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Power Allocation in Primary User-Assisted Multi-Channel Cognitive Radio Networks

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ABSTRACT

This paper addresses power allocation problem for spectrum sharing multi-band cognitive radio networks, where the primary user (PU) allows secondary users (SUs) to transmit simultaneously with it by coding SU's signal together with its own signal. The PU acts as the relay for the SUs and sells its transmit power to the SUs to increase its benefit, and the SUs bid for the PU's transmit power for maximizing their utilities. We propose a power allocation scheme based on traditional ascending clock auction, in which the SUs iteratively submit the optimal power demand to the PU according to the PU's announced price, and the PU updates that price based on all SUs' total power demands. Then we mathematically prove the convergence property of the proposed auction algorithm (i.e., the auction algorithm converges in a finite number of clocks), and show that the proposed power auction algorithm can maximize the social welfare. Finally, the performance of the proposed scheme is verified by the simulation results.

Keywords: Power Allocation; Cognitive Radio; Auction; Cooperative Communications

1. Introduction

With the rapid deployment of wireless services in the last decade, the scarcity in radio spectrum emerges a critical issue for wireless communications. However, the report from the Federal Communications Commissions shows that most of the licensed spectrum is severely underutilized in traditional fixed spectrum allocation [1]. Cognitive radio (CR) is a technology that can deal with the dilemma between spectrum scarcity and spectrum underutilization [2]. It allows the unlicensed users (secondary users (SUs)) to access licensed bands owned by the licensed users (primary users (PUs)) without interfering with them.

As described in [3], SUs can access the spectrum owned by the primary user using spectrum sharing, where the SU coexists with the PU and transmits with power constraints to guarantee the quality of service (QoS) of the PU. To improve the efficiency of resource utilization, cooperative communications has been introduced in CR networks. In [4], the SU transmitter allocates only part of its power to deliver its own messages, and uses the remaining power to forward PU's messages so as to compensate the interference at the PU receiver. A dynamic spectrum leasing architecture was proposed in [5], which allows PUs to reduce their power expenditure by using the SUs as relay nodes. The authors in [6] formulated the resource allocation problem as a two-tier game, in which each PU acts as a relay for multiple SUs and sells the unused radios to SUs. However, the studies in the literature on PU-assisted cooperative communications in spectrum sharing CR networks are still relatively sparse, and how to control the transmit power at the PU for secondary transmissions remains an open problem.

Network Coding has been proved to be a promising approach to reduce time-slot overhead for cooperative communications in wireless networks [7]. In [8], the exchange of independent information between two nodes in a wireless network has been analyzed. It demonstrated that information exchange can be efficiently performed by exploiting network coding and the broadcast nature of the wireless medium. The authors in [9] addressed the power allocation problem in a network-coded multiuser two-way relaying network, where multiple pairs of users communicate with their partners via a common relay node. In cognitive radio networks, the authors in [10] showed that distributed network users can automatically adjust their coding structure, then collaborate together to avoid the degrading effects of signal fading. In this study, we consider the scenario that PU helps SUs by forwarding their combining signals to improve transmission efficiency.

The researches on power control for CR networks have been conducted most recently [11-13]. Most existing studies focused on centralized schemes that often need high requirements on network infrastructure, and their computational complexity scales up with the network size. Game theory has been widely recognized as a powerful tool for distributed resource allocation in interactive multiuser systems. In order to address both system efficiency and user fairness issues of CR networks, the authors in [14] proposed a distributed power control strategy by using a cooperative Nash bargaining game model. In [15], a joint power and rate control strategy were presented for SUs on the basis of a cooperative game theoretic framework. Three auction-based schemes were proposed in [16] for multimedia streaming over CR networks. In [17], the authors considered auction-based power allocation in multi-band CR networks, where multiple SUs transmit via a common relay, and bid for the transmit power of the relay.

