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An Improved Chirplet Transform and Its Application for Harmonics Detection

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Abstract

The chirplet transform is the generalization form of fast Fourier Transform, short-time Fourier transform, and wavelet transform. It has the most flexible time frequency window and successfully used in practices. However, the chirplet transform has not inherent inverse transform, and can not overcome the signal reconstructing problem. In this paper, we proposed the improved chirplet transform (ICT) and constructed the inverse ICT. Finally, by simulating the harmonic voltages, the power of the improved chirplet transform are illustrated for harmonic detection. The contours clearly showed the harmonic occurrence time and harmonic duration.

1. Introduction

In many power quality analysis and disciplines, the concept of a stationary time series is a mathematical idealization that is never realized and is not particularly useful in the detection of power quality disturbances in power systems. Although the Fourier transform of the entire time series does contain information about the spectral components in a time series, for a large class of practical applications such as voltage signals in power systems, this information is inadequate. So in the year of 1996, Stockwell, Mansinha and Lowe presented a new S transform that provides a joint time-frequency representation (TFR) with frequency-dependent resolution [1] while, at the same time, maintaining the direct relationship, through time-averaging, with the Fourier spectrum. Several have been proposed in the past; among them are the Gabor transform [2], the related short-time Fourier transforms [3], the continuous wavelet transform (CWT) [4], and the bilinear class of time-frequency distributions known as Cohen's class [5], of which the Wigner distribution [6] is a member. The S transform was defined as following [1].

$$S(\tau, f) = \int_{-\infty}^{+\infty} h(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau-t)^2 f^2}{2}} e^{-j2\pi ft} dt. \qquad (1)$$

It is easy to prove that

$$\int_{-\infty}^{+\infty} S(\tau, f) d\tau = H(f). \qquad (2)$$

where $H(f)$ is the Fourier transform of $h(t)$. It follows that $h(t)$ is exactly recoverable from $S(\tau, f)$. Thus

$$h(t) = \int_{-\infty}^{+\infty} \left\{ \int_{-\infty}^{+\infty} S(\tau, f) d\tau \right\} e^{j2\pi ft} df. \qquad (3)$$

The *S*-transforms have successfully been used in power system disturbance detection and identification [7]. Another time-frequency representation, Chirplet transform (CT), was defined by Mann Steven in 1992 [8].

$$C(b, \omega) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{+\infty} h(t) e^{-\frac{(t-b)^2}{2\sigma^2}} e^{-i(\omega t + qt^2)} dt. \qquad (4)$$

From (4), it is easy to show that Chirplet transform is an extension of Gabor transform, short-time Fourier transform and continuous wavelet transform, furthermore, the time-frequency window of CT is more flexible than that of CWT. CT has been used successfully to classify power quality disturbance (including voltage sag, voltage swell, voltage interruption, and linear time-varying harmonics and nonlinear time-varying harmonics) [9], denoise the motor fault signals [10], and detect slight fault of electrical machines [11].

However, CT does not satisfy basic characters (2) and (3) and inconvenience in practical applications. So, in this paper, we present an improved Chirplet transform satisfying (2) and (3), moreover, the novel Chirplet transform is an extension of the *S*-transform.

This paper is organized as follows. The improved Chirplet transform is presented in Section 2, and the numerical algorithm is introduced in Section 3; Several simulated power quality harmonic waveforms are detected and identified using the proposed method in Section 4; Finally, The conclusions and references are given.

2. The Improved Chirplet Transform

Chirplet transform is considered as the "phase correction" of the CWT. The CWT $W(b,a)$ of a function $h(t)$ is defined by [4]

$$W(b,a) = \int_{-\infty}^{+\infty} h(t)w(t-b,a)dt. \qquad (5)$$

where $w(\bullet)$ is a scaled replica of the fundamental mother wavelet. The dilation a determines the "width" of the wavelet $w(\bullet)$ and thus controls the resolution. Along with (5), there exists an admissibility condition on the mother wavelet $w(\bullet)$ [4] that $w(\bullet)$ must have zero mean.

