

# A New Chaotic Behavior from Lorenz and Rossler Systems and Its Electronic Circuit Implementation

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## Abstract

This paper presents a new three-dimensional continuous autonomous chaotic system with ten terms and three quadratic nonlinearities. The new system contains five variational parameters and exhibits Lorenz and Rossler like attractors in numerical simulations. The basic dynamical properties of the new system are analyzed by means of equilibrium points, eigenvalue structures. Some of the basic dynamic behavior of the system is explored further investigation in the Lyapunov Exponent. The new system examined in Matlab-Simulink and Orcad-PSpice. An electronic circuit realization of the proposed system is presented using analog electronic elements such as capacitors, resistors, operational amplifiers and multipliers.

**Keywords:** Chaos, Lorenz System, Rossler System, Lyapunov Exponent, Bifurcation

## 1. Introduction

The science of nonlinear dynamics and chaos theory has sparked many researchers to develop mathematical models that simulate vector fields of nonlinear chaotic physical systems. Nonlinear phenomena arise in all fields of engineering, physics, chemistry, biology, economics, and sociology. Examples of nonlinear chaotic systems include planetary climate prediction models, neural network models, data compression, turbulence, nonlinear dynamical economics, information processing, preventing the collapse of power systems, high-performance circuits and devices, and liquid mixing with low power consumption [1-3].

The Lorenz system of differential equations arose from the work of meteorologist/mathematician Edward N. Lorenz, who was studying thermal variations in an air cell underneath a thunderhead.

The Lorenz equations are a fairly simple model in which to study chaos [3].

$$\left. \begin{aligned} \dot{x} &= \sigma(y-x) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= xy - \beta z \end{aligned} \right\} \quad (1)$$

The arbitrary parameters  $\sigma$ ,  $r$  and  $\beta > 0$  and for this example are  $\sigma = 10$ ,  $r = 28$  and  $\beta = 8/3$ . The Rossler system has only one quadratic nonlinearity  $xz$  numerical

integration shows that this system has a strange attractor for  $a = b = 0.2$ ,  $c = 5.7$  [2].

$$\left. \begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c) \end{aligned} \right\} \quad (2)$$

This paper propose a new chaotic system based on adding two chaotic system (Lorenz and Rossler) and it compares the results with the chaotic system and an electronic circuit realization of the proposed system is presented using analog electronic elements, The remainder of the paper is organized as follows: Section 2 discusses the proposal of a new chaotic system and its analysis, section 3 present deal with circuit realization of the new attractor and section 4 discusses and examines a new scheme [4].

## 2. A New Chaotic System and Its Analysis

Most researchers developed a new chaotic system depending on one chaotic system like Lorenz or Rossler systems the proposed scheme in this paper based on merging two chaotic systems Lorenz chaotic system and Rossler chaotic system. Therefore will be added two chaotic systems in (1) and (2), a new system is shown in (3).

$$\left. \begin{aligned} \dot{x} &= \sigma(y-x) - y - z \\ \dot{y} &= rx - y - xz + x + ay \\ \dot{z} &= xy - \beta z + b + z(x - c) \end{aligned} \right\} \quad (3)$$

We note after adding the two chaotic systems, it is noticed (1) and (2) that the control parameter increased from three ( $\delta, r, \beta$ ) to six ( $\delta, r, \beta, b, a, c$ ) but to check the new system is suitable for achieving the chaotic requirements, by plot phase plane for a new system we note a new system loss chaotic behavior shown in **Figure 1**.

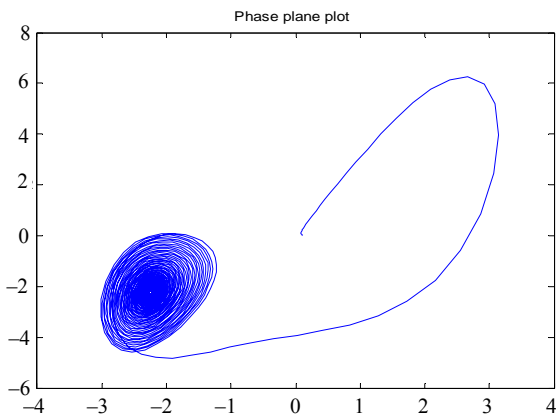
Therefore we try to manipulate the above equation to achieve a chaotic behavior, so we will add Cuomo Circuit shown in (4) (linear transformation of Lorenz equations with a new scale) [2,5] to Rossler equations after changing  $z$  instead  $x$  in last equation of Rossler system, the final system is shown below in (5) and (6).