In this paper, we consider the power allocation problem for spectrum sharing multi-band CR networks, where the PU allows the SUs to transmit simultaneously with it by coding SU's signal together with its own signal. The PU acts as the relay for the SUs and sells its transmit power to the SUs to increase its benefit, and the SUs bid for the PU's transmit power for maximizing their utilities. Our main contributions are as follows: First, we propose a power allocation scheme based on traditional ascending clock auction (ACA-T), in which the SUs iteratively submit the optimal power demand to the PU according to the PU's announced price, and the PU updates that price based on all SUs' total power demands. Second, we mathematically prove the convergence property of the proposed auction algorithm (i.e., the auction algorithm converges in a finite number of clocks), and show that the proposed power auction algorithm can maximize the social welfare. Finally, the performance of the proposed scheme is verified by the simulation results.

2. Network Modeling and Notations

Consider a CR system consisting of a primary user divided into M non-overlapping narrowband subchannels and N secondary users. The primary user transmits signals from the primary transmitter (PT) to the primary receiver (PR). Each secondary user i sends messages from secondary transmitter s_i to secondary receiver d_i. The PT acts as the relay and assists SUs' transmissions. We employ analog network coding and the amplify-and-forward relaying protocol at the PT. Assume that each sub-channel of the PT can be accessed by only one SU, and the channel occupancy by the SUs is maintained by the PT. For simplicity, we consider the scenario where $N=M$. The cases with $N < M$ and $N > M$ can be analyzed in a similar way.

The structure of a CR frame under spectrum sharing consists of a channel allocation slot, an auction slot and a data transmission slot. In channel allocation slot, the SUs who intend to send data to their receivers submit their transmission requests to the PT. The PT then randomly assigns a sub-channel to each SU. As shown in **Figure 1**, SU i is designated to the j_{th} sub-channel of the PT, and the transmission time period is divided into three phases. Here the solid lines indicate the intended communications, while the dotted lines represent the interference.

In the first phase: At each sub-channel, the PT transmits its data to its destination PR with power Pu. Assume that the total transmit power of the PT is Pt, then we have $Pu=Pt/M$. As in the wireless environment, the data will be transmitted in a broadcast way, so all the receivers in the system will overhear and receive the data. The signals received at the PR and the secondary receivers are respectively given by

$$Y_{PT}^{PR} = \sqrt{P_u G_{PT}^{PR}} X_u + n_{PT}^{PR} \tag{1}$$

$$Y_{PT}^{d_i} = \sqrt{P_u G_{PT}^{d_i}} X_u + n_{PT}^{d_i} \tag{2}$$

where $Y_{\{\cdot\}}^{\{\cdot\cdot\}}$ represents the signal received at $\{\cdot\cdot\}$ from $\{\cdot\}$, X_u is the information symbol transmitted by PT with $E[|X_u|^2]=1$, $n_{\{\cdot\}}^{\{\cdot\cdot\}}$ is the additive white Gaussian noise (AWGN) with variance σ^2. And $G_{\{\cdot\}}^{\{\cdot\cdot\}}$ denotes the fading channel gain from $\{\cdot\}$ to $\{\cdot\cdot\}$, where its amplitude $|G_{\{\cdot\}}^{\{\cdot\cdot\}}|^2$ is exponentially distributed. Assume all the channel gains remain static during each transmission frame, and are available to the PT.

The signal-to-interference-plus-noise-ratio (SINR) of X_u at the PT in the first phase is

$$\Gamma_{PT}^{PR}(1) = P_u G_{PT}^{PR} / \sigma^2 \tag{3}$$

In the second phase: SU i transmits its signal with power P_i. The signals received by d_i, PT and PR are

$$Y_{s_i}^{d_i} = \sqrt{P_i G_{s_i}^{d_i}} X_{s_i} + n_{s_i}^{d_i} \tag{4}$$

$$Y_{s_i}^{PT} = \sqrt{P_i G_{s_i}^{PT}} X_{s_i} + n_{s_i}^{PT} \tag{5}$$

$$Y_{s_i}^{PR} = \sqrt{P_i G_{s_i}^{PR}} X_{s_i} + n_{s_i}^{PR} \tag{6}$$

where X_{s_i} is the signal transmitted by s_i in this phase with $E[|X_{s_i}|^2]=1$.

