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New Generalized (G'/G)-Expansion Method Applications to Coupled Konno-Oono Equation

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Abstract

The new generalized (G'/G)-expansion method is one of the powerful and competent methods that appear in recent time for establishing exact solutions to nonlinear evolution equations (NLEEs). We apply the new generalized (G'/G)-expansion method to solve exact solutions of the new coupled Konno-Oono equation and construct exact solutions expressed in terms of hyperbolic functions, trigonometric functions, and rational functions with arbitrary parameters. The significance of obtained solutions gives credence to the explanation and understanding of related physical phenomena. As a newly developed mathematical tool, this method efficiency for finding exact solutions has been demonstrated through showing its straightforward nature and establishing its ability to handle nonlinearities prototyped by the NLEEs whether in applied mathematics, physics, or engineering contexts.

1. Introduction

Various physical, mechanical, chemical, biological, engineering and some economic laws and relations appear

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mathematically in the form of differential equations which are linear or nonlinear, homogeneous or inhomogeneous. Almost all differential equations relating physical phenomena are nonlinear. Methods of solutions of linear differential equations are reasonably easy and well avowed. In contrast, the techniques of solutions of nonlinear differential equations are less obtainable and in general, approximations are generally used. Nonlinearity is a fascinating element of nature, today; many scientists observe nonlinear science as the most important frontier for the fundamental understanding of nature. The analytical solutions of such equations are of fundamental importance to reveal the inner structure of the phenomena. The world around us is inherently nonlinear. For instance, nonlinear evolution equations (NEEs) are widely used as models to describe complex physical phenomena in various fields of sciences, especially in fluid mechanics, solid-state physics, plasma physics, plasma waves and biology, etc. One of the basic physical problems for those models is to obtain their travelling wave solutions. In particular, various methods have been utilized to explore different kinds of solutions of physical models described by nonlinear partial differential equations (NPDEs). In the numerical methods, stability and convergence should be considered so as to avoid divergent or inappropriate results. However, in recent years, a variety of effective analytical and semi-analytical methods have been developed to be used for solving NLEEs, such as the inverse scattering transform method [1], the $(G'/G, 1/G)$-expansion method [2] [3], the modified simple equation method [4] [5], the Sumudu transform method [6]-[8], the homogeneous balance method [9] [10], the Darboux transformation method [11], the Backlund transformation method [12], the complex hyperbolic function method [13] [14], the (G'/G)-expansion method [15]-[25], the improved (G'/G)-expansion method [26], the collocation method [27] [28], the similarity reductions method [29] [30], the homotopy analysis method [31] [32], the spectral-homotopy analysis method [33]-[35], the Hermite-Pade approximation method [36] and so on.

Naher and Abdullah [37] introduced a new approach of (G'/G)-expansion method and a new approach of generalized (G'/G)-expansion method for a reliable treatment of the nonlinear evolution equations. Afterwards, many researchers investigated many nonlinear PDEs to construct traveling wave solutions via this powerful (G'/G)-expansion method. For example, Alam and Akbar [38] [39] applied this method for finding traveling wave solutions of the KP-BBM equation, the (3 + 1)-dimensional potential-YTSF equation, the (2 + 1)-dimensional Zakharov-Kuznetsov equation. Alam *et al*. [40] [41] concerned about this method to construct traveling wave solutions of the strain wave equation in microstructured solids, the (3 + 1)-dimensional mKdV-ZK equation and the (1 + 1)-dimensional compound KdVB equations. The objective of this article is to look for new study relating to the new generalized (G'/G) expansion method for solving the new coupled Konno-Oono equuation to make the goodwill and helpfulness of the method obvious.

Our aim in this paper is to present an application of the new generalized (G'/G)-expansion method to the new coupled Konno-Oono equation to be solved by this method for the first time.

The rest of the paper is organized as follows: In Section 2, we give the description of the new generalized (G'/G)-expansion method. In Section 3, we apply this method to the new coupled Konno-Oono equation with discussion and Graphical representations of the solutions. Conclusions are given at last.

2. Description of the Method

Let us consider a general nonlinear PDE in the form

$$P\left(u, u_t, u_x, u_{tt}, u_{tx}, u_{xx}, \cdots\right) = 0,\qquad(1)$$

where $u = u(x, t)$ is an unknown function, P is a polynomial in $u(x, t)$ and its derivatives in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives.

