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Calculus for Coloring

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Professors who administer the differential and integral calculus classes in the hard sciences courses and in the most diverse kinds of engineering, not rarely, are faced by the lack of motivation to learning and difficulties in absorbing the concepts of the discipline. With the purpose of contributing to the teaching-learning process related to calculus classes, this paper looks for synthesizing a process of elaboration and trial of an artwork destined for coloring, which content is specifically related to the discipline "Calculus with Applications IV", from the Biosystems Engineering course, of the University of Sao Paulo (USP)-Brazil. The material, always prepared in the form of pairs of pages, one displaying the picture to be colored (the artwork) and the other one like its twin (the text), was individually rated by the apprentices students. The obtained results point to the possibility of facing the paradigm and showing that projects like "calculus for coloring" are not only possible, but also a quality complement in the learning-teaching process.

Keywords: Calculus; Coloring; Education; Learning Process; Mathematics

Introduction

The mathematics education is highlighted by its complexities and challenges. It is possible to think about different views/ aspects of mathematics education implying challenges that can to call attention as a common theme. This is addressed by (Chapman, 2012). Chapman makes an analysis based on articles that deal on different challenges involving mathematics education such as: implementing a new curriculum (Ponte, 2012), to establish connections between real-world contexts and mathematics (Taylor, 2012), sustainable change in learning process (Zoest et al., 2012) and implementing of technology (Doğan, 2012). Ponte's article (Ponte, 2012), provides insights about a national professional development program that was used in the preparatory stages of the introduction of a new mathematics curriculum in Portugal indicating two main purposes for mathematics education: to promote positive attitudes and appreciation of mathematics that emphasizes three mathematical processes: problem solving, mathematical reasoning and mathematical communication. Edd Taylor (Taylor, 2012), addresses connections to students' out-of-school experiences and provides insights of challenges with teachers' learning and a possible way of addressing them. On the other hand, (Zoest et al., 2012), provides insights regarding sustainability of learning trough the use of sociomathematical and professionals norms. Describes a study that investigated the extent to which three sociomathematical and four professional norms intentionally fostered in an early mathematics pedagogy course re-emerged in a similar context. In his article, (Doğan, 2012), Doğan deals with prospective teachers views of technology (computer in mathematics education) in their learning and teaching. He considered mathematics teachers from two different universities in Turkey.

Even in accordance, in greater part, with Chapman's analysis, our experience in Brazil has permitted to note a low motivation generalized of the mathematics apprentices since primary school and even in university degree. For this reason, we have researched about the stimulation and motivation in mathematics education.

We believe that our experience contributes with the challenge of provide a complementary alternative in order to facilitating learning in mathematics education. In this paper, we applied the idea, particularly, involving the differential and integral calculus. However, we believe that the experiment could be applied with a similar success to other levels of mathematics education. This is the idea or experiment, "Calculus for Coloring", that can be extended in the near future, for a "Mathematics for Coloring", also considering other level of mathematics education.

The practice of coloring has been proved itself a very efficient method on facilitating learning and memorization, such as a visualization method. Visualization seems indeed to result in a positive effect on the learning of students (Brandt et al., 2001). In fact, there are a great number of the works involving research on visualization in learning and teaching Mathematics. In the Handbook edited by Gutierrez and Boero (Gutiérrez & Boero, 2006), for instance, N. Presmeg wrote a chapter dedicated exclusively to this theme. Mayer (1997) has elaborated the dual coding theory in order to explain why pictures support under specific conditions the understanding of technical or physical phenomena. Mayer assumes that verbal and pictorial explanations are processed in different cognitive subsystems and that they result in the construction of different mental models. In the Mayer's work, students who received coordinated presentation of explanations in verbal and visual format (multiple representation group) generated a median of over 75% more creative solutions on problem-solving transfer tests than did students who received verbal explanations alone (single representation group).

We believe that the using of a color code raises the consciousness of connections, makes the visualization of complexes designs easier and can contribute for, the almost always required, process of abstraction of concepts and ideas through symbols.

When the act of coloring is requested to the apprentice, as a complementary or additional job in the process of learning about a determined concept, such act implies in a really active way of consolidating these concepts. With hands in activity, there is a large chance of focusing attention in a much more intense way while concentrating on simple topics, on at a time. At the same time, the mind associates shapes with concepts, transferring them from the short time memory to a more lasting time memory.

