = 0$ and $\nu^-(x) \neq 0$, it denotes that x does not satisfy the property of $B = \{(x; \mu^+(x), \nu^-(x)) \mid x \in X\}$ but somewhat satisfies the counter property of $B = \{(x; \mu^+(x), \nu^-(x)) \mid x \in X\}$. It is possible for an element x to be such that $\mu^+(x) \neq 0$ and $\nu^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X . For the sake of simplicity, the symbol $B = (X; \mu^+, \nu^-)$ shall be used for the bipolar valued fuzzy set $B = \{(x; \mu^+(x), \nu^-(x)) \mid x \in X\}$.

Definition 2.8. A BVFS B in a set X with the positive membership $\mu^+ : X \rightarrow [0, 1]$ and negative membership $\nu^- : X \rightarrow [0, 1]$ is indicated to have Sup-Inf property, if for any subset T of X , there exists $x_0 \in T$ such that $\mu^+(x_0) = \sup_{t \in T} \mu^+(t)$ and $\nu^-(x_0) = \inf_{t \in T} \nu^-(t)$.

Definition 2.9. Let $f : X \rightarrow Y$ be a function and let $A = (X; \mu_A^+, \nu_A^-)$ and $B = (Y; \mu_B^+, \nu_B^-)$ be the bipolar valued fuzzy sets of X and Y , respectively. Then, the image of A under f is defined as $f(A) = (Y; \mu_{f(A)}^+, \nu_{f(A)}^-)$ such that

$$\mu_{f(A)}^+(y) = \begin{cases} \sup_{z \in f^{-1}(y)} \mu_A^+(z) & \text{if } f^{-1}(y) = \{x : f(x) = y\} \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

and

$$v_{f(A)}^-(y) = \begin{cases} \inf_{z \in f^{-1}(y)} v_A^-(z) & \text{if } f^{-1}(y) = \{x : f(x) = y\} \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.10. Let $f : X \rightarrow Y$ be a function and let $A = (X; \mu_A^+, \nu_A^-)$ and $B = (Y; \mu_B^+, \nu_B^-)$ be the bipolar valued fuzzy sets of X and Y , respectively. Then, the inverse image of B under f is defined as

$$f^{-1}(B) = (X; \mu_{f^{-1}(B)}^+, \nu_{f^{-1}(B)}^-)$$

such that $\mu_{f^{-1}(B)}^+(x) = \mu_B^+(f(x))$ and $\nu_{f^{-1}(B)}^-(x) = \nu_B^-(f(x)) \forall x \in X$.

3. Bipolar Valued Fuzzy α -Ideal

In this section, Bipolar valued fuzzy α -ideal of a BF-algebra is defined. It is also proved that any Bipolar valued fuzzy α -ideal in X is a Bipolar valued fuzzy BF-ideal and the sufficient condition is derived for the converse.

Definition 3.1. A BVFS $B = (X; \mu^+, \nu^-)$ in X is called a bipolar valued fuzzy subalgebra of X , if it satisfies:

$$\mu^+(x * y) \geq \mu^+(x) \wedge \mu^+(y)$$

$$\nu^-(x * y) \leq \nu^-(x) \vee \nu^-(y) \quad \forall x, y \in X \tag{6}$$

Definition 3.2. A BVFS $B = (X; \mu^+, \nu^-)$ in X is called a bipolar valued fuzzy ideal (BVF-ideal) of X , if it satisfies:

$$\mu^+(0) \geq \mu^+(x) \text{ and } \nu^-(0) \leq \nu^-(x) \tag{7}$$

$$\mu^+(x) \geq \mu^+(x * y) \wedge \mu^+(y)$$

$$\nu^-(x) \leq \nu^-(x * y) \vee \nu^-(y) \quad \forall x, y \in X. \tag{8}$$

Definition 3.3. A BVFS B in a BF-algebra X , is to be a Bipolar Valued Fuzzy Closed BF-ideal (BVFC-BF-ideal) of X , if

- $\mu^+(x) \geq \mu^+(x * y) \wedge \mu^+(y)$
- $\nu^-(x) \leq \nu^-(x * y) \vee \nu^-(y)$
- $\mu^+(0 * x) \geq \mu^+(x)$
- $\nu^-(0 * x) \leq \nu^-(x) \quad \forall x, y \in X$.