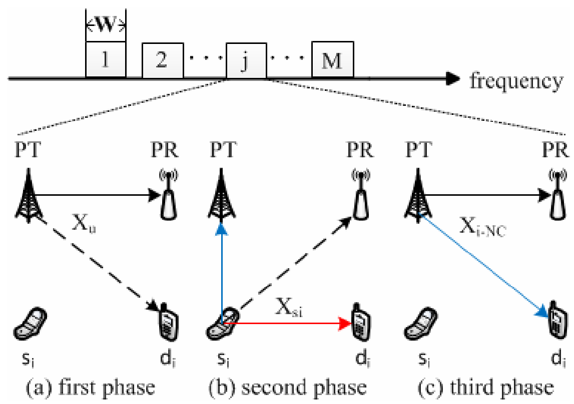

Figure. 1. Transmissions in three phases.

Thus, the SINR of X_{s_i} at SR d_i in the second phase is

$$\Gamma_{s_i}^{d_i}(2) = P_i G_{s_i}^{d_i} / \sigma^2 \quad (7)$$

In the third phase: In this case, the PT makes a combination of $Y_{s_i}^{PT}$ and its own signal X_u with network coding [18], then amplifies and forwards the combined signal $X_{i\text{-}NC}$. Then the received signals are

$$Y_{PT-NC}^{PR} = \sqrt{P_{ui} G_{PT}^{PR}} X_{i-NC} + n_{PT}^{PR} \quad (8)$$

$$Y_{PT-NC}^{d_i} = \sqrt{P_{ui} G_{PT}^{d_i}} X_{i-NC} + n_{PT}^{d_i} \quad (9)$$

where $Y_{\{\cdot\}-NC}^{\{\cdot,\cdot\}}$ represents the signal received by using network coding, P_{ui} is the PT's transmit power for SU i, and

$$X_{i-NC} = \frac{X_u + Y_{s_i}^{PT}}{| X_u + Y_{s_i}^{PT} |} \quad (10)$$

Substituting (5) into (10), we can rewrite (8) and (9) as

$$Y_{PT-NC}^{PR} = \frac{\sqrt{P_{ui} G_{PT}^{PR}}}{\sqrt{1 + P_i G_{s_i}^{PT} + \sigma^2}} (X_u + X_{s_i} + n_{s_i}^{PT}) + n_{PT}^{PR} \quad (11)$$

$$Y_{PT-NC}^{PR} = \frac{\sqrt{P_{ui} G_{PT}^{d_i}}}{\sqrt{1 + P_i G_{s_i}^{PT} + \sigma^2}} (X_u + X_{s_i} + n_{s_i}^{PT}) + n_{PT}^{d_i} \quad (12)$$

In the previous two time phases, PT and s_i have transmitted their data respectively. And we assume that the destination nodes PR and d_i know exactly the useful messages from their source nodes, where X_u is for the PR from PT in the first phase and X_{s_i} is for d_i from s_i in the second phase. So after they have received signals $X_{i\text{-}NC}$ in the third phase, each destination node can perfectly distinguish its useful signal from the combined signals.

Thus, the PT can completely extract X_u from $X_{i\text{-}NC}$ and we can get the required signal $\hat{Y}_{PT-NC}^{PR}$

$$\hat{Y}_{PT-NC}^{PR} = \frac{\sqrt{P_{ui} G_{PT}^{PR}}}{\sqrt{1 + P_i G_{s_i}^{PT} + \sigma^2}} (X_u + n_{s_i}^{PT}) + n_{PT}^{PR} \quad (13)$$

And d_i extracts X_{s_i} from $X_{i\text{-}NC,}$ then we have the needed signal $\hat{Y}_{PT-NC}^{d_i}$

$$\hat{Y}_{PT-NC}^{d_i} = \frac{\sqrt{P_{ui} G_{PT}^{d_i}}}{\sqrt{1 + P_i G_{s_i}^{PT} + \sigma^2}} (\sqrt{P_i G_{s_i}^{PT}} X_{s_i} + n_{s_i}^{PT}) + n_{PT}^{d_i} \quad (14)$$