It is well known that we are visual apprentices, in other words, a lot of what we learn originates from our sense of vision. Even tasks like, for instance, those executed by a sportsman, must be "visualized" before being fulfilled.

That is the reason why it is so important to "visualize concepts" to understand and consolidate contents of a pre-established discipline, for example.

It is possible to find in the Brazilian publishing market three excellent books of the series "the coloring book": Anatomy (Elson & Kapit, 2001), Physiology (Kapit, Macey, & Meisami, 1999) and Microbiology (Alcamo & Elson, 2004). On this works, authors manage to teach successfully to the lay public the essence of math concepts with a series of sheets and posters.

Traditionally, courses as Anatomy or Microbiology can be thought as more "visual" than courses like Physics and, above all, Mathematics. Notwithstanding, an example that other courses or disciplines are capable of "visualization" is the case of the volume "The Physiology Coloring Book" already mentioned above. Physiology may be understood, in certain way, as the application of Physics, Chemistry and Math to Medicine. In this case, is not about just describing static figures as in Anatomy. It is substantial to explain process as forces, chemical reactions, fluxes, states, signals and feedbacks.

In this work, we challenge the paradigm and show that a "Calculus for Coloring" isn't only possible as it can also be a quality complement on the teaching-learning process.

If we compare old math textbooks to recent ones, it will be immediate to note that, a major increase in the number and quality of pictures to motivate the readers. In this project, we believe that the pictures should be almost 50% of the content of the textbook.

Having this in mind, the objective of this work is developing teaching and researching skills and then testing the graphical for coloring material created and the explanatory text about the topics of the discipline "Calculus with Applications IV" of the Biosystems Engineering course, at the Pirassununga campus, from University of São Paulo (USP-Brazil).

Furthermore, this work has as its main goal to contribute to the developing of knowledge in the teaching-learning field in Calculus. On the edge, this work may be faced as an attempt of planting an embryo that would contribute to the Brazil's imperative challenge to improve the quality of teaching, especially in mathematics education.

Methods

Selected Topics and Layouts

In the Biossystems Engineering course of the University of São Paulo, the discipline "Calculus with Applications IV" has on its syllabus, the study of ordinary differential equations (Boyce & Di Prima, 2008)—solved by series of potentials method—as well as the study of partial differential equation (Kreyszig, 2006) and the Laplace transform. In the preparation of this project, from the contents of the referred discipline, the main topics were selected and subdivided in eleven parts, which were named "sheets". A sheet was composed of two pages.

The left page displays the illustration to be colored about the topic of the referred sheet (the artwork page). The right page (the text page), on the other hand, has a "summary text" containing the main information about the topic covered. Each sheet refers to one of the selected topics of the discipline, namely:

1) Laplace Transform;
2) Special functions;
3) Power series;
4) Fourier series;
5) Euler Equations;
6) Legendre Equations;
7) Bessel Equation;
8) Partial Differential Equations;
9) Heat Equation;
10) Wave Equation;
11) Laplace Equation.

The transfiguration of mathematical concepts relatively abstract in illustrations which can be colored and quickly understood didn't come as a trivial task. Aiming the maximization of the receptivity by its potential users (the apprentices), the material was built in order to attract the more attention and interest as possible.

In a way of attending those demands, a bibliographical research (with emphasis on the classical books of calculus) was made.

Some sheets created on this project were submitted to students that attended the referred discipline on the last semester, for the purpose of evaluating the efficiency of the work. Armed with crayons, the students painted each illustration and read all the informative texts, establishing relations between the contents and, at the very end, filled a form writing their individual opinions about the project. The results of this form may be observed on details in Section 3.2.

The Artwork

In the confection of each sheet, the start point was always the artwork (illustration). From the main concept about the subject matter, a sketch was drawn with the intention of expressing the central idea of the topic or some important particularities. At the same time that the art confection was made, annotations were also written with the objective of connecting it to the artwork next.

The Text-Page

After the sketch of the artwork, its twin part was elaborated. That means the written part that, besides bringing the most important information as objectively as possible relatively to the sketched artwork, also brings orientations related to the painting procedure suggested to the subject matter.

The written part, as the informative text present in each sheet, was organized in five sections (questions), which are: a) What is it? b) How does it work? c) What is it for? d) Curiosities, e) Did you know? (or Stay alert!).

Each one of these questions is described as follow:

a) What is it?

The answers to this question aimed to provide the student, in a few lines, the basic concept and the nature of the topic covered by the sheet. This way, the apprentice gets a quick notion about the subject immediately in the first lines of reading.

b) How does it work?