$$\left. \begin{aligned} \dot{u} &= \sigma(v-u) \\ \dot{v} &= ru - v - 20uw \\ \dot{w} &= 5uv - \beta w \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} \dot{x} &= \sigma(y-x) - y - z \\ \dot{y} &= rx - y - 20xz + x + ay \\ \dot{z} &= 5xy - \beta z + b + x(z-c) \end{aligned} \right\} \quad (5)$$

$$\left. \begin{aligned} \dot{x} &= (\delta-1)y - \delta x - z \\ \dot{y} &= (r+1)x - (1-a)y - 20xz \\ \dot{z} &= 5xy - \beta z + b + xz - cx \end{aligned} \right\} \quad (6)$$

To check that the new system has a chaotic behavior or not, no definition of the term chaos has been universally accepted yet but most researchers agree on the three ingredients used in following definition “Chaos is aperiodic long term behavior in a deterministic system that exhibits dependence on initial condition” [1-4,6]. Even though the definition of chaos has not been agreed upon by mathematicians, two properties that are generally agreed to characterize it are sensitivity to initial conditions and the presence of period-doubling cycles leading to chaos.



**Figure 1.** Phase plane at adding Lorenz with Rossler systems.

The new system has six terms, two quadratic nonlinearities ( $xz, xy$ ) and six real constant parameters ( $\delta, r, a, b, \beta$  and  $c$ ). The state variables of the system are  $x, y,$  and  $z$ . The new system equations have one equilibrium point. This point which satisfies this requirement is found by setting  $x, y, z = 0,$  in (5), and solving for  $x, y$  and  $z$ :

$$\left. \begin{aligned} 0 &= \sigma(y-x) - y - z \\ 0 &= rx - y - 20xz + x + ay \\ 0 &= 5xy - \beta z + b + x(z-c) \end{aligned} \right\} \quad (7)$$

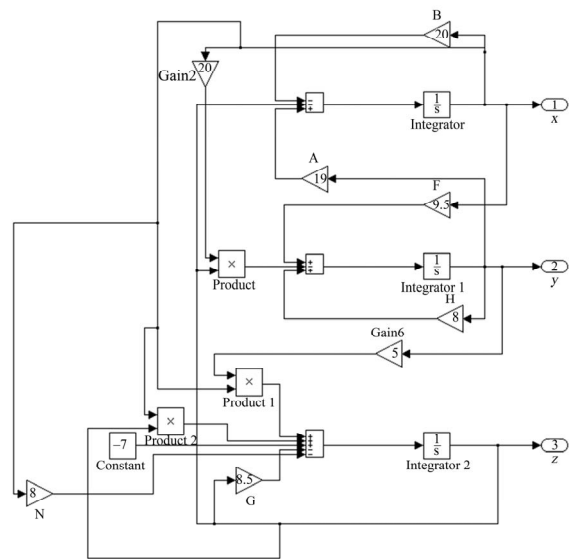
The fixed point just we have one point  $(0,0,0),$  The Jacobian of the system is:

$$J = \begin{bmatrix} -\delta & (\delta-1) & -1 \\ (r+1-20z) & (a-1) & -20x \\ 5y+z-c & 5x & -\beta+x \end{bmatrix}$$

For the case when the fixed point is  $(x^*, y^*, z^*) = (0,0,0),$  the Jacobian becomes

$$J = \begin{bmatrix} -\delta & (\delta-1) & -1 \\ (r+1) & (a-1) & 0 \\ -c & 0 & -b \end{bmatrix}$$

The eigenvalues are found by solving the characteristic equation,  $|J - \lambda I| = 0,$  for which is yielding eigenvalues  $\lambda_1 = 18.4561, \lambda_2 = -30.6770$  and  $\lambda_3 = -8.279$  the equilibrium points are unstable and this implies chaos. Thus, the system orbits around the unstable equilibrium point. Using a Matlab-Simulink model as shown in **Figure 2**. The  $xy, xz,$  and  $yz$  phase portraits of the new system achieved are shown in **Figure 3, Figure 4,** and **Figure 5,** also the time series for the new chaotic system is shown in **Figure 6**.