From (13), the PU's SINR in the third phase is

$$\Gamma_{PT}^{PR}(3) = \frac{P_{ui} G_{PT}^{PR}}{(1 + P_{ui} G_{PT}^{PR} + P_i G_{s_i}^{PT} + \sigma^2)\sigma^2} \quad (15)$$

Using (14), the SINR at node d_i is given by

$$\Gamma_{PT}^{d_i}(3) = \frac{P_{ui} G_{PT}^{d_i}}{(1 + P_{ui} G_{PT}^{d_i} + P_i G_{s_i}^{PT} + \sigma^2)\sigma^2} \quad (16)$$

Therefore, with (3) and (15), the PU's achievable rate in the j_{th} sub-channel is

$$\begin{aligned} R_{PU}^{j} &= W \log_2(1 + \Gamma_{PT}^{PR}(1) + \Gamma_{PT}^{PR}(3))/3 \\ &= \frac{W}{3} \log_2\left(1 + \frac{P_u G_{PT}^{PR}}{\sigma^2} + \frac{P_u G_{PT}^{PR}}{(1 + P_{ui} G_{PT}^{PR} + P_i G_{s_i}^{PT} + \sigma^2)\sigma^2}\right) \end{aligned} \quad (17)$$

As for SU i, its achievable rate is

$$\begin{aligned} R_i^C &= W \log_2(1 + \Gamma_{s_i}^{d_i}(2) + \Gamma_{PT}^{d_i}(3))/3 \\ &= \frac{W}{3} \log_2\left(1 + \frac{P_i G_{s_i}^{d_i}}{\sigma^2} + \frac{P_u G_{PT}^{d_i} P_i G_{s_i}^{PT}}{(1 + P_{ui} G_{PT}^{d_i} + P_i G_{s_i}^{PT} + \sigma^2)\sigma^2}\right) \end{aligned} \quad (18)$$

where W is the sub-channel's bandwidth. The coefficient 1/3 dues to the fact that cooperative transmission uses one third of the resources.

3. Problem Formulation

There are two fundamental questions on power allocation to be addressed: 1) The incentives of PU and SUs for using cooperative transmissions; 2) The optimal power P_{ui} allocated to SU i. In this section, we present a game-theoretic framework of the transmit power allocation at the PU for the SUs.

3.1. SUs's Utility Function

To depict a SU's satisfaction with the received relay power from the PT, we define a utility function for SU i as:

$$U_i^C = g R_i^C - \lambda P_{ui} \quad (19)$$

where in the right side of the equation, the first term is the gain and the second term is the cost by using cooperative transmissions. g is a positive constant providing conversion of units, R_i^C is the achievable rate in (18), λ represents the price per unit of power charged by the PT, and P_{ui} denotes how much power SU i will buy from the PT.

Each SU aims at maximizing its own utility, which subjects to the total available transmit power P_t of the PT. Thus, we can model the optimal cooperative power demand of each SU as

$$\begin{aligned} &\max_{P_{ui}} \ U_i^C = g R_i^C - \lambda P_{ui} \\ &\text{s.t.} \quad 0 \le P_{ui} \le P_t \end{aligned} \quad (20)$$

Clearly, the utility function U_i^C is concave in P_{ui}, so we can solve the optimal power P_{ui} by taking the derivative of U_i^C in (19) with the respect to P_{ui} as

$$\frac{\partial U_i^C}{\partial P_{ui}} = g \frac{\partial R_i^C}{\partial P_{ui}} - \lambda = 0 \quad (21)$$

For simplicity, we define

$$W' = \frac{gW}{3\ln(2)};\ A_i = 1 + \frac{P_i G_{s_i}^{d_i}}{\sigma^2}$$
$$B_i = \frac{P_i G_{s_i}^{PT}}{\sigma^2};\ C_i = \frac{1 + P_i G_{s_i}^{PT} + \sigma^2}{G_{PT}^{d_i}} \quad (22)$$

Substituting (22) into (21), we can get the optimal cooperative power $P_{ui}(\lambda)$ expressed in (23) for maximizing the utility function U_i^C.