The second section of the informative text is, frequently, the longest and most elaborated. In this section, the most important equations and mathematical definitions of the subject matter are exposed.

c) What is it for?

In this section, an answer to one of the most frequent questionings of the students of the courses of Engineering is provided. Allowing the students to get to know the usefulness of a determined subject may be and additional motivation for studying.

d) Curiosities

In this section, like its own name suggests, complementary information about thetopic may be found. Usually, it's in this part of the text that the pioneer author(s) of the subject of the sheet is/are reminded, with a short description of her major works. Other characteristics about the subject matter may be found in this section.

e) Did you know? (or Stay alert!)

The last section is practically a footnote, with different colors and font. The section "Did you know?" always contains something relevant to the apprentice, many times connecting the subject matter in the sheet to different topics located in other sheets. This section can also have the name "Stay alert!". On this case, the section contains tips about other contents which may improve performance of the apprentice on the subjects referred to that sheet.

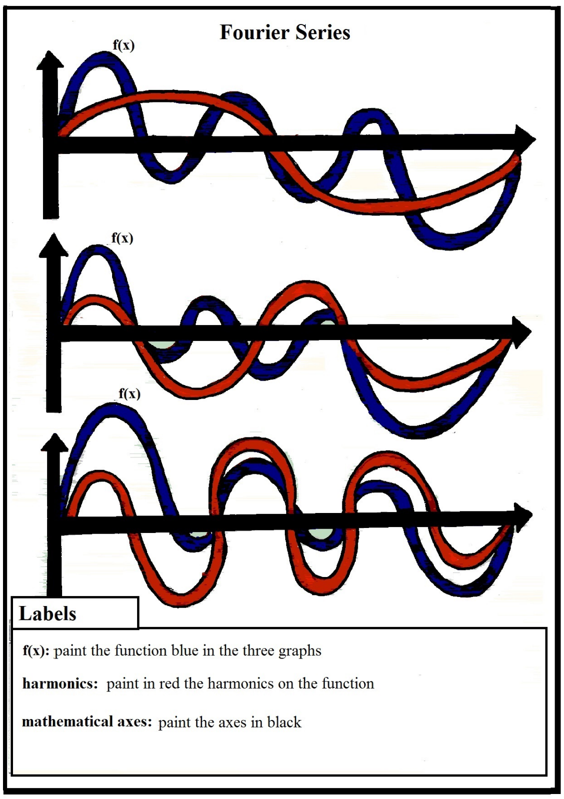
It's worth remembering that distributed on these five sections there are always information connecting the illustration of the left page (the artwork) to the corresponding theory on the right page (the text). In Section 3.1, both parts can be viewed in detail.

Results

In this section, two sheets are presented, as examples, as well as the results of the experimental evaluation submitted to the students of the referred discipline. More specifically, in the Section 3.1, two examples of the set—artwork and text—in its final state (already colored) may be viewed. In Section 3.2, in the means of Figures, a brief of the impression and evaluation that the students of Biosystems Engineering, of the University of São Paulo, provided about the material.

The Sheets-Two Examples

We would like show two sheets examples provided to the apprentices, such as: Fourier Series (**Figure 1**) and Heat Equation (**Figure 2**).



FOURIER SERIES

1. What is it?

a) An efficient method of representation of any periodic function by adding sines and cosines.

b) It is considered the most generic tool in the representation of functions by series, accomplishing better results than other methods.

2. How does it work?

a) A function can be expanded in a Fourier series as the definition:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

b) However, what actually defines the Fourier Series are the values of the coefficients **a_n** e **b_n**. In order to find the values of these coefficients and replace them in the previous formula, the following definitions must be used:

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos(\frac{n\pi x}{L})dx \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin(\frac{n\pi x}{L})dx$$

c) In **a_n** the integral sums all cosines of the function, while **b_n** sums all sines, both at the **-L** to **L** range. Like in the power series, the bigger the number of terms **n** is, the bigger is the approximation to the required function.

d) A symbolic representation of the operation of the Fourier series can be seen on the illustration on the opposite page. The Fourier function's behavior is simulated by its harmonics (first, second and third, respectively).

3. What is it for?

a) The primary objective of the application of Fourier Series is the simplification of complex periodic functions into functions easier to visualize and manipulate. Actually, practically any function can be represented by one of these series.

b) In the physics, it was created for studying the propagation of heat in solid bodies, but in the future showed itself even more useful, in the simulation of numerous types of waves and in the modern physics.