The improved chirplet transform of a function $h(t)$ is defined as a CWT with a specific mother wavelet defined as

$$w(t,f) = \frac{\sqrt{f^2+iq}}{\sqrt{2\pi}} e^{-\frac{t^2(f^2-iq)}{2}} e^{-i(2\pi ft + qt^2)}. \qquad (6)$$

note that the frequency f and constant q.

The improved chirplet wavelet in (6) does not satisfy the condition of zero mean for an admissible wavelet; therefore, it is not strictly a CWT, Written out explicitly, the improved chirplet transform is

$$IC(b,f)$$
$$= \frac{\sqrt{f^2+iq}}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} h(t)e^{-\frac{(t-b)^2(f^2-iq)}{2}} e^{-i(2\pi f+q(t-b)^2)}dt. \qquad (7)$$

It is obvious that Equation (7) degenerates to be an *S*-transform as $q = 0$ [1].

If the improved chirplet transform is indeed a representation of the local spectrum, one would expect a simple operation of averaging the local spectra over time to give the Fourier spectrum. It is shown as follows

$$\int_{-\infty}^{+\infty} IC(b,f)db = H(f). \qquad (8)$$

where $H(f)$ is the Fourier transform of $h(t)$. It follows that $h(t)$ is exactly recoverable from $IC(b,f)$. Thus

$$h(t) = \int_{-\infty}^{+\infty} \left\{ \int_{-\infty}^{+\infty} IC(b,f)db \right\} e^{i2\pi ft}df. \qquad (9)$$

Proof: from

$$\int_{-\infty}^{+\infty} e^{-\frac{(t-b)^2(f^2-iq)}{2}} e^{-iq(t-b)^2}db = \int_{-\infty}^{+\infty} e^{-\frac{(t-b)^2(f^2+iq)}{2}}db$$

$$= \frac{1}{\sqrt{f^2+iq}} \int_{-\infty}^{+\infty} e^{-\frac{u^2}{2}}du \left(set \quad u = \sqrt{f^2+iq}(b-t) \right)$$

$$= \frac{\sqrt{2\pi}}{\sqrt{f^2+iq}} \left(\because \int_{-\infty}^{+\infty} e^{-\frac{u^2}{2}}du = \sqrt{2\pi} \right).$$

we can get

$$\int_{-\infty}^{+\infty} IC(b,f)db$$

$$= \int_{-\infty}^{+\infty} \frac{\sqrt{f^2+iq}}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} h(t)e^{-\frac{(t-b)^2(f^2-iq)}{2}} e^{-i(2\pi ft+q(t-b)^2)}dtdb$$

$$= \frac{\sqrt{f^2+iq}}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} h(t)e^{-i2\pi ft}dt \int_{-\infty}^{+\infty} e^{-\frac{(t-b)^2(f^2-iq)}{2}} e^{-i(q(t-b)^2)}db.$$

$$= \int_{-\infty}^{+\infty} h(t)e^{-i2\pi ft}dt = H(f).$$

Then (9) is the inverse of the above equation.

Alike FFT, STFT, WT, and CT, the improved chirplet transform has the linear property. This is an advantage over the bilinear class of time-frequency representations (TFR's). The presence of the cross terms makes it difficult to reliably estimate the signal. The improved chirplet transform can be written as operations on the Fourier spectrum $H(f)$ of $h(t)$

$$IC(b,f) = \int_{-\infty}^{+\infty} H(\alpha+f)e^{-\frac{2\pi^2\alpha^2}{f^2+iq}} e^{i2\pi\alpha b}d\alpha \qquad (11)$$