Definition 3.4. A BVFS A in a BF-algebra X is called to be a Bipolar Valued Fuzzy α -ideal (BVF- α -ideal) of X , if

- $\mu_A^+(0) \geq \mu_A^+(x) \text{ and } \nu_A^-(0) \leq \nu_A^-(x)$
- $\mu_A^+(x) \geq \mu_A^+((x * z) * (y * z)) \wedge \mu_A^+(y)$
- $\nu_A^-(x) \leq \nu_A^-((x * z) * (y * z)) \vee \nu_A^-(y) \quad \forall x, y, z \in X$.

Example 3.5. The BF-algebra $X = \{0, 1, 2, 3\}$ is considered with the Cayley table as given below.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

$A = \{ \langle x, \mu_A^+(x), \nu_A^-(x) \rangle \mid x \in X \}$ is the BVFS of X defined as

$$\mu_A^+(x) = \begin{cases} 1; & x = 0,1 \\ 0.5; & x = 2,3 \end{cases} \text{ and } \nu_A^-(x) = \begin{cases} -1; & x = 0,1 \\ -0.6; & x = 2,3 \end{cases}$$

is a BVF- α -ideal of X .

Definition 3.6. A BVFS A in a BF-algebra X is considered to be a Bipolar valued fuzzy closed α -ideal (BVFC- α -ideal) of X , if

- $\mu_A^+(x) \geq \mu_A^+((x * z) * (y * z)) \wedge \mu_A^+(y)$
- $\nu_A^-(x) \leq \nu_A^-((x * z) * (y * z)) \vee \nu_A^-(y)$
- $\mu_A^+(0 * x) \geq \mu_A^+(x)$
- $\nu_A^-(0 * x) \leq \nu_A^-(x) \quad \forall x, y, z \in X.$

Example 3.7. Consider the BF-algebra $X = \{0, 1, 2, 3\}$ with the Cayley table given below.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

$A = \{ \langle x, \mu_A^+(x), \nu_A^-(x) \rangle \mid x \in X \}$ is the BVFS of X defined as

$$\mu_A^+(x) = \begin{cases} 0.8; & x = 0,1 \\ 0.1; & x = 2,3 \end{cases} \text{ and } \nu_A^-(x) = \begin{cases} -0.5; & x = 0,1 \\ -0.1; & x = 2,3 \end{cases}$$

is a BVFC- α -ideal of X .

Trivially, the following can be proved:

Proposition 3.8. Every BVFC- α -ideal is a BVF- α -ideal.

In general, the converse of the above proposition is not true from the following:

Example 3.9. Consider the BF-algebra $X = \{0, 1, 2, 3\}$ with the Cayley table given below

*	0	1	2	3
0	0	3	0	1
1	1	0	1	3
2	2	3	0	1
3	3	1	3	0

$A = \{ \langle x, \mu_A^+(x), \nu_A^-(x) \rangle \mid x \in X \}$ is the BVFS of X defined as

$$\mu_A^+(x) = \begin{cases} 0.6; & x = 0,1 \\ 0.2; & x = 2,3 \end{cases} \text{ and } \nu_A^-(x) = \begin{cases} -0.7; & x = 0,1 \\ -0.3; & x = 2,3 \end{cases}$$

is a BVF- α -ideal of X but not BVFC- α -ideal.

Since $\mu_A^+(0 * 1) < \mu_A^+(1)$ and $\nu_A^-(0 * 1) \leq \nu_A^-(1)$.

Proposition 3.10. If A is Bipolar valued fuzzy α -ideal of X with $x \leq y$ for any $x, y \in X$, then $\mu_A^+(x) \geq \mu_A^+(y)$ and $\nu_A^-(x) \leq \nu_A^-(y)$. That is μ_A^+ is order-reversing and ν_A^- is order-preserving.

Proof: Let $x, y, z \in X$ such that $x \leq y \leq z$.

Then, by the partial ordering if \leq is defined in X , $x * y = 0$ and $y * z = 0$.

Thus,
$$\begin{aligned} \mu_A^+(x) &\geq \mu_A^+((x * y) * (y * z)) \wedge \mu_A^+(y) \\ &\geq \mu_A^+((0 * 0) \wedge \mu_A^+(y)) = \mu_A^+(0) \wedge \mu_A^+(y) = \mu_A^+(y). \end{aligned}$$

And
$$\begin{aligned} \nu_A^-(x) &\leq \nu_A^-((x * y) * (y * z)) \vee \nu_A^-(y) \\ &\leq \nu_A^-((0 * 0) \vee \nu_A^-(y)) = \nu_A^-(0) \vee \nu_A^-(y) = \nu_A^-(y). \end{aligned}$$

It completes the proof.