4. Curiosities

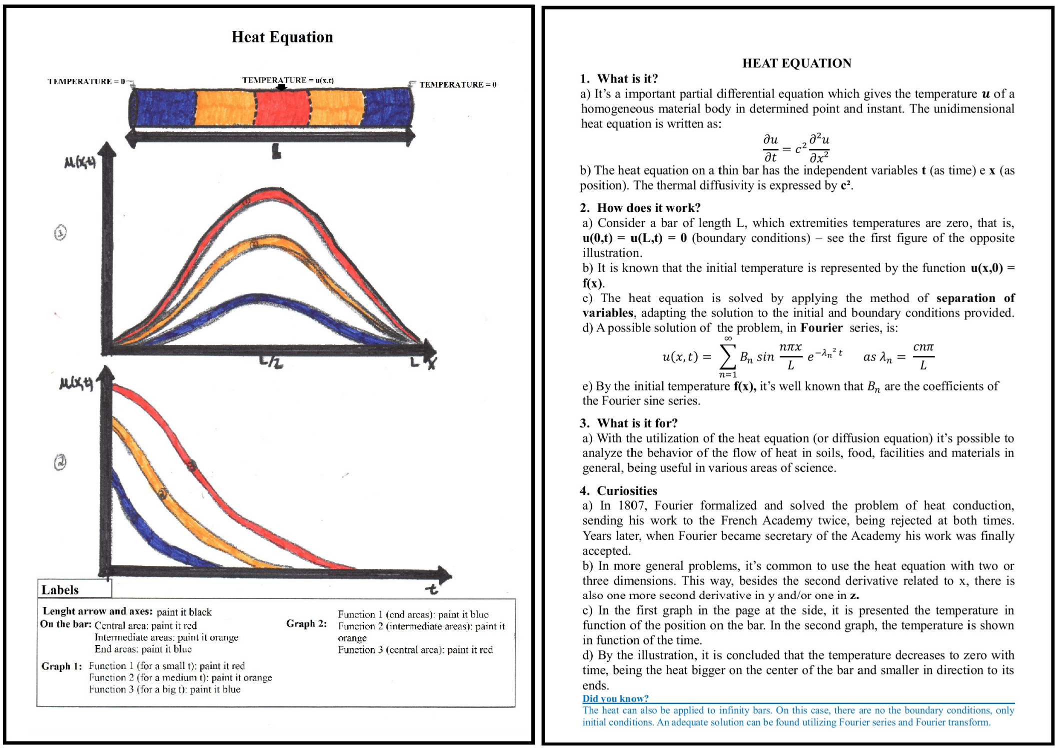
a) Jean Baptiste Joseph Fourier was a French mathematician born in 1768. He studied archeology in Egypt and worked for the army of Napoleon. He became famous for his studies about the flux of heat leading him to create the series which would be named after him.

b) The Fourier method was so effective that if the represented function is continuous and with continuous derivatives, only a few terms are enough for a good approximation.

Did you know?

Fourier series were elaborated to solve a physical problem. However, the method created by the French mathematician was so efficient that it was used for hundreds of others applications, even after his death. In the history of Physics, many new mathematical boundaries were broken in order to solve problems of the time.

Figure 1.
Example of the sheet "Fourier Series".



HEAT EQUATION

1. What is it?

a) It's a important partial differential equation which gives the temperature ***u*** of a homogeneous material body in determined point and instant. The unidimensional heat equation is written as:

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

b) The heat equation on a thin bar has the independent variables **t** (as time) e **x** (as position). The thermal diffusivity is expressed by $\mathbf{c^2}$.

2. How does it work?

a) Consider a bar of length L, which extremities temperatures are zero, that is, **u(0,t) = u(L,t) = 0** (boundary conditions) – see the first figure of the opposite illustration.

b) It is known that the initial temperature is represented by the function **u(x,0) = f(x)**.

c) The heat equation is solved by applying the method of **separation of variables**, adapting the solution to the initial and boundary conditions provided.

d) A possible solution of the problem, in **Fourier** series, is:

$$u(x,t) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{L} e^{-\lambda_n{}^2 t} \qquad as\ \lambda_n = \frac{cn\pi}{L}$$

e) By the initial temperature **f(x),** it's well known that B_n are the coefficients of the Fourier sine series.

