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The Study of the Polignac Numbers

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Abstract

In this paper we give an algorithm for polignac numbers. That algorithm is also the base of the proof of the de Polignac's conjecture. The examples of the application present algorithm for twin, cousin, and sexy primes are included.

Keywords

Polignac's Conjecture, Twin Primes, Cousin Primes, Sexy Primes

1. Introduction

Polignac's conjecture was made by Alphonse de Polignac in 1849 and it states: there are infinitely many consecutive primes p and p', such that

$$p' - p = 2l \tag{1}$$

where $l > 0$ an integer. In other words: for any positive number, l there are infinitely many prime gaps of the size of $2l$. These numbers are called polignac numbers.

If $l = 1$ then we have twin primes, for $l = 2$ we have cousin primes and if $l = 3$ then we have sexy primes.

Inspired by groundbreaking paper of Zhang Yitang in Annals of Mathematics [1], Polignac conjecture and its generalization were the subject of intensive studies. The beginning gap of size $2l < 70000000$ has been reduced to 246 one year after Zhang (according to the Polymath project wiki) [2]. More bout history of Polignac numbers we can find in [3].

Unfortunately, the Polignac's conjecture has not yet been proven. In this article we present entirely different method for tackling Polignac's conjecture then the authors in articles cited above. We give an algorithm for polignac numbers which is the base of the proof of Polignac's conjecture.

2. Preliminaries

Let N and l be the natural numbers and $2l \leq N$. Let r_i, $i = 1$, ..., k be the remainders of the number n by dividing by the consecutive odd prime numbers p_i, $i = 1$, ..., k, $p_i < \sqrt{N}$.

If

$$r_i = p_i - 2l \tag{2}$$

then

$$r_i + 2l = 0 \tag{3}$$

(in modular arithmetic $modp_i$). Thus if we eliminate all primes $p \leq N$ with remainders $p_i - 2l$, $i = 1$, ... , k then $p + 2l$ will be allways the prime number.

This allows us to give the algorithm for all pairs of the prime numbers $(p,\ p + 2l)$.

3. Algorithm

From the set of all primes in interval $(\sqrt{N},\ N]$ we eliminate primes congruent to $(p_i - 2l)(modp_i)$, $i = 1$, ... , k. This means we eliminate all primes p such that $p + 2l$ is a number divided by 3 or 5, ..., or by $p_k \leq \sqrt{N}$. Thus if p is remaining prime then $p + 2l$ is always a prime. The set of remaining primes is not empty.

Otherwise, $p + 2l$ for each prime p from the set of all primes in interval $(\sqrt{N},\ N]$ will be a prime number. But It is not true. Every remaining prime number will be the first of the pairs of prime numbers $(p,\ p + 2l)$.

4. Proof of the Polignac's Conjecture

We will show: there are infinity many pairs of the primes $(p,\ p + 2l)$, $l = 1, 2,$ ... . We note that described algorithm gives the set of prime pairs $(p,\ p+2l)$, $l = 1, 2,$ ... in interval $(\sqrt{N},\ N]$. If we apply algorithm in the interval $(N + 1,\ (N + 1)^2]$ then each of prime pairs $(p,\ p + 2l)$ in this interval will be different of those pairs in the interval $(\sqrt{N},\ N]$. Repeating procedure ad infinium we get thesis.

5. Examples

The following examples explain how described algorithm works.

Example 1. Let $N = 128$, $l = 1$, (twin primes) $\sqrt{N} = 11.3137...$
$p_1 - 2 = 3 - 2 = 1$, $p_2 - 2 = 5 - 2 = 3$, $p_3 - 2 = 7 - 2 = 5$,
$p_4 = 11 - 2 = 9$.

Note: the number 128 used in examples is casual.

We will consider the set of primes in interval [13, 128] 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127.

We have

primes$\equiv (1)(\text{mod}3)$
13, 19, 31, 37, 43, 61, 67, 73, 79, 97, 103, 109, 127,
primes$\equiv (3)(\text{mod}5)$
13, 23, 43, 53, 73, 83, 103, 113,
primes$\equiv (5)(\text{mod}7)$
19, 47, 61, 83, 89, 103,

primes$\equiv (9)(\text{mod}11)$

31, 53, 97.

Removing these primes from the set of all primes in interval [13, 128] we get 17, 29, 41, 59, 71, 101, 107, and the set of twin primes (17, 19), (29, 31), (41, 43), (59, 61), (71, 73), (101, 103), (107, 109)

Example 2. We put $l = 2$ (cousin primes).

We have to remove all primes which are congruent to $(p_i - 4)(modp_i)$, $i = 1, 2, 3, 4$. Because $p_1 - 4 = 3 - 4 = 2$, $p_2 - 4 = 5 - 4 = 1$, $p_3 - 4 = 7 - 4 = 3$, $p_4 - 4 = 11 - 4 = 7$, in modular arithmetic (modulo 3, 5, 7, 11 suitable), we have to remove all primes $x \equiv (2)(\text{mod}3)$, $x \equiv (1)(\text{mod}5)$, $x \equiv (3)(\text{mod}7)$, $x \equiv (7)(\text{mod}11)$.

primes$\equiv (2)(\text{mod}3)$

17, 23, 29, 41, 47, 53, 59, 71, 83, 89, 101, 107, 113,

primes$\equiv (1)(\text{mod}5)$

31, 41, 61, 71, 101,

primes$\equiv (3)(\text{mod}7)$

17, 31, 59, 73, 101,

primes$\equiv (7)(\text{mod}11)$

29, 73

After removing these primes we get 13, 19, 37, 43, 67, 79, 97, 103, 109, and the set of cousin primes (13, !7), (19, 23), (37, 41), (43, 47), (67, 71), (79, 83), (97, 101), (103, 107), (109, 113)

Example 3. Let $l = 3$ (sexy primes).

Proceeding similarly as in examples above we have to remove from the set of primes in interval [13, 113] primes congruent to $(p_i - 6)(\text{mod}p_i)$, $i = 1, 2, 3, 4$. Because $3 - 6 = 0$, $5 - 6 = 4$, $7 - 6 = 1$, $11 - 6 = 5$, in modular arithmetic (modulo 3, 5, 7, 11, suitable), we have to remove all primes $x \equiv (0)(\text{mod}3)$, $x \equiv (4)(\text{mod}5)$, $x \equiv (1)(\text{mod}7)$, $x \equiv (5)(\text{mod}11)$,

primes$\equiv (0)(\text{mod}3)$

none

primes$\equiv (4)(\text{mod}5)$

19, 29, 59, 79, 89, 109,

primes$\equiv (1)(\text{mod}7)$

29, 43, 71, 113,

primes$\equiv (5)(\text{mod}11)$

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After removing these primes we get 13, 17, 23, 31, 37, 41, 47, 53, 61, 67, 73, 83, 97, 101, 103, 107, and the set of sexy primes (13, 19), (17, 23), (23, 29), (31, 37), (37, 43), (41, 47), (47, 53) (53, 59), (61, 67), (67, 73), (73, 79), (83, 89), (97, 103), (101, 107), (103, 109), (107, 113).

6. Conclusion

The aim of this article is to present an algorithm for polignac numbers and to prove that there is an infinity of these numbers.

Conflict of Interest

The author declares no conflict of interest regarding the publication of this paper.

References

[1] Zhang, Y.T. (2014) Bonded Gaps between Primes. *Annals of Mathematics*, **179**, 1121-1174.

[2] Polymath (2014) http://michaelnielsen.org/polymath1/index.php?title=Bounded_gaps_between_primes

[3] https://en.wikipedia.org/wiki/Polignac_conjecture