Proof:

$$\int_{-\infty}^{+\infty} H(\alpha+f)e^{-\frac{2\pi^2\alpha^2}{f^2+iq}} e^{i2\pi\alpha b}d\alpha$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(t)e^{-i2\pi(\alpha+f)t} e^{-\frac{2\pi^2\alpha^2}{f^2+iq}} e^{i2\pi\alpha b}dtd\alpha$$

$$= \int_{-\infty}^{+\infty} h(t)e^{-i2\pi ft} e^{-\left(\frac{f^2+iq}{2}(t-b)^2\right)}dt \int_{-\infty}^{+\infty} e^{-\left(\frac{\sqrt{2\pi}\alpha}{\sqrt{f^2+iq}}\right)^2}d\alpha$$

$$= \frac{\sqrt{f^2+iq}}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} h(t) \cdot e^{-\left(\frac{(t-b)^2(f^2+iq)}{2}\right)} e^{-i(2\pi ft+q(t-b)^2)}dt$$

$$= IC(b,f).$$

The discrete analog of (11) is used to compute the discrete improved chirplet transform by taking advantage of the efficiency of the Fast Fourier transform (FFT) and the convolution theorem.

3. The Discrete Improved Chirplet Transform

Let $h[kT]$, $k = 0,1,2,\cdots,N-1$ denote a discrete time series corresponding to signal $h(t)$ with a time sampling interval of T. The discrete Fourier transform is given by

$$H\left[\frac{n}{NT}\right] = \frac{1}{N}\sum_{k=0}^{N-1}h[kT]e^{-\frac{i2\pi nk}{N}}.$$ (12)

where $n = 0,1,2,\cdots,N-1$. In the discrete case, the improved chirplet transform is the projection of the vector defined by the time series $h[kT]$ onto a spanning set of vectors. The spanning vectors are not orthogonal, and the elements of the improved chirplet transform are not independent. Each basis vector (of the Fourier transform) is divided into N localized vectors by an element-by-element product with the N shifted Gaussians such that the sum of these N localized vectors is the original basis vector.

Using (11), the improved chirplet transform of a discrete time series $h[kT]$ is given by (letting $f \rightarrow n/NT$ and $\tau \rightarrow jT$)

$$IC\left[jT,\frac{n}{NT}\right] = \sum_{m=0}^{N-1}H\left[\frac{m+n}{NT}\right]e^{-\frac{2\pi^2 m^2}{n^2+i\tilde{q}}}e^{\frac{i2\pi mj}{N}}.$$ (13)

where $n \neq 0$, $\tilde{q} = q(NT)^2$. For the $n = 0$, it is equal to the constant defined as

$$IC[jT,0] = \frac{1}{N}\sum_{m=0}^{N-1}h\left(\frac{m}{NT}\right).$$ (14)

with j, m, and $n = 0,1,2,\cdots,N-1$. The discrete improved chirplet transform suffers the familiar problems from sampling and finite length, giving rise to implicit periodicity in the time and frequency domains. The discrete inverse of the improved chirplet transforms (13) and (14) is

$$h[kT] = \sum_{k=0}^{N-1}\left\{\frac{1}{N}\sum_{j=0}^{N-1}IC\left[jT,\frac{n}{NT}\right]\right\}e^{\frac{j2\pi nk}{N}}.$$ (15)