Theorem 3.11. If A is BVFC- α -ideal of X , then the sets $J = \{x \in X; \mu_A^+(x) = \mu_A^+(0)\}$ and $K = \{x \in X; \nu_A^-(x) = \nu_A^-(0)\}$ are α -ideals of X .

Proof: Clearly, $0 \in J$ and $0 \in K$. Hence, $J \neq \emptyset$ and $K \neq \emptyset$.

Let $(x * y) * (y * z) \in J$ and $y \in J$.

$$\Rightarrow \mu_A^+((x * y) * (y * z)) = \mu_A^+(y) = 0$$

$$\Rightarrow \mu_A^+(x) \geq \mu_A^+((x * y) * (y * z)) \wedge \mu_A^+(y) = 0 \wedge 0 = 0.$$
 But $\mu_A^+(0) \geq \mu_A^+(x) \Rightarrow \mu_A^+(x) = 0$.

Hence, J is an α -ideal of X . Similarly, it can be proved that K is an α -ideal of X .

Theorem 3.12. Any BVF- α -ideal of X is a Bipolar valued fuzzy BF-ideal of X .

Proof: It is trivial by putting $z = 0$ in the definition of BVF- α -ideal.

The converse of the above theorem may not be true.

Now, a sufficient condition is derived for a Bipolar valued fuzzy BF-ideal to be a BVF- α -ideal as follows:

Theorem 3.13. Let A be a BVF-BF-ideal of X . If $\mu_A^+(x * y) \geq \mu_A^+((x * z) * (y * z))$ and $\nu_A^-(x * y) \leq \nu_A^-((x * z) * (y * z)) \forall x, y, z \in X$, then A is BVF- α -ideal of X .

Proof: Let A be a Bipolar valued fuzzy BF-ideal of X and assign $x, y, z \in X$.

So, we have $\mu_A^+(0) \geq \mu_A^+(x)$ and $\nu_A^-(0) \leq \nu_A^-(x)$.

Then,

$$\mu_A^+(x) \geq \mu_A^+(x * y) \wedge \mu_A^+(y) \geq \mu_A^+((x * z) * (y * z)) \wedge \mu_A^+(y)$$

and

$$\nu_A^-(x) \leq \nu_A^-(x * y) \vee \nu_A^-(y) \leq \nu_A^-((x * z) * (y * z)) \vee \nu_A^-(y).$$

Hence, A is BVF- α -ideal of X .

Theorem 3.14. The intersection of any two Bipolar valued fuzzy α -ideals of X is also a Bipolar valued fuzzy α -ideal.

Proof: Let A and B be any two Bipolar valued fuzzy α -ideals of X .

Let $A = \{ \langle x, \mu_A^+(x), \nu_A^-(x) \rangle \mid x \in X \}$ and $B = \{ \langle x, \mu_B^+(x), \nu_B^-(x) \rangle \mid x \in X \}$.

Consider

$$C = A \cap B = \{ \langle x, \mu_C^+(x), \nu_C^-(x) \rangle \mid x \in X \}$$

where $\mu_C^+(x) = \mu_A^+(x) \wedge \mu_B^+(x)$ and $\nu_C^-(x) = \nu_A^-(x) \vee \nu_B^-(x)$.

Let $x, y \in X$.

Now, $\mu_C^+(0) = \mu_A^+(0) \wedge \mu_B^+(0) \geq \mu_A^+(x) \wedge \mu_B^+(x) = \mu_C^+(x)$ and

$$\nu_C^-(0) = \nu_A^-(0) \vee \nu_B^-(0) \leq \nu_A^-(x) \vee \nu_B^-(x) = \nu_C^-(x).$$

$$\begin{aligned} \mu_C^+(x) &= \mu_A^+(x) \wedge \mu_B^+(x) \\ &\geq (\mu_A^+((x * z) * (y * z)) \wedge \mu_A^+(y)) \wedge (\mu_B^+((x * z) * (y * z)) \wedge \mu_B^+(y)) \\ &= (\mu_A^+((x * z) * (y * z)) \wedge \mu_B^+((x * z) * (y * z))) \wedge (\mu_A^+(y) \wedge \mu_B^+(y)) \\ &= \mu_C^+((x * z) * (y * z)) \wedge \mu_C^+(y) \end{aligned}$$

Similarly, $\nu_C^-(x) \leq \nu_C^-((x * z) * (y * z)) \vee \nu_C^-(y)$, and it completes the proof.

