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Erratum to "The Riemann Hypothesis-Millennium Prize Problem" [Advances in Pure Mathematics 6 (2016) 915-920]

A. A. Durmagambetov

L. N. Gumilyov Eurasian National University
Email: aset.durmagambet@gmail.com

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The original online version of this article (Durmagambetov, A.A. (2016) The Riemann Hypothesis-Millennium Prize Problem. Advances in Pure Mathematics, 6, 915-920. 10.4236/apm.2016.612069) unfortunately contains a mistake. The author wishes to correct the errors in Theorem 2 of the result part.

2. Results

These are the well-known Abel's results.

Theorem 1. Let the function $\phi(x)$ be limited on every finite interval, and $\frac{d\phi}{dx}(x)$ is continuous and limited on every finite interval then

$$\sum_{a<n\le b}\phi(n)=\int_a^b\phi(x)dx+\int_a^b\left(x-[x]-1/2\right)\frac{d\phi}{dx}dx+\left(a-[a]-1/2\right)\phi(a)-\left(b-[b]-1/2\right)\phi(b) \quad (1)$$

Corollary 1. Let the function $s>1$, $\phi(x)=x^{-s}$, $a,b\in N$ then

$$\sum_{a<n\le b}n^{-s}=\frac{b^{1-s}-a^{1-s}}{1-s}-s\int_a^b\frac{\left(x-[x]-1/2\right)}{x^{s+1}}dx+\frac{1}{2}\left(b^{-s}-a^{s}\right) \quad (2)$$

$$\sum_{1<n<\infty}n^{-s}=-\frac{1}{1-s}-s\int_1^\infty\frac{\left(x-[x]-1/2\right)}{x^{s+1}}dx+\frac{1}{2}\left(b^{-s}-a^{s}\right) \quad (3)$$

Our goal is to use this theorem on the analogs of zeta functions. We are interested in the analytical properties of the following generalizations of zeta functions:

$$P(s)=\sum_p\frac{1}{p^s},Q(s)=\sum_p\frac{1}{(p-1)^s} \quad (4)$$

$$P_m(s)=\sum_{p\le m}\frac{1}{p^s},Q_m(s)=\sum_{p\le m}\frac{1}{(p-1)^s} \quad (5)$$

$$P^m(s)=\sum_{p>m}\frac{1}{p^s},Q^m(s)=\sum_{p>m}\frac{1}{(p-1)^s} \quad (6)$$

$$\varsigma_p^m(s)=\varsigma(s)-P^m(s) \quad (7)$$

Let N be the set of all natural numbers and $N_p^m=(n\in N,n\geq m,n-\text{prime number})$ $NP_m=N/N_p^m$ —the set of all natural numbers without N_p^m

Below we will always let $m>3$, this limitation is introduced only to simplify the calculations. Considering all the information above let us rewrite

$$\varsigma_p^m(s)=\sum_{n\in NP_m}\frac{1}{n^s}.$$

For the function $\varsigma_p^m(s)=\varsigma(s)-P^m(s)$, let us apply the results obtained by Muntz for the zeta function representation. With the help of the given definitions we formulate the analog of Muntz theorem.

Lemma 1. Let the function

$$\delta(s)=P^m(s)-Q^m(s),\quad\text{then} \quad (8)$$

$$\delta(s)=-sP^m(s+1)+s^2O(P^m(s+2)). \quad (9)$$

PROOF: According to the theorem conditions we have

$$\begin{aligned}\delta(s)&=\sum_{p\in N_p^m}\left[\frac{1}{p^s}-\frac{1}{(p-1)^s}\right]=\sum_{p\in N_p^m}\frac{1}{p^s}\left[1-\frac{1}{(-1/p+1)^s}\right]\\&=-s\sum_{p\in N_p^m}\frac{1}{p^{s+1}}+s^2O(P^m(s+2)).\end{aligned} \quad (10)$$