$$P_{ui}(\lambda) = \frac{1}{2(A_i + B_i)} \times \left[\sqrt{B_i^2 C_i^2 + \frac{4W'}{\lambda}(A_i B_i C_i + B_i^2 C_i)} - (2A_i C_i + B_i C_i)\right], \quad \forall i \in i, 2, \ldots, N \quad (23)$$

3.2. PU's Utility Function

The PU's utility is based on the rate it achieves and the gain of selling its power P_t to the SUs. Thus, the PU's utility function is given by

$$U_{PU}^C = gR_{PU} + \lambda P_t \quad (24)$$

where $R_{PU} = \sum_{j=1}^{M} R_{PU}^j$ is the total achievable rate of the PU in all M sub-channels, and λP_t is the total payment from the SUs.

The choice of price λ is important to the PU. Since the PU can choose either to use the power itself or to share the power with SUs. For the PU, we define the reservation price of the power as the expense of relaying for the SUs. This reservation price is also called cost, which represents the adverse effects of SUs' transmissions on PU's performance. It may consist of device depreciation, power consumption, performance degradation of PU, etc. We denote the reservation price of PU as λ_0.

4. Power Auction Mechanism

We model a single-auctioneer, multiple-bidder power trading market, in which the PU wants to sell and the SUs want to buy the relay power. And we discuss how the PU sells its power to the SUs by using an auction mechanism. The auction procedures can be briefly described as follows: the PT, i.e. the "auctioneer", announces a price, the SUs, i.e. the "bidders", report to the auctioneer their demanded power at that price. The auctioneer updates the price and the process repeats until the total demanded power meets the maximum available supply power of the PT.

The proposed scheme is based on the traditional ascending clock auction (ACA-T) [16]. As shown in Algorithm 1, before the auction, the PT initially sets clock index $\tau = 0$, the step size $\mu > 0$, and a reserved price $\lambda = \lambda_0$. For each auction clock $\tau = 0, 1, \ldots,$ there is a specific price λ_τ corresponds to it. The PT announces the price to the SUs. Based on the announced price λ_τ, each SU submits its optimal power demand $P_{ui}(\lambda_\tau)$, which refers to (23), to the PT. After receiving all the demands, the PT sums up all the bids $P_{tal}(\lambda_\tau) = \sum_{i=1}^{N} P_{ui}(\lambda_\tau)$ and compares it with P_t. If $P_{tal}(\lambda_\tau) > P_t$, the auction continues to time $\tau + 1$. Then the PT raises the price $\lambda_{\tau+1} = \lambda_\tau + \mu$ and announces the new price to all SUs. Otherwise, the auction concludes and the current time denoted as T. Since the price increases discretely, the demanded power of each SU may decrease, and we might have $P_{tal}(\lambda_T) < P_t$. In order to fully utilize the power P_t (i.e. $P_{tal}(\lambda_T) = P_t$), we apply a proportional rationing rule [19] and the final allocated power is given by

$$P_{ui}^* = P_{ui}(\lambda_T) + \frac{P_{ui}(\lambda_{T-1}) - P_{ui}(\lambda_T)}{\sum_{i=1}^{N} P_{ui}(\lambda_{T-1}) - \sum_{i=1}^{N} P_{ui}(\lambda_T)} \left[P_t - \sum_{i=1}^{N} P_{ui}(\lambda_T)\right] \quad (25)$$

Algorithm 1 Ascending Clock Power Auction Algorithm

1. Initialization

PT initializes clock index $\tau = 0$, step size $\mu > 0$, gives the available power P_t, and announces the initial price λ_0 with the reserve price.

Each SU i computes its optimal power demand $P_{ui}(\lambda_0)$, and submits the bid to PT.

PT sums up all the bids $P_{tal}(\lambda_0) = \sum_{i=1}^{N} P_{ui}(\lambda_0)$, and compares it with P_t.

2. Bid Update

WHILE ($P_{tal}(\lambda_\tau) > P_t$)

· PT sets $\lambda_{\tau+1} = \lambda_\tau + \mu$, $\tau = \tau + 1$;

· PT announces λ_τ to all the SUs;

· Each SU computes its $P_{ui}(\lambda_\tau)$ based on (23), and submits the best bid to PT;

· PT sums up all the bids $P_{tal}(\lambda_\tau) = \sum_{i=1}^{N} P_{ui}(\lambda_\tau)$.