The above theorem can be generalized as follows.

Theorem 3.15. The intersection of a family of Bipolar valued fuzzy α -ideals of X is a Bipolar valued fuzzy α -ideal of X .

The following can be analogously proved.

Theorem 3.16. Intersection of any two Bipolar valued fuzzy closed α -ideal of X is also a Bipolar valued fuzzy closed α -ideal of X . Hence, the intersection of a family of Bipolar valued fuzzy closed α -ideal of X is also a Bipolar valued fuzzy closed α -ideal of X .

Remark 3.17. $B = (X; \mu^+, \nu^-)$ is a BVFS defined on any universe X , if and only if μ^+ and $-\nu^-$ are the fuzzy subsets of X .

Theorem 3.18. A BVFS $B = (X; \mu^+, \nu^-)$ is a BVF- α -ideal of X , if and only if the fuzzy subsets μ^+ and $-\nu^-$ are fuzzy α -ideals of X .

Proof: Let $B = (X; \mu^+, \nu^-)$ be a BVF- α -ideal of X .

Further, clearly μ^+ is a fuzzy α -ideal of X .

Also $\nu^-(0) \leq \nu^-(x)$ and $\nu_A^-(x) \leq \nu_A^-((x * z) * (y * z)) \vee \nu_A^-(y) \forall x, y, z \in X$.

$$\Rightarrow -\nu^-(0) \geq -\nu^-(x) \text{ and } -\nu^-(x) \geq -\nu_A^-((x * z) * (y * z)) \wedge -\nu_A^-(y)$$

Therefore, $-\nu^-$ is a fuzzy α -ideal of X

Conversely, assume μ^+ and $-\nu^-$ are fuzzy α -ideals of X .

It is enough to prove that, $\nu^-(0) \leq \nu^-(x)$ and $\nu_A^-(x) \leq \nu_A^-((x * z) * (y * z)) \vee \nu_A^-(y) \forall x, y, z \in X$.

For, $-\nu^-(0) \geq -\nu^-(x)$ and $-\nu^-(x) \geq -\nu_A^-((x * z) * (y * z)) \wedge -\nu_A^-(y)$.

$$\Rightarrow \nu^-(0) \leq \nu^-(x) \text{ and } \nu_A^-(x) \leq \nu_A^-((x * z) * (y * z)) \vee \nu_A^-(y).$$

It fulfills the proof.

The following can be obtained using this theorem.

Theorem 3.19. A BVFS $B = \{ \langle x, \mu_A^+(x), \nu_A^-(x) \rangle \mid x \in X \}$ is a BVF- α -ideal of X , if and only if

- $\square B = (X; \mu^+, -\mu^+)$ and
- $\diamond B = (X; -\nu^-, \nu^-)$ are also BVF- α -ideals of X .

Proof: $A = \{ \langle x, \mu_A^+(x), \nu_A^-(x) \rangle \mid x \in X \}$ is a Bipolar valued fuzzy α -ideal of X , if and only if, the fuzzy subsets μ^+ and $-\nu^-$ are fuzzy α -ideals of X by the theorem 3.18.

That is, if and only if, $\square B$ and $\diamond B$ are also Bipolar valued fuzzy α -ideal of X by the definition of $\square B$ and $\diamond B$.

The following is analogously true.

Theorem 3.20. A BVFS $B = \{ \langle x, \mu_A^+(x), \nu_A^-(x) \rangle \mid x \in X \}$ is a BVFC- α -ideal of X if and only if

- $\square B = (X; \mu^+, -\mu^+)$ and
- $\diamond B = (X; -\nu^-, \nu^-)$ are also BVFC- α -ideals of X .

4. Homomorphism on Bipolar Valued Fuzzy α -Ideal

Here, the image and pre-image of Bipolar valued fuzzy α -ideals under the action of homomorphism and anti-homomorphism on BF-algebras are discussed.