Lemma 2. Let the function

$$\gamma1(s)=\sum_{p\in N_p^m}\int_{p-1}^{p}\frac{x}{x^{s+1}}\mathrm{d}x,\gamma2(s)=-\sum_{p\in N_p^m}\int_{p-1}^{p}\frac{[x]}{x^{s+1}}\mathrm{d}x,\gamma3(s)=-\sum_{p\in N_p^m}\int_{p-1}^{p}\frac{1/2}{x^{s+1}}\mathrm{d}x, \quad (11)$$

then

$$\gamma1(s)=\frac{1}{1-s}\sum_{p\in N_p^m}\left[\frac{1}{p^{s-1}}-\frac{1}{(p-1)^{s-1}}\right]=\frac{\delta(s-1)}{1-s} \quad (12)$$

$$\gamma2(s)=-\frac{1}{s}\sum_{p\in N_p^m}\left[\frac{p-1}{p^s}-\frac{p-1}{(p-1)^s}\right]=-\frac{\delta(s-1)}{s}+\frac{P^m(s)}{s} \quad (13)$$

$$\gamma3(s)=-\frac{1}{2s}\sum_{p\in N_p^m}\left[\frac{1}{p^s}-\frac{1}{(p-1)^s}\right]=-\frac{\delta(s)}{2s}. \quad (14)$$

$$s[\gamma1(s)+\gamma2(s)+\gamma3(s)]=s\left[\frac{\delta(s-1)}{s-1}-\frac{\delta(s-1)}{s}+\frac{P^m(s)}{s}-\frac{\delta(s)}{2s}\right] \quad (15)$$

PROOF: Follows from computing of integrals.

Lemma 3. Let the function

$$\phi(x)=x^{-s}, s>1,\quad a,b,m-\text{prime numbers}$$

$$(a,b)\bigcap N_p^m=\varnothing,\quad \{a,a\geq m\}=N_p^m \quad \text{then}$$

$$-\delta(s-1)-m^{1-s}=\sum_{a,b\in N_p^m}\left[(b-1)^{1-s}-a^{1-s}\right] \tag{16}$$

$$\sum_{a,b\in N_p^m} s\int_a^{b-1}\frac{(x-[x]-1/2)}{x^{s+1}}\mathrm{d}x = s\int_m^{\infty}\frac{(x-(x)-1/2)}{x^{s+1}}\mathrm{d}x - s\left[\gamma1(s)+\gamma2(s)+\gamma3(s)\right]; \tag{17}$$

PROOF: Computing the sums , we have

$$\sum_{a,b\in N_p^m}\left[(b-1)^{1-s}-a^{1-s}\right]=-m^{1-s}+\sum_{p\in N_p^m}\left[(p-1)^{1-s}-p^{1-s}\right]=-m^{1-s}-\delta(s-1) \tag{18}$$

Theorem 2. Let the function

$$\phi(x)=x^{-s}, s>1,\quad a,b,m-\text{prime numbers}$$

$$(a,b)\bigcap N_p^m=\varnothing,\quad \{a,a\geq m\}=N_p^m \quad \text{then}$$

$$sP^m(s)=\varsigma(s)-\left[-m^{1-s}-s\int_m^{\infty}\frac{(x-[x]-1/2)}{x^{s+1}}\mathrm{d}x-\delta(s)-m^{-s}+O\left(P^m(s+1)\right)\right] \tag{19}$$

PROOF: Using Corollary 1. we have

$$\begin{aligned}\varsigma_p^m(s)&=\sum_{a,b\in N_p^m}\sum_{a<n<b}n^{-s}\\&=\sum_{a,b\in N_p^m}\frac{(b-1)^{1-s}-a^{1-s}}{1-s}-\sum_{a,b\in N_p^m}s\int_a^{b-1}\frac{(x-[x]-1/2)}{x^{s+1}}\mathrm{d}x\\&\quad+\frac{1}{2}\sum_{a,b\in N_p^m}\left((b-1)^{-s}-a^{-s}\right)\end{aligned} \tag{20}$$

$$\begin{aligned}\varsigma_p^m(s)&=-\delta(s-1)-m^{1-s}-s\int_m^{\infty}\frac{(x-[x]-1/2)}{x^{s+1}}\mathrm{d}x\\&\quad+s\left[\gamma1(s)+\gamma2(s)+\gamma3(s)\right]-\delta(s)-m^{-s}\\&=-\delta(s-1)-m^{1-s}-s\int_m^{\infty}\frac{(x-[x]-1/2)}{x^{s+1}}\mathrm{d}x\\&\quad+s\left[\frac{\delta(s-1)}{s-1}-\frac{\delta(s-1)}{s}+\frac{P^m(s)}{s}-\frac{\delta(s)}{2s}\right]-\delta(s)-m^{-s}\end{aligned} \tag{21}$$