END

3. Power Allocation

Conclude the auction, set $T = \tau$, and according to (25), allocate P_{ui}^* to SU i.

4. Payment

Finally, each SU i pays $\lambda_T P_{ui}^*$ to PT.

where $\sum_{i=1}^{N} P_{ui}^* = P_t$. Consequently, the payment from SU i to the PT is $\lambda_T P_{ui}^*$. Note that, the power constraint in the auction is $0 \le \sum_i P_{ui} \le P_t$, and we can get the optimal strategy for SU i, $i \in \{1, 2, \ldots, N\}$ *as*

$$P_{opt}^{i}=\min(P_t,\max(P_{ui}^{*},0)) \quad (26)$$

Theorem 1. *The proposed auction game in Algorithm1 will conclude in a finite number of clocks.*

Proof: It is straightforward to see that the optimal bid $P_{ui}(\lambda_\tau)$ is a non-increasing function in λ_τ, i.e. $P_{ui}(\lambda_{\tau+1})\le P_{ui}(\lambda_\tau)$, and when $P_{ui}(\lambda_{\tau+1})=P_{ui}(\lambda_\tau)=P_t$ or $P_{ui}(\lambda_{\tau+1})=P_{ui}(\lambda_\tau)=0,\forall\tau$, the equality occurs. Cause λ increases with a fixed index $\mu>0$, and for a sufficient large τ, there will be $P_{ui}(\lambda_{\tau+1})<P_{ui}(\lambda_\tau)<P_t$. Then, there exists a finite large number T makes $\sum_{i=1}^{N}P_{ui}(\lambda_T)<P_t$, which means that the auction concludes at clock T.

From theorem1, we can see that the proposed power auction algorithm has the convergence property.

Theorem 2. *When μ is sufficiently small, the proposed ascending-clock auction will converge to $(P_{u1}^{*},P_{u2}^{*},\ldots,P_{uN}^{*})$, which maximizes the social welfare.*

Proof: The proposed distributed algorithm can maximize the sum of rates, i.e. $(P_{u1}^{*},P_{u2}^{*},\ldots,P_{uN}^{*})$ is the solution to the following optimization problem:

$$\begin{aligned}&\max_{P_{ui}}\ \sum_{i=1}^{N}R_i^C(P_{ui})\\ &\text{s.t.}\ \sum_{i=1}^{N}P_{ui}\le P_t\\ &\quad 0\le P_{ui}\le P_t,\quad \forall i=1,2,\ldots,N\end{aligned} \quad (27)$$

which is convex in terms of P_{ui}, since R_i^C is concave in P_{ui}. We find the optimal P_{ui} by solving the Karush-Kuhn-Tucker (KKT) conditions, and we formulate the Lagrangian of problem (28) as [20]:

$$\begin{aligned}L(P_{ui},\xi,\rho_i,v_i)=&-\sum_{i=1}^{N}R_i^C(P_{ui})+\xi\Big(\sum_{i=1}^{N}P_{ui}-P_t\Big)\\ &+\sum_{i=1}^{N}\rho_i(P_{ui}-P_t)-\sum_{i=1}^{N}v_iP_{ui}\end{aligned} \quad (28)$$

Then, the KKT conditions are given by:

$$\begin{aligned}&-\frac{W}{3\ln 2}\frac{B_iC_i}{(A_iC_i+A_iP_{ui}+B_iP_{ui})(C_i+P_{ui})}+\xi+\rho_i-v_i=0,\\ &\xi\Big(\sum_{i=1}^{N}P_{ui}-P_t\Big)=0,\\ &\rho_i(P_{ui}-P_t)=0,\forall i=1,2,\ldots,N\\ &v_iP_{ui}=0,\forall i=1,2,\ldots,N\\ &0\le P_{ui}\le P_t,\forall i=1,2,\ldots,N\\ &\sum_{i=1}^{N}P_{ui}\le P_t\end{aligned} \quad (29)$$