Step 1: We combine the real variables x and t by a complex variable η

$$u(x, t) = u(\eta), \quad \eta = x \pm Vt,\qquad(2)$$

where V is the speed of the traveling wave. The traveling wave transformation (2) converts Equation (1) into an ordinary differential equation (ODE) for $u = u(\eta)$:

$$Q\left(u, u', u'', u''', \cdots\right) = 0,\qquad(3)$$

where Q is a polynomial of u and its derivatives and the superscripts indicate the ordinary derivatives with respect to η.

Step 2: According to possibility Equation (3) can be integrated term by term one or more times, yields constant(s) of integration. The integral constant may be zero, for simplicity.

Step 3: Suppose the traveling wave solution of Equation (3) can be expressed as follows:

$$u(\eta) = \sum_{i=0}^{N} \alpha_i (d+H)^i + \sum_{i=1}^{N} \beta_i (d+H)^{-i}, \tag{4}$$

where either α_N or β_N may be zero, but both α_N or β_N could be zero at a time, α_i $(i=0,1,2,\cdots,N)$ and β_i $(i=1,2,\cdots,N)$ and d are arbitrary constants to be determined later and $H(\eta)$ is

$$H(\eta) = (G'/G) \tag{5}$$

where $G = G(\eta)$ satisfies the following auxiliary ordinary differential equation:

$$AGG'' - BGG' - EG^2 - C(G')^2 = 0 \tag{6}$$

where the prime stands for derivative with respect to η; A, B, C and E are real parameters.

Step 4: To determine the positive integer N, taking the homogeneous balance between the highest order nonlinear terms and the derivatives of the highest order appearing in Equation (3).

Step 5: Substitute Equation (4) and Equation (6) including Equation (5) into Equation (3) with the value of N obtained in Step 4, we obtain polynomials in $(d+H)^N$ $(N=0,1,2,\cdots)$ and $(d+H)^{-N}$ $(N=0,1,2,\cdots)$. Then, we collect each coefficient of the resulted polynomials to zero, yields a set of algebraic equations for α_i $(i=0,1,2,\cdots,N)$ and β_i $(i=1,2,\cdots,N)$, d and V.

Step 6: Suppose that the value of the constants α_i $(i=0,1,2,\cdots,N)$, β_i $(i=1,2,\cdots,N)$, d and V can be found by solving the algebraic equations obtained in Step 5. Since the general solution of Equation (6) is well known to us, inserting the values of α_i $(i=0,1,2,\cdots,N)$, β_i $(i=1,2,\cdots,N)$, d and V into Equation (4), we obtain more general type and new exact traveling wave solutions of the nonlinear partial differential Equation (1).

Using the general solution of Equation (6), we have the following solutions of Equation (5):

Family 1: When $B \neq 0$, $\psi = A - C$ and $\Omega = B^2 + 4E(A-C) > 0$,

$$H(\eta) = \left(\frac{G'}{G}\right) = \frac{B}{2\psi} + \frac{\sqrt{\Omega}}{2\psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Omega}}{2A}\eta\right) + C_2 \cosh\left(\frac{\sqrt{\Omega}}{2A}\eta\right)}{C_1 \cosh\left(\frac{\sqrt{\Omega}}{2A}\eta\right) + C_2 \sinh\left(\frac{\sqrt{\Omega}}{2A}\eta\right)} \tag{7}$$

Family 2: When $B \neq 0$, $\psi = A - C$ and $\Omega = B^2 + 4E(A-C) < 0$,

$$H(\eta) = \left(\frac{G'}{G}\right) = \frac{B}{2\psi} + \frac{\sqrt{-\Omega}}{2\psi} \frac{-C_1 \sin\left(\frac{\sqrt{-\Omega}}{2A}\eta\right) + C_2 \cos\left(\frac{\sqrt{-\Omega}}{2A}\eta\right)}{C_1 \cos\left(\frac{\sqrt{-\Omega}}{2A}\eta\right) + C_2 \sin\left(\frac{\sqrt{-\Omega}}{2A}\eta\right)} \tag{8}$$

Family 3: When $B \neq 0$, $\psi = A - C$ and $\Omega = B^2 + 4E(A-C) = 0$,

$$H(\eta) = \left(\frac{G'}{G}\right) = \frac{B}{2\psi} + \frac{C_2}{C_1 + C_2\eta} \tag{9}$$