4. Examples

4.1. The Signal TFR Figure Using the Improved Chirplet Transform

Equations (7) and (9) are the improved chirplet transform (ICT) for the time-frequency representation (TFR) and its inverse ICT for the signal reconstruction. In the follow figures, **Figure 1(a)** is the A-phase current signal of the inductor motor with single phase grounding. **Figure 2(b)** shows the TFR of the A-phase current signal of the motor using (7). The reconstruction signal using (9) is illustrate in **Figure 2(c)**. From **Figure 1**, we know (7) and (9) are very effective for representing a time frequency feature of a signal and reconstructing from TFR.
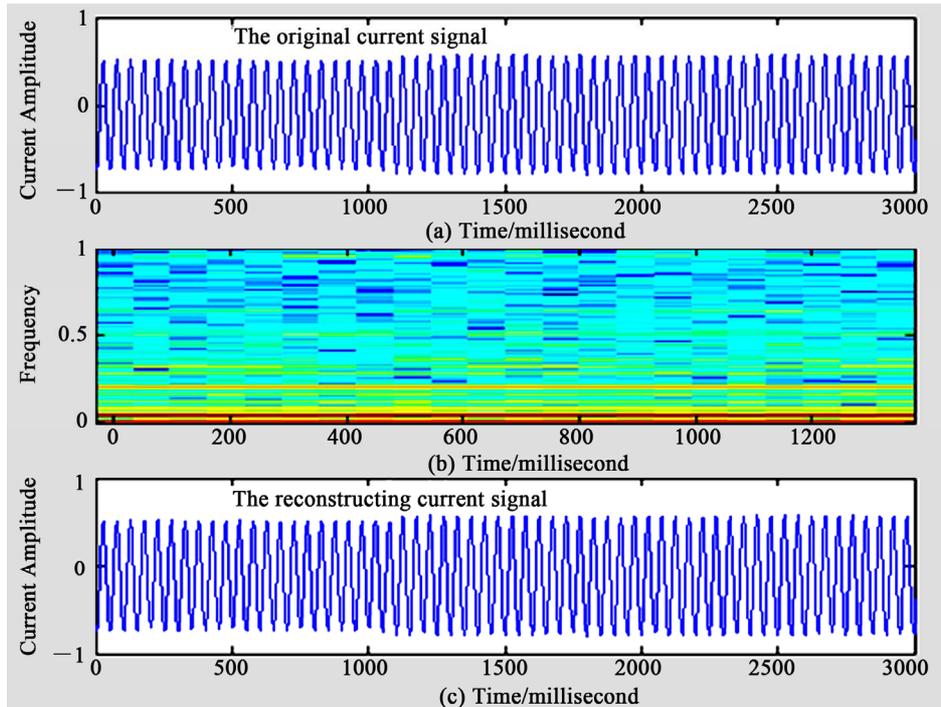

Figure 1. The motor A-phase current signal (a), its TFR (b), and the reconstructing signal.

4.2. The harmonics Detection

Figures 2 and **3** demonstrate the class of time series for which the improved chirplet transform would be useful;

they highlight the advantages of such an approach as compared with other techniques.

Considering a simulating segment harmonic voltage with zero initial phase as follows.

$$f(t) = \begin{cases} \sin(2*pi*50*t) & (0 \leq t \leq 0.3) \\ \sin(2*pi*50*t) + \sin(2*pi*150*t) & (0.3 < t \leq 0.6) \\ \sin(2*pi*(50+0.2*t)*t) & (0.6 < t \leq 1.4) \\ \sin(2*pi*50*t) + \sin(2*pi*350*t) & (1.4 < t \leq 1.7) \\ \sin(2*pi*50*t) & (1.7 \leq t \leq 2) \end{cases} \tag{16}$$
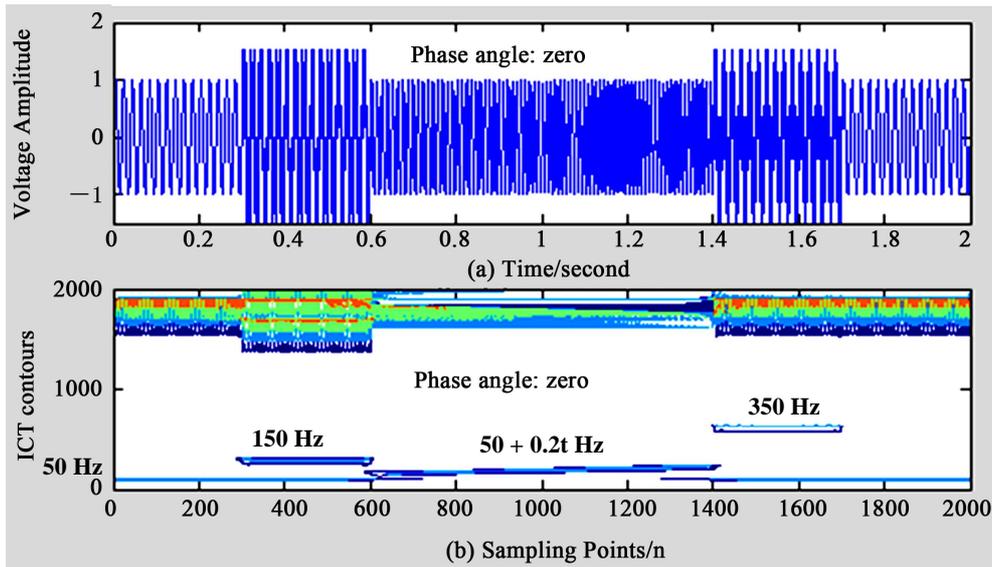