Theorem 4.1. Let f be a homomorphism from BF-algebras X onto Y . A be a bipolar valued fuzzy α -ideal of X with Sup-Inf property. Then, the image of A , $f(A) = \{ \langle y, \mu_{f(A)}^+(y), \nu_{f(A)}^-(y) \rangle \mid y \in Y \}$ is a bipolar valued fuzzy α -ideal of Y .

Proof: Let $a, b, c \in Y$ with $x_0 \in f^{-1}(a), y_0 \in f^{-1}(b)$ and $z_0 \in f^{-1}(c)$ such that

$$\mu_A^+(x_0) = \sup_{t \in f^{-1}(a)} \mu_A^+(t); \mu_A^+(y_0) = \sup_{t \in f^{-1}(b)} \mu_A^+(t); \mu_A^+(z_0) = \sup_{t \in f^{-1}(c)} \mu_A^+(t)$$

and

$$v_A^-(x_0) = \inf_{t \in f^{-1}(a)} v_A^-(t); v_A^-(y_0) = \inf_{t \in f^{-1}(b)} v_A^-(t); v_A^-(z_0) = \inf_{t \in f^{-1}(c)} v_A^-(t)$$

Now, by the definitions 2.8, 2.9 and 2.4, the following is framed

$$\mu_{f(A)}^+(0) = \sup_{t \in f^{-1}(0)} \mu_A^+(t) \geq \mu_A^+(0) \geq \mu_A^+(x_0) = \sup_{t \in f^{-1}(a)} \mu_A^+(t) = \mu_{f(A)}^+(a)$$

and

$$v_{f(A)}^-(0) = \inf_{t \in f^{-1}(0)} v_A^-(t) \leq v_A^-(0) \leq v_A^-(x_0) = \inf_{t \in f^{-1}(a)} v_A^-(t) = v_{f(A)}^-(a)$$

$$\mu_{f(A)}^+((a * c) * (b * c)) \wedge \mu_{f(A)}^+(b) = \sup_{t \in f^{-1}((a * c) * (b * c))} \mu_A^+(t) \wedge \sup_{t \in f^{-1}(b)} \mu_A^+(t)$$

Now,

$$\leq \mu_A^+((x_0 * z_0) * (y_0 * z_0)) \wedge \mu_A^+(y_0)$$

$$\leq \mu_A^+(x_0) = \sup_{t \in f^{-1}(a)} \mu_A^+(t) = \mu_{f(A)}^+(a)$$

$$\begin{aligned} v_{f(A)}^-((a * c) * (b * c)) \vee v_{f(A)}^-(b) &= \inf_{t \in f^{-1}((a * c) * (b * c))} v_A^-(t) \vee \inf_{t \in f^{-1}(b)} v_A^-(t) \\ &\geq v_A^-((x_0 * z_0) * (y_0 * z_0)) \vee v_A^-(y_0) \\ &\geq v_A^-(x_0) = \inf_{t \in f^{-1}(a)} v_A^-(t) = v_{f(A)}^-(a) \end{aligned}$$

Hence, the image $f(A)$ is a bipolar valued fuzzy α -ideal of Y .

Theorem 4.2. Let f be a homomorphism from BF-algebras X onto Y and A be a Bipolar valued fuzzy closed α -ideal of X with Sup-Inf property. Then, the image of A , $f(A) = \{ \langle y, \mu_{f(A)}^+(y), v_{f(A)}^-(y) \rangle \mid y \in Y \}$ is a bipolar valued fuzzy closed α -ideal of Y .

Proof: Let $x \in Y$ with $x_0 \in f^{-1}(x)$ such that

$$\mu_A^+(x_0) = \sup_{t \in f^{-1}(x)} \mu_A^+(t); v_A^-(x_0) = \inf_{t \in f^{-1}(x)} v_A^-(t).$$

Then, we have

$$\mu_{f(A)}^+(x) = \sup_{t \in f^{-1}(x)} \mu_A^+(t) \leq \mu_A^+(x_0) \leq \mu_A^+(0 * x_0) = \sup_{t \in f^{-1}(0 * x)} \mu_A^+(t) = \mu_{f(A)}^+(0 * x)$$

$$v_{f(A)}^-(x) = \inf_{t \in f^{-1}(x)} v_A^-(t) \geq v_A^-(x_0) \geq v_A^-(0 * x_0) = \inf_{t \in f^{-1}(0 * x)} v_A^-(t) = v_{f(A)}^-(0 * x)$$

Hence, by the above theorem, the image $f(A)$ is considered as a Bipolar valued fuzzy closed α -ideal of Y .