$$\varsigma_p^m(s)=\delta(s-1)\left[\frac{s-2}{1-s}\right]+P^m(s)-m^{1-s}-s\int_m^{\infty}\frac{(x-[x]-1/2)}{x^{s+1}}\mathrm{d}x-\delta(s)-m^{-s} \tag{22}$$

$$\begin{aligned}\varsigma(s)-P^m&=(s-2)P^m(s)+P^m(s)-m^{1-s}-s\int_m^{\infty}\frac{(x-[x]-1/2)}{x^{s+1}}\mathrm{d}x\\&\quad-\delta(s)-m^{-s}+O\left(P^m(s+1)\right).\end{aligned} \tag{23}$$

$$\varsigma(s)=sP^m(s)-m^{1-s}-s\int_m^{\infty}\frac{(x-[x]-1/2)}{x^{s+1}}dx-\delta(s)-m^{-s}+O\left(P^m(s+1)\right). \tag{24}$$

$$sP^{m}(s)=\varsigma(s)-\left[-m^{1-s}-s\int_{m}^{\infty}\frac{(x-[x]-1/2)}{x^{s+1}}\mathrm{d}x-\delta(s)-m^{-s}+O\left(P^{m}(s+1)\right)\right] \tag{25}$$

From the last equation we obtain the regularity of the function $\varsigma_{p}^{m}(s),P^{m}(s)$ as s satisfied $1/2<\mathrm{Re}(s)<1$.

Theorem 3. The Riemann's function has nontrivial zeros only on the line $\mathrm{Re}(s)=1/2$;

PROOF: For $R2(s)=\sum_{m=2}^{\infty}P(ms)/m$, we have

$$|R2(s)|=\left|\sum_{m=2}^{\infty}P(ms)/m\right|\le\sum_{m=2}^{\infty}|P(ms)/m|\le C_{\delta}\sum_{m=2}^{\infty}\left|-2^{m\delta}/m\right|<CC_{\delta}<\infty \tag{26}$$

Applying the formula from the theorem 2

$$\ln(\varsigma(s))=P(s)+\sum_{m=2}^{\infty}P(ms)/m=P(s)+R2(s)=\varsigma(s)-\varsigma_{p}^{m}(s)-P_{m}(s)+R2(s) \tag{27}$$

estimating by the module

$$|\ln(\varsigma(s)|\le|\varsigma(s)|+|\varsigma_{p}^{m}(s)|+|R2(s)|+|P_{m}(s)|. \tag{28}$$

Estimating the zeta function, potentiating, we obtain

$$|(\varsigma((s))|\ge\exp\left[-|\varsigma(s)|-|\varsigma_{p}^{m}(s)|-|R2(s)|-|P_{m}(s)|\right] \tag{29}$$

According to the theorem 1 $|\varsigma(s)|$ limited for z from the following multitude

$$(s,|s|<R,|s|>1+\delta,\delta>0) \tag{30}$$

similarly, applying the theorem 2 for $|\varsigma_{p}^{m}(s)|$ we obtain its limitation in the same multitude. For the function $|R2(s)|$ we have a limitation for all z, belonging to the half-plane Re(s) > 1/2 + 1/R. similarly, applying the theorem 2 for $|\varsigma_{p}^{m}(s)|$ we obtain its limitation in the same multitude and finally we obtain:

$$|(\varsigma(s))|\ge\exp[-C_{R}],\ \mathrm{Re}(s)>1/2+1/R,|s|<R,|s|>1+\delta,\delta>0 \tag{31}$$

These estimations for $|P(s)|,|R2(s)|,|P_{m}(s)|$ prove that zate function does not have zeros on the half-plane $\mathrm{Re}(s)>1/2+1/R$ due to the integral representation (3) these results are projected on the half-plane $\mathrm{Re}(s)<1/2$ for the case of nontrivial zeros. The Riemann's hypothesis is proved.

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