where $\xi\ge0,\rho_i\ge0,v_i\ge0,\forall i\in1,2,\ldots,N$ are the lagrangian multiplier with the relevant of power constraints. By solving the optimal convex problem above, we can get the solution that $P_{ui}(\xi)$ is in the form in (23), and ξ makes $\sum_{i=1}^{N}P_{ui}(\xi)=P_t$. Thus, the outcome $(P_{u1}^{*},P_{u2}^{*},\ldots,P_{uN}^{*})$ is the solution that maximizes the social warfare of all the SUs when PU sells out its cooperative transmit power P_t.

5. Simulation Results

In this section, we present simulation results to demonstrate the performance of the proposed power allocation algorithm. We consider a scenario as shown in **Figure 2**, where three secondary links ($s_1\to d_1,s_2\to d_2,s_3\to d_3$) want to be relayed by PU. The channel gains are $(0.097/d^{\alpha})$, where d is the distance between two nodes, and the path-loss exponent is $\alpha=4$. We assume that the various units are positioned such that d does not approach zero. The transmit power of each SU is P_i=0.01W, $\forall i\in1,2,3$, the transmit power budget of PU is fixed, the reserve price $\lambda_0=1$, the conversion factor is g = 0.01, and the noise variance is $\sigma^2=10^{-13}$.

Figure 3 verifies the convergence of the proposed power allocation algorithm. When P_t is identified (P_t= 2), the different step sizes will reach the same price λ, and the convergence speed increases with the increasing of the step size μ. This proves that, the power auction process will conclude in finite clocks and finally reaches to

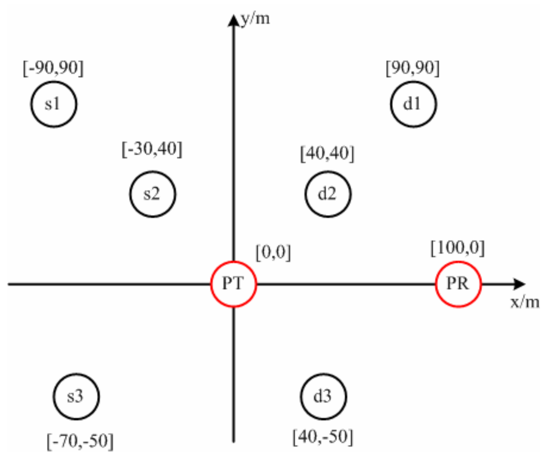

Figure 2. A three-user simulation network.

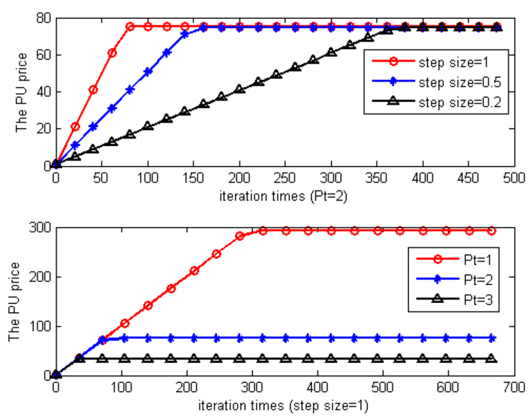

Figure 3. Convergence performance.

the optimal power allocation. From the below sub-fig, we can see that with the same step size ($\mu = 1$), the iteration times T decreases with the increasing of P_t. This is because that when the total relay power P_t of PT is sufficiently large, and the total demanded cooperative power by SUs is less than P_t, then the relay will sell the power early in the auction with a relatively lower price.

Figure 4 shows the allocated power of each SU. It's evident that with the increase of PU's power P_t, the cooperative power of each SU increases, with SU 2 has the largest power while SU 3 has the smallest. This attributed to the local information of the SUs, such as location, their transmit power, the path-loss.