Family 4: When $B = 0$, $\psi = A - C$ and $\Delta = \psi E > 0$,

$$H(\eta) = \left(\frac{G'}{G}\right) = \frac{\sqrt{\Delta}}{\psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Delta}}{A}\eta\right) + C_2 \cosh\left(\frac{\sqrt{\Delta}}{A}\eta\right)}{C_1 \cosh\left(\frac{\sqrt{\Delta}}{A}\eta\right) + C_2 \sinh\left(\frac{\sqrt{\Delta}}{A}\eta\right)} \tag{10}$$

Family 5: When $B = 0$, $\psi = A - C$ and $\Delta = \psi E < 0$,

$$H(\eta) = \left(\frac{G'}{G}\right) = \frac{\sqrt{-\Delta}}{\psi} \frac{-C_1 \sin\left(\frac{\sqrt{-\Delta}}{A}\eta\right) + C_2 \cos\left(\frac{\sqrt{-\Delta}}{A}\eta\right)}{C_1 \cos\left(\frac{\sqrt{-\Delta}}{A}\eta\right) + C_2 \sin\left(\frac{\sqrt{-\Delta}}{A}\eta\right)} \tag{11}$$

3. The New Coupled Konno-Oono Equation

The new coupled Konno-Oono equation: In this section, we will put forth the new generalized (G'/G) expansion method to construct many new and more general traveling wave solutions of the new coupled Konno-Oono equation. Let us consider the new coupled Konno-Oono equation [42] [43],

$$u_{xt} - 2uv = 0, \quad v_t + 2uu_x = 0. \tag{12}$$

Now let us suppose that the traveling wave transformation equation be

$$u(\eta) = u(x,t), \quad v(\eta) = v(x,t), \quad \eta = x - Vt. \tag{13}$$

The Equation (13) reduces Equation (12) into the following ODEs

$$-Vu'' - 2uv = 0. \tag{14}$$

$$-Vv' + 2uu' = 0. \tag{15}$$

By integrating (15) with respect to η, we obtain

$$v = \frac{1}{V}(u^2 + P), \tag{16}$$

where P is a constant of integration.

Substituting Equation (16) into Equation (14), we get

$$V^2 u'' + 2uP + 2u^3 = 0. \tag{17}$$

Taking the homogeneous balance between highest order nonlinear term u^3 and linear term of the highest order u'' in Equation (17), we obtain $N = 1$. Therefore, the solution of Equation (17) is of the form:

$$v(\eta) = \alpha_0 + \alpha_1(d + M) + \beta_1(d + M)^{-1}, \tag{18}$$

where $\alpha_0, \alpha_1, \beta_1$ and d are constants to be determined.

Substituting Equation (18) together with Equations (5) and (6) into Equation (17), the left-hand side is converted into polynomials in $(d + H)^N$ $(N = 0, 1, 2, \cdots)$ and $(d + H)^{-N}$ $(N = 1, 2, \cdots)$. We collect each coefficient of these resulted polynomials to zero yields a set of simultaneous algebraic equations (for simplicity, the equations are not presented) for α_0, α_1, β_1, d, P and V. Solving these algebraic equations with the help of computer algebra, we obtain following:

$$P = -\frac{\alpha_1^2}{4\psi^2}(4E\psi + B^2), \quad V = -\frac{Ai\alpha_1}{\psi}, \quad d = d, \quad \alpha_0 = -\frac{\alpha_1}{2\psi}(B + 2d\psi), \quad \alpha_1 = \alpha_1, \quad \beta_1 = 0. \tag{19}$$

where $\psi = A - C$, d, A, B, C, E are free parameters.

Substituting Equation (19) into Equation (18), along with Equation (7) and simplifying, our traveling wave solutions become, if $C_1 = 0$ but $C_2 \neq 0$; $C_2 = 0$ but $C_1 \neq 0$ respectively:

$$u_{1_1}(\eta) = \frac{\alpha_1}{2\psi}\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2A}\eta\right),$$

and

$$v_{1_1}(\eta) = -\frac{\alpha_1}{4iA\psi}\left[\Omega\coth^2\left(\frac{\sqrt{\Omega}}{2A}\eta\right) - (4E\psi + B^2)\right],$$

$$u_{1_2}(\eta) = \frac{\alpha_1}{2\psi}\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2A}\eta\right),$$

and

$$v_{1_2}(\eta) = -\frac{\alpha_1}{4iA\psi}\left[\Omega\tanh^2\left(\frac{\sqrt{\Omega}}{2A}\eta\right) - \left(4E\psi + B^2\right)\right],$$