Figure 2. The simulating voltage with zero initial phase (a), and its ICT contour (b).
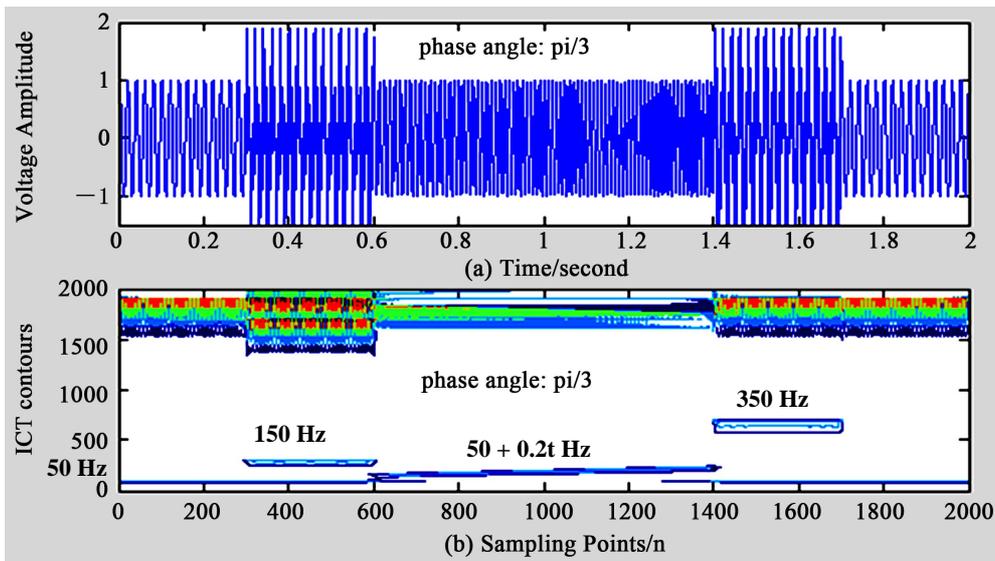

Figure 3. The simulating voltage with pi/3 initial phase (a), and its ICT contour (b).

The voltage signal divided into 5 time segments. In the first time interval, the voltage constrains only a frequency: 50 Hz. At time 0.3, the voltage constrains another harmonic: 150 Hz. In the time interval $0.6 < t \leq 1.4$, the voltage constrains a linear time changing harmonic. Then at 1.4 s, the harmonic, 350 Hz, added to the signal. Finally, the voltage retain normally at 1.7 s.

The sampling frequency is 1000 Hz. **Figure 2 (a)** is the improved chirplet transform plot of (16). **Figure 2(b)** is the contour plot of the signal (a) using (13) and (14). Form **Figure 2(b)**, we find the ICT contour illustrates the work frequency 50 Hz, two harmonic frequency 150 Hz and 350 Hz, and the linear time changing frequency 50 + 0.2 t Hz.

Moreover, the contour in **Figure 2(b)** clearly shows the harmonic occurrence times and durations.

In order to investigate the influence of initial phase, we modulating the above simulating voltage signal with pi/3 phase. From **Figure 3**, we find that the initial phase does not influence the harmonic detection.

5. Conclusions

The chirplet transform is the generalization form of fast Fourier transform, short-time Fourier transform, and wavelet transform. It has the most flexible time frequency window and successfully used in practices. However, the chirplet transform has not inherent reconstructing formulae. So we proposed the improved chirplet transform (ICT) and constructed the inverse ICT. Finally, the power of the improved chirplet transform is apparent from the above examples.

6. References

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