**Figure 2.** The Matlab-Simulink model of the new system.

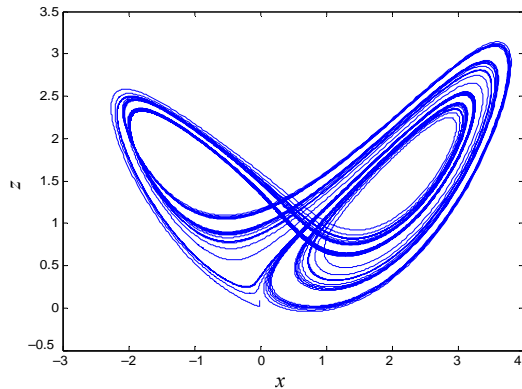


Figure 3. xz phase portrait of the new system.

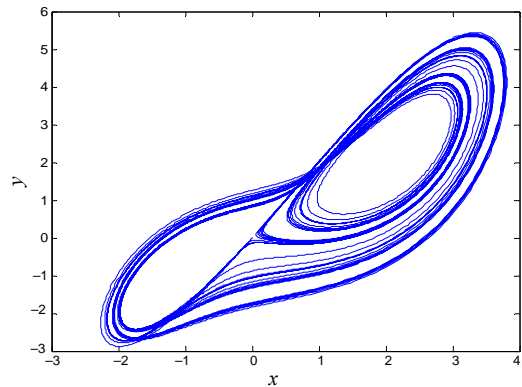


Figure 4. xy phase portrait of the new system.

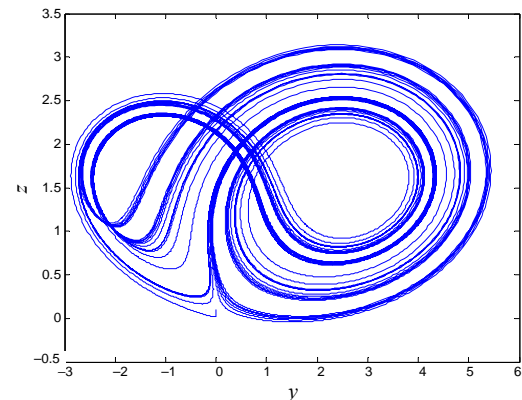


Figure 5. yz phase portrait of the new system.

Another important test is the Lyapunov exponents, which measures the exponential rates of divergence and convergence of nearby trajectories in state space, and the Lyapunov exponent spectrum provides additional useful information about the system as shown in **Figure 7**. A positive and zero Lyapunov exponent indicates chaos, two zero Lyapunov exponents indicate a bifurcation, and a zero and a negative Lyapunov exponent indicates periodicity, however as noticed from Lyapunov exponent the sum of the Lyapunov exponents must be negative. A positive Lyapunov exponent reflects a “direction” of stret-

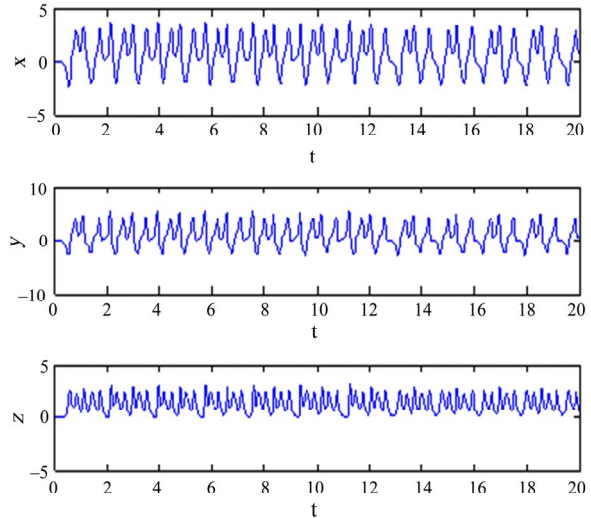


Figure 6. The time series for new chaotic system.