Substituting Equation (19) into Equation (18), along with Equation (8) and simplifying yields exact solutions, if $C_1 = 0$ but $C_2 \neq 0$; $C_2 = 0$ but $C_1 \neq 0$ respectively:

$$u_{1_3}(\eta) = \frac{i\alpha_1}{2\psi}\sqrt{\Omega}\cot\left(\frac{\sqrt{-\Omega}}{2A}\eta\right),$$

and

$$v_{1_3}(\eta) = \frac{\alpha_1}{4iA\psi}\left[\Omega\cot^2\left(\frac{\sqrt{-\Omega}}{2A}\eta\right) + \left(4E\psi + B^2\right)\right],$$

$$u_{1_4}(\eta) = -\frac{i\alpha_1}{2\psi}\sqrt{\Omega}\tan\left(\frac{\sqrt{-\Omega}}{2A}\eta\right),$$

and

$$v_{1_4}(\eta) = \frac{\alpha_1}{4iA\psi}\left[\Omega\tan^2\left(\frac{\sqrt{-\Omega}}{2A}\eta\right) + \left(4E\psi + B^2\right)\right],$$

Substituting Equation (19) into Equation (18), along with Equation (9) and simplifying, our obtained solution becomes:

$$u_{1_5}(\eta) = \alpha_1\left(\frac{C_2}{C_1 + C_2\eta}\right),$$

and

$$v_{1_5}(\eta) = -\frac{\alpha_1}{4iA\psi}\left[\left(\frac{2\psi C_2}{C_1 + C_2\eta}\right)^2 - \left(4E\psi + B^2\right)\right],$$

Substituting Equation (19) into Equation (18), together with Equation (10) and simplifying, yields following traveling wave solutions, if $C_1 = 0$ but $C_2 \neq 0$; $C_2 = 0$ but $C_1 \neq 0$ respectively:

$$u_{1_6}(\eta) = -\frac{\alpha_1}{2\psi}\left(B - 2\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{A}\eta\right)\right).$$

and

$$v_{1_6}(\eta) = -\frac{\alpha_1}{4iA\psi}\left[\left(B - 2\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{A}\eta\right)\right)^2 - \left(4E\psi + B^2\right)\right]$$

$$u_{1_7}(\eta) = -\frac{\alpha_1}{2\psi}\left(B - 2\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{A}\eta\right)\right).$$

and

$$v_{1_7}(\eta) = -\frac{\alpha_1}{4iA\psi}\left[\left(B - 2\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{A}\eta\right)\right)^2 - \left(4E\psi + B^2\right)\right]$$

Substituting Equation (19) into Equation (18), along with Equation (11) and simplifying, our exact solutions become, if $C_1 = 0$ but $C_2 \neq 0$; $C_2 = 0$ but $C_1 \neq 0$ respectively:

$$u_{1_8}(\eta) = -\frac{\alpha_1}{2\psi}\left(B - 2i\sqrt{\Delta}\cot\left(\frac{\sqrt{-\Delta}}{A}\eta\right)\right)$$

and

$$v_{1_8}(\eta) = -\frac{\alpha_1}{4iA\psi}\left[\left(B - 2i\sqrt{\Delta}\cot\left(\frac{\sqrt{-\Delta}}{A}\eta\right)\right)^2 - \left(4E\psi + B^2\right)\right]$$

$$u_{1_9}(\eta) = -\frac{\alpha_1}{2\psi}\left(B + 2i\sqrt{\Delta}\tan\left(\frac{\sqrt{-\Delta}}{A}\eta\right)\right),$$

and

$$v_{1_9}(\eta) = -\frac{\alpha_1}{4iA\psi}\left[\left(B + 2i\sqrt{\Delta}\tan\left(\frac{\sqrt{-\Delta}}{A}\eta\right)\right)^2 - \left(4E\psi + B^2\right)\right].$$

where $\eta = x - \left(\frac{Aia_1}{\psi}\right)t$.