We further show the rates and utilities of the users in the CR network in **Figure 5** and **Figure 6**. **Figure 5** shows the rates in cooperative transmission (CT) in this paper versus the direct transmission (DT). It is obvious that the cooperative way can greatly improve the rates of SUs. And with the relay power increases, the rates of each SU increase slowly. The sum rates of this cooperative communication can effectively maximize the the social welfare. The order of magnitude of the rates is big, so the growth in the image is not that obvious. In **Figure 6**, we can see that the utilities of SUs have the same trend with rates, which demonstrates that the auction-based power allocation algorithm in this work can greatly improve the SUs' utilities.

6. Conclusions

In this article, we tackled the power allocation problem for the relay-assisted SUs' transmission, where the PU acts as the relay and the SUs transmitting in spectrum underlay mode. We proposed a distributively power auction algorithm, which based on the traditional ascending clock auction (ACA-T). The convergence and social welfare properties are investigated and simulated. Future work can be extended to the case with many PUs in the network.

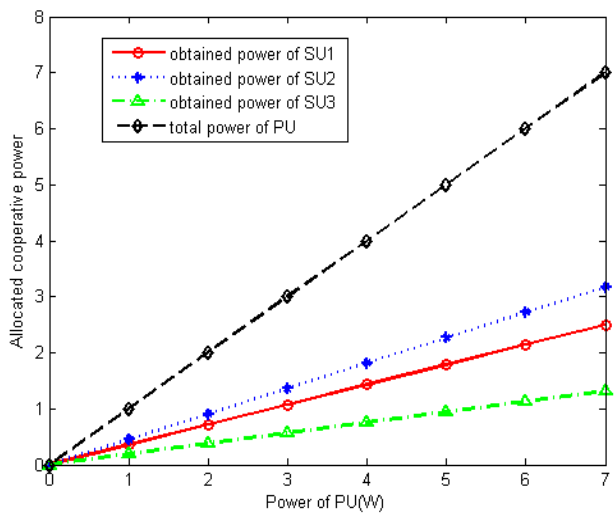

Figure 4. The obtained transmit power of the SUs.

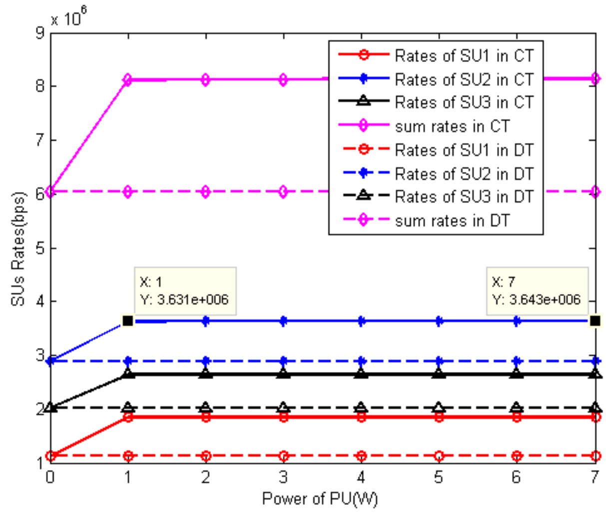

Figure 5. The achievable rates of the SUs.

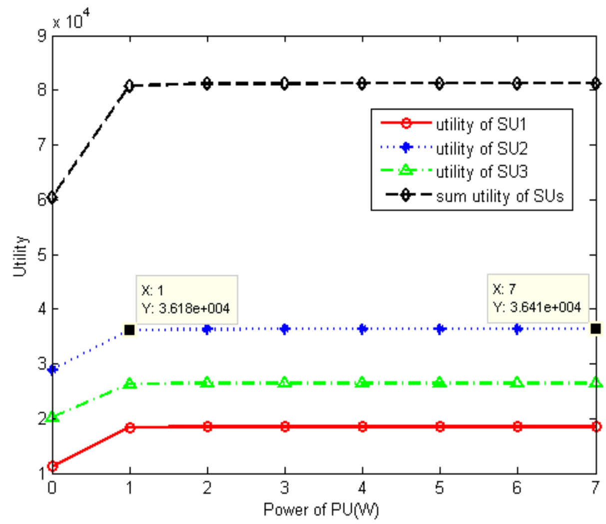

Figure 6. The utility achieved by the SUs.

7. Acknowledgements

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