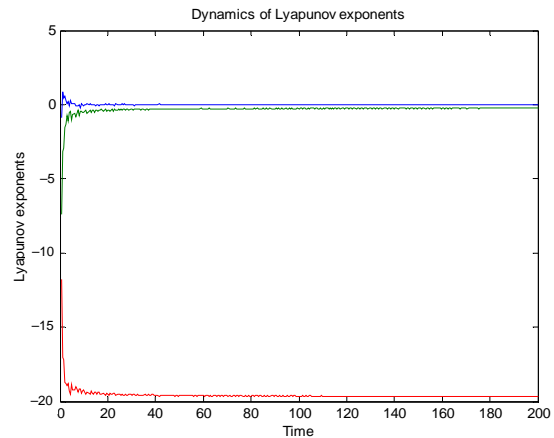


Figure 7. Dynamics Lyapunov exponent of the new system.

ching and folding and therefore determines chaos in the system, 3D continuous dissipative  $(\lambda_1, \lambda_2, \lambda_3)$ ,  $(+, 0, -)$ —A strange attractor;  $(0, 0, -)$ —A two-torus;  $(0, -, -)$ —A limit cycle;  $(-, -, -)$ —A fixed point [7-11].

### 3. Circuit Realization of the New Attractor

A simple electronic circuit is designed, so that it can be used to study chaotic phenomena. The circuit employs simple electronic elements, such as resistors, and operational amplifiers, the operational amplifiers and associated circuitry perform the operations of addition, subtraction, and integration. Analog multipliers implement the nonlinear terms in the circuit equations, and is easy to construct [12]. Circuit schematic for implementing the new chaotic system in (6). By applying standard node analysis techniques to the circuit of **Figure 8**, a set of state equations that govern the dynamical behavior of the circuit can be obtained. This set of equations is given by

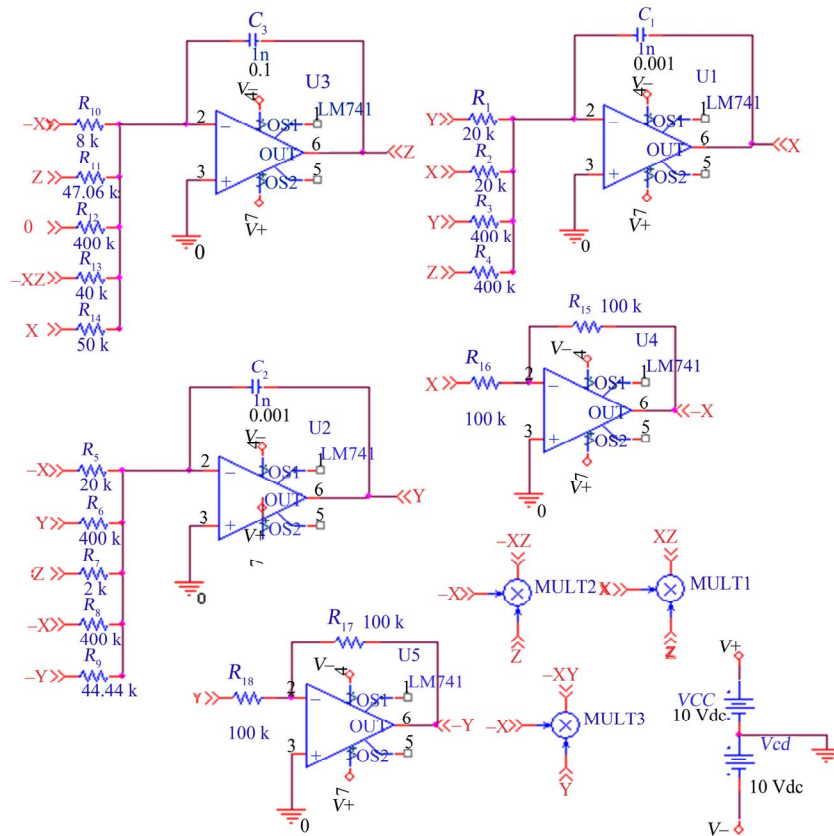


Figure 8. The electronic circuit schematic of the new chaotic system.