Theorem 4.3. Let f be a homomorphism from BF-algebras X onto Y and B be a bipolar valued fuzzy α -ideal of Y . Then, the inverse image of B , $f^{-1}(B)$ is a bipolar valued fuzzy α -ideal of X .

Proof: Let $x, y \in X$.

Now, it is clear that

$$\mu_{f^{-1}(B)}^+(0) = \mu_B^+(f(0)) \geq \mu_B^+(f(x)) = \mu_{f^{-1}(B)}^+(x) \quad \text{and} \quad v_{f^{-1}(B)}^-(0) = v_B^-(f(0)) \leq v_B^-(f(x)) = v_{f^{-1}(B)}^-(x)$$

Then,

$$\begin{aligned} \mu_{f^{-1}(B)}^+(x) &= \mu_B^+(f(x)) \geq \mu_B^+((f(x) * f(z)) * (f(y) * f(z))) \wedge \mu_B^+(f(y)) \\ &= \mu_{f^{-1}(B)}^+((x * z) * (y * z)) \wedge \mu_{f^{-1}(B)}^+(y) \end{aligned}$$

Also

$$\begin{aligned} v_{f^{-1}(B)}^-(x) &= v_B^-(f(x)) \leq v_B^-((f(x) * f(z)) * (f(y) * f(z))) \vee v_B^-(f(y)) \\ &= v_{f^{-1}(B)}^-((x * z) * (y * z)) \vee v_{f^{-1}(B)}^-(y) \end{aligned}$$

Then, the inverse image of B , $f^{-1}(B)$ is a bipolar valued fuzzy α -ideal of X .

Theorem 4.4. Let f be a homomorphism from BF-algebras X onto Y and B be a Bipolar valued fuzzy closed α -ideal of Y . Then the inverse image of B , $f^{-1}(B)$ is a Bipolar valued fuzzy closed α -ideal of X .

Proof: Let $x \in X$. Then, we have

$$\begin{aligned}\mu_{f^{-1}(B)}^+(0 * x) &= \mu_B^+(f(0 * x)) = \mu_B^+(f(0) * f(x)) \geq \mu_B^+(f(x)) = \mu_{f^{-1}(B)}^+(x) \\ \nu_{f^{-1}(B)}^-(0 * x) &= \nu_B^-(f(0 * x)) = \nu_B^-(f(0) * f(x)) \leq \nu_B^-(f(x)) = \nu_{f^{-1}(B)}^-(x).\end{aligned}$$

Hence, through the above theorem, the inverse image $f^{-1}(B)$ becomes a Bipolar valued fuzzy closed α -ideal of X .

In the same way, the following can be proved.

Theorem 4.5. Let f be an anti-homomorphism from X onto Y and A be a bipolar valued fuzzy α -ideal of X with Sup-Inf property. Then, the image of A , $f(A)$ is a bipolar valued fuzzy α -ideal of Y .

Theorem 4.6. Let f be an anti-homomorphism from X onto Y and B be a bipolar valued fuzzy α -ideal of Y . Then, the inverse image of B , $f^{-1}(B)$ is a bipolar valued fuzzy α -ideal of X .

Theorem 4.7. Let f be an anti-homomorphism from X onto Y and A be a bipolar valued fuzzy closed α -ideal of X with Sup-Inf property. Then, the image of A , $f(A)$ is a bipolar valued fuzzy closed α -ideal of Y .

Theorem 4.8. Let f be an anti-homomorphism from X onto Y and B be a bipolar valued fuzzy closed α -ideal of Y . Then, the inverse image of B , $f^{-1}(B)$ is a bipolar valued fuzzy closed α -ideal of X .

5. Conclusion

From the preliminaries of this research work, Bipolar valued fuzzy sets of various researchers are analyzed. Especially, for the present work stated in this paper, an investigation on the Bipolar valued fuzzy α -ideals of BF-algebras has been carried out. From the investigation, several interesting results are observed. As a result, the research has been focused on this way and all the possible ways are found out to prove this strategy. The surprising point is that in [7] Andrzej Walendziak, theorem 2.11 says that the structure of BF algebra becomes a BG-algebra and the proof is followed directly from the definition. Hence, it is concluded that all the results prove here for BF-algebras can directly be carried over to BG-algebras.

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