Khan and Akbar [43] investigated solutions of the the new coupled Konno-Oono equation by the modified simple equation method and obtained only eight solutions (A1)-(A8) (see appendix). Moreover, in this article eighteen solutions of the new coupled Konno-Oono equation are constructed by applying the new approach of generalized (G'/G)-expansion method. But by means of the new approach of generalized (G'/G)-expansion method we obtained solutions are different to Khan and Akbar [43] solutions. Furthermore, we obtain solutions $u_{1_1}(\eta) - u_{1_9}(\eta)$, $v_{1_1}(\eta) - v_{1_9}(\eta)$. These solutions are new and were not obtained by Khan and Akbar [43]. On the other hand, the auxiliary equation used in this paper is different, so obtained solutions is also different.

4. Graphical Representations of the Solutions

The graphical illustrations of the solutions are depicted in the **Figures 1-6** with the aid of commercial software Maple.
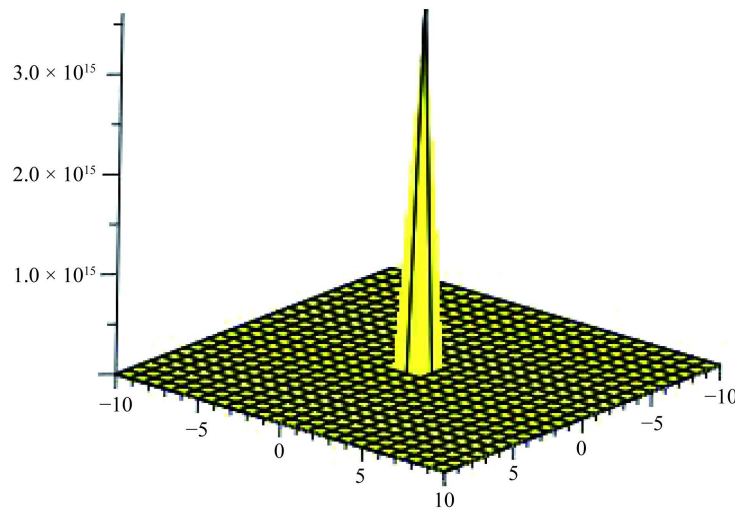

Figure 1. Single soliton of $u_{1_1}(\eta)$ when $A = 4$, $B = 1$, $C = 1$, $E = 1$, $d = 1$, $\alpha_1 = 1$ and $-10 \leq x, t \leq 10$.
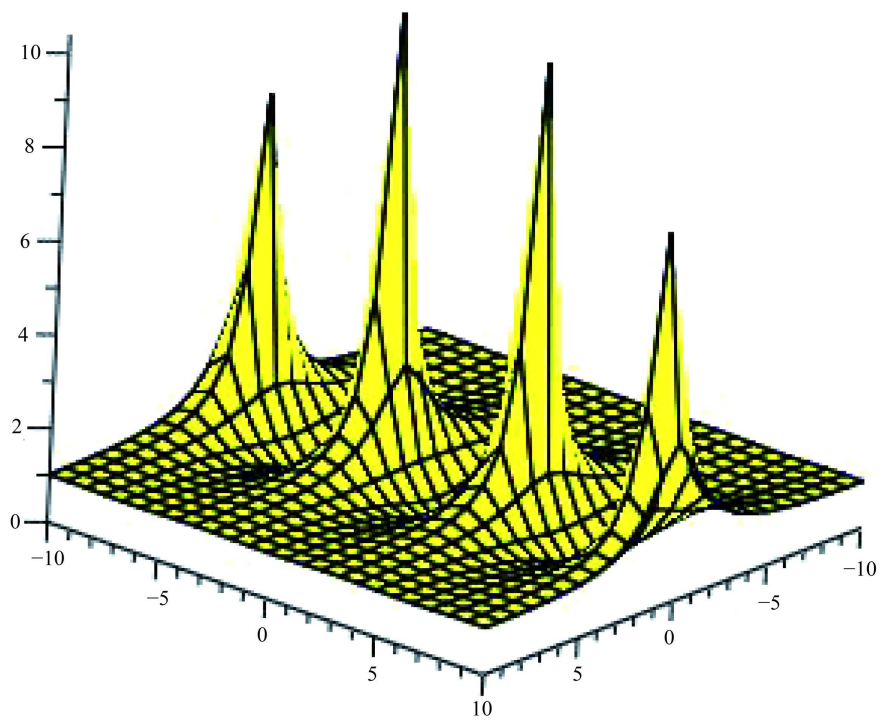
Figure 2. Multiple soliton of $u_{1_7}(\eta)$ when $A = 2$, $B = 0$, $C = 1$, $E = 1$, $d = 1$, $\alpha_1 = 1$ and $-10 \leq x,t \leq 10$.
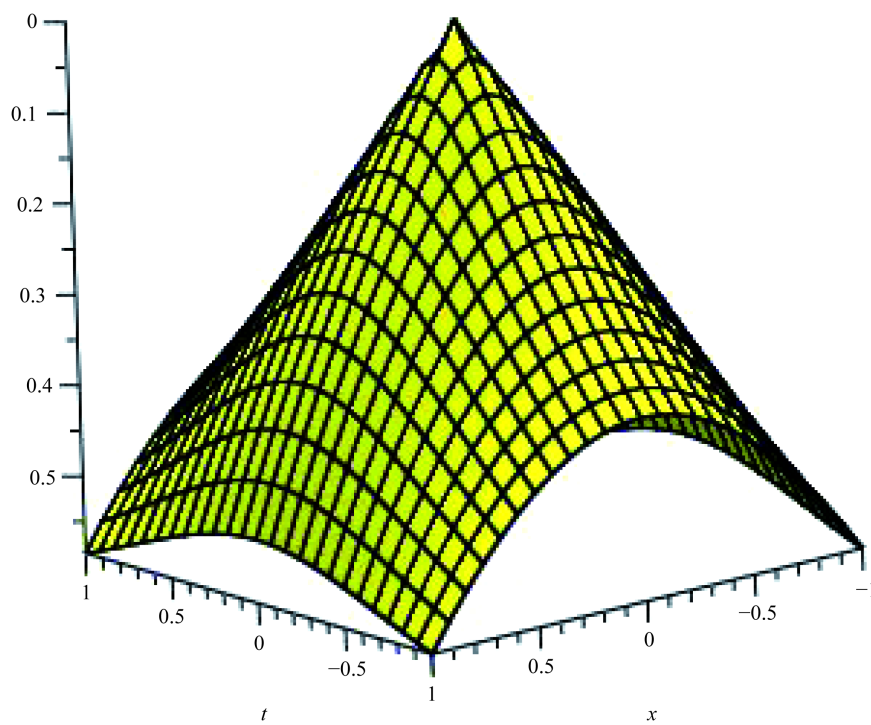