3. What is it for?

a) With the utilization of the heat equation (or diffusion equation) it's possible to analyze the behavior of the flow of heat in soils, food, facilities and materials in general, being useful in various areas of science.

4. Curiosities

a) In 1807, Fourier formalized and solved the problem of heat conduction, sending his work to the French Academy twice, being rejected at both times. Years later, when Fourier became secretary of the Academy his work was finally accepted.

b) In more general problems, it's common to use the heat equation with two or three dimensions. This way, besides the second derivative related to x, there is also one more second derivative in y and/or one in **z.**

c) In the first graph in the page at the side, it is presented the temperature in function of the position on the bar. In the second graph, the temperature is shown in function of the time.

d) By the illustration, it is concluded that the temperature decreases to zero with time, being the heat bigger on the center of the bar and smaller in direction to its ends.

Did you know?

The heat can also be applied to infinity bars. On this case, there are no the boundary conditions, only initial conditions. An adequate solution can be found utilizing Fourier series and Fourier transform.

Figure 2.
Example of the sheet "Heat Equation".

Experimental Response

In order to test the efficiency of the completed job, a copy of the sheets referring to the "Fourier Series" and "Heat Equation" was given to the students of the course of Biosystems Engineering, of the University of São Paulo, that were attending to the discipline of "Calculus with Applications IV". Together with the sheets, a questioning with three questions regarding the quality of the work was also given to the students. It is important to highlight that, to the volunteer students nothing was offered in exchange of the notes of their impressions and evaluations. It's worth saying also that the professor who coordinated this work and the professor of the referred class that evaluated the material were different.

The obtained results on this evaluation are listed below:

Question 1: Did the illustrations presented in this material contribute to the absorption of the concepts?

Question 2: Are the pictures chosen appropriately to illustrate the proposed concepts in each topic?

Question 3: Would the idea of implementing this complementary material with all topics ministered in calculus be interesting?

The answers obtained and represented by the **Figures 3-5** may be synthesized in a pretty objective way. Almost 85% of the students believe that the sheets contribute significantly to the absorption of contents, what leads us to conclude that the method of coloring has great chances of obtaining efficiency in its objective of improving and making easier the absorption of basic concepts. Half the students (50%) classify the choice of the images presented in the sheets for the representation of the concepts as "Excellent", while 36% acknowledge the choice as "Very good". This positive reception is and indicative that the correct choices regarding the illustrations of each sheet are indispensable. Finally, the expressive parcel of 72.8% classifies as "Excellent" the idea of extrapolating this complementary material to all the other topics in the area of Calculus.

Final Considerations

There are several methodologies of teaching and entertaining at the same time. We believed that "Calculus for Coloring" is among these methodologies. The act of coloring, in addition to providing an approach relatively easy compared to the traditional methods of study, intends to provide a greater interest and amplify the capacity of absorption of concepts.

Considering, fundamentally, the results obtained by the answers that the apprentice students provided, it can be realized that that the initial impression that it is possible to face the paradigm and show that "Calculus for Coloring", is not only possible, it is also a good complement in the process of teaching-learning.

Unfortunately, to our knowledge, until now only two professors

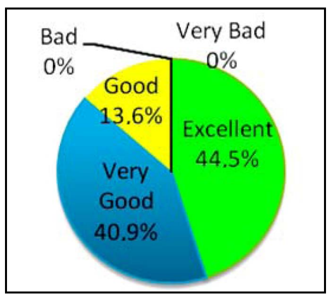


Figure 3.
Result of question 1.

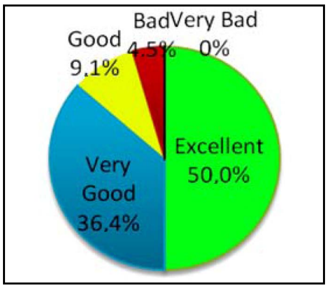


Figure 4.
Result of question 2.

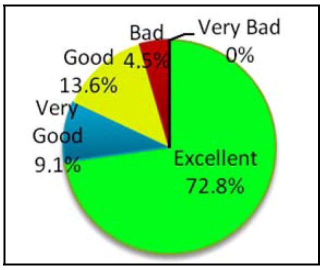


Figure 5.
Result of question 3.

(the authors) tested this method, which limit the conclusions stated. It would be wishful a major enrolment of the math teacher community to further verify the validity of the thesis presented here.

Finally, it's worth highlighting that the results reached so far brave and stimulate to give sequence and amplification to this project.

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