(8):

$$\left. \begin{aligned} \dot{x} &= \left( \frac{1}{R_1 C_1} - \frac{1}{R_3 C_1} \right) y - \frac{1}{R_1 C_1} x - \frac{1}{R_4 C_1} z \\ \dot{y} &= \left( \frac{1}{R_5 C_2} - \frac{1}{R_8 C_2} \right) x - \left( \frac{1}{R_6 C_2} - \frac{1}{R_9 C_2} \right) y - \frac{1}{R_7 C_2} xz \\ \dot{z} &= \frac{1}{R_{10} C_3} xy - \frac{1}{R_{11} C_3} z + \frac{1}{R_{12} C_3} d + \frac{1}{R_{13} C_3} xz - \frac{1}{R_{14} C_3} x \end{aligned} \right\} \quad (8)$$

For the chosen component value is equivalent to after rescaling time by a factor of 1500. An electronic circuit of the new chaotic system is implemented with parameters of ( $\delta = 20, r = 20, a = 9, \beta = 8.5, b = 0$  and  $c = 8$ ) and initial conditions  $x_0 = 0.0010, y_0 = 0.001, z_0 = 0.1$  LM741 opamps, and the analog multipliers are used with  $R_1 = R_2 = R_5 = 20$  K,  $R_3 = R_4 = R_6 = R_8 = R_{12} = 400$  K,  $R_9 = 44.44$  K,  $R_{10} = 8$  K,  $R_{11} = 47.06$ ,  $R_7 = 2$  K,  $R_{13} = 40$  K and  $R_{14} = 50$  K and  $C_1 = C_2 = C_3 = 1$  nF. The output voltage is the products of the inputs multiplied by 10 V. PSpice simulations of the new chaotic system are also attained in **Figure 9**, **Figure 10**, and **Figure 11** for  $xy$ ,  $xz$ , and  $yz$  attractors, respectively. In this simulation, the parameters ( $\delta, r, a, \beta, b$  and  $c$ ) are set at a value of 20,20,9,8.5,0 and 8.

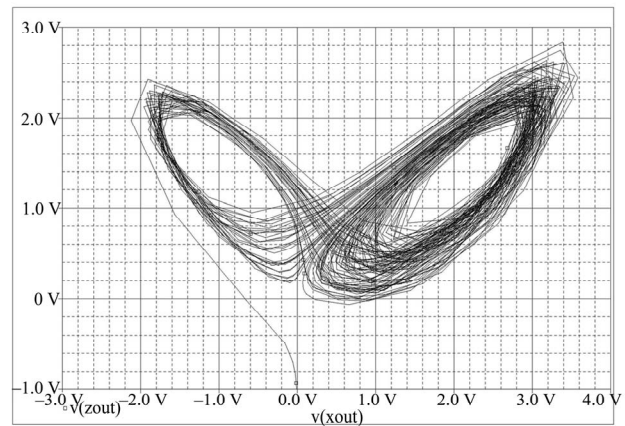
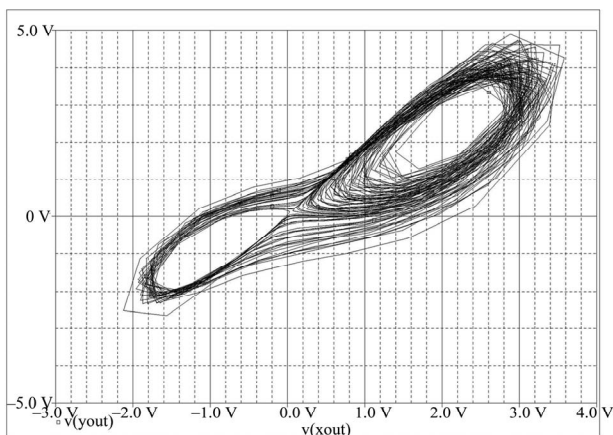


Figure 9. PSpice simulation result of the new chaotic system's electronic oscillator (Figure 3) for  $xz$  strange attractor.

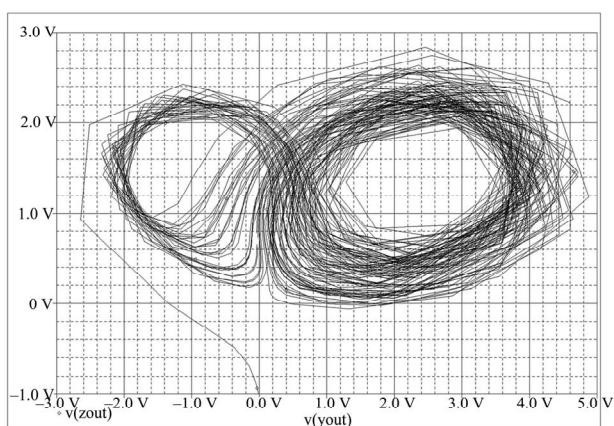
### 4. Conclusions

In this paper, we have displayed a three-dimensional continuous autonomous chaotic system modified from the Lorenz system and Rössler system, which the first equation has not non-linear cross-product term but the second equation has one non-linear cross-product term





**Figure 10.** PSpice simulation result of the new chaotic system's electronic oscillator (Figure 4) for  $xy$  strange attractor.



**Figure 11.** PSpice simulation result of the new chaotic system's electronic oscillator (Figure 5) for  $yz$  strange attractor.

and the third one has two non-linear cross-product term. Part of the basic dynamic behavior of the system is explored further investigation in the Lyapunov Exponent and bifurcation diagrams. Moreover, this was the new system also physically realized using analogue electronic circuits.

## 5. References

- [1] G. Chen and X. Dong, "From Chaos to Order: Methodologies, Perspectives and Applications," World Scientific Publishing, Singapore, 1998.  
[doi:10.1142/9789812798640](https://doi.org/10.1142/9789812798640)
- [2] K. M. Cuomo, A. V. Oppenheim and S. H. Strogatz, "Synchronization of Lorenz-Based Chaotic Circuits with Applications to Communications," *IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing*, Vol. 40, No. 10, 1993, pp. 626-633.  
[doi:10.1109/82.246163](https://doi.org/10.1109/82.246163)
- [3] E. N. Lorenz, "Deterministic Nonperiodic Flow," *Journal of Atmospheric Sciences*, Vol. 20, No. 2, 1963, pp. 130-141.  
[doi:10.1175/1520-0469\(1963\)020<0130:DNF>2.0.CO;2](https://doi.org/10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2)
- [4] O. E. Rossler, "An Equation for Continuous Chaos," *Physics Letters A*, Vol. 57, No. 5, 1976, pp. 397-398.  
[doi:10.1016/0375-9601\(76\)90101-8](https://doi.org/10.1016/0375-9601(76)90101-8)
- [5] W. Yu, J. Cao, K. Wong and J. Lu, "New Communication Schemes Based on Adaptive Synchronization," *Chaos*, Vol. 17, No. 3, 2007, pp. 33-114.
- [6] J. H. Lü, G. Chen, D. Cheng and S. Celikovsky, "Bridge the Gap between the Lorenz System and the Chen System," *International Journal of Bifurcation and Chaos*, Vol. 12, No. 12, 2002, pp. 2917-2926.
- [7] S. Celikovsky and G. Chen, "On a Generalized Lorenz Canonical Form of Chaotic Systems," *International Journal of Bifurcation and Chaos*, Vol. 12, No. 8, 2002, pp. 1789-1812. [doi:10.1142/S0218127402005467](https://doi.org/10.1142/S0218127402005467)
- [8] J. Lü, G. Chen and S. Zhang, "Dynamical Analysis of a New Chaotic Attractor," *International Journal of Bifurcation and Chaos*, Vol. 12, No. 5, 2002, pp. 1001-1015.
- [9] S. Nakagawa and T. Saito, "An RC OTA Hysteresis Chaos Generator," *IEEE Transactions on Circuits Systems Part I: Fundamental Theory and Applications*, Vol. 43, No. 12, 1996, pp. 1019-1021. [doi:10.1109/81.545846](https://doi.org/10.1109/81.545846)
- [10] J. Lü and G. Chen, "Generating Multiscroll Chaotic Attractors: Theories, Methods and Applications," *International Journal of Bifurcation and Chaos*, Vol. 16, No. 4, 2006, pp. 775-858. [doi:10.1142/S0218127406015179](https://doi.org/10.1142/S0218127406015179)
- [11] M. E. Yalcin, J. A. K. Suykens, J. Vandewalle and S. Ozoguz, "Families of Scroll Grid Attractors," *International Journal of Bifurcation and Chaos*, Vol. 12, No. 1, 2002, pp. 23-41. [doi:10.1142/S0218127402004164](https://doi.org/10.1142/S0218127402004164)
- [12] K. M. Cuomo and A. V. Oppenheim, "Circuit Implementation of Synchronized Chaos with Applications to Communications," *Physical Review Letters*, Vol. 71, No. 65, 1993, pp. 65-68. [doi:10.1103/PhysRevLett.71.65](https://doi.org/10.1103/PhysRevLett.71.65)