Figure 3. Compacton of $u_{1_4}(\eta)$ when $A = 2$, $B = 1$, $C = 4$, $E = 1$, $d = 1$, $\alpha_1 = 1$ and $-1 \leq x,t \leq 1$.
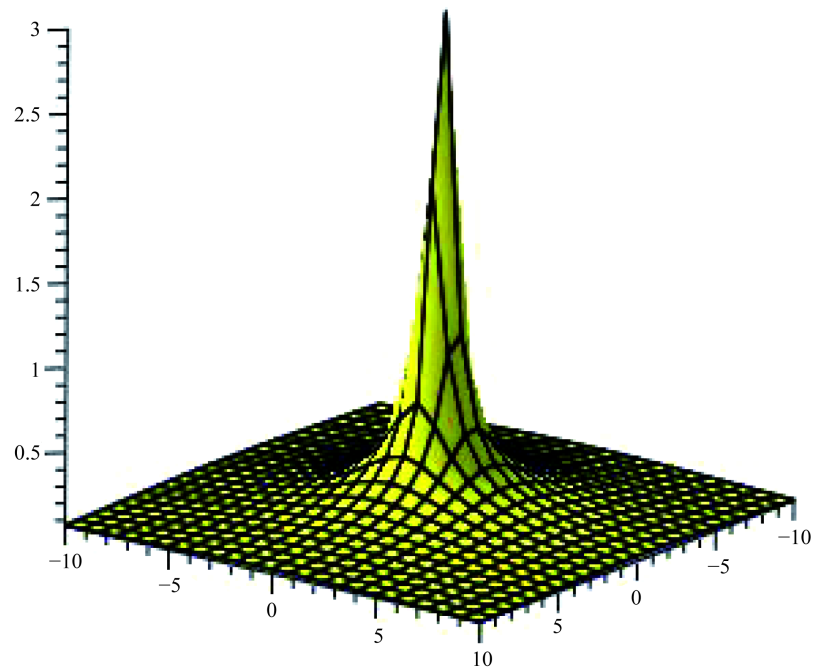
Figure 4. Singular soliton of $u_{1_5}(\eta)$ when $C_1 = 1$, $C_2 = 2$, $A = 1$, $B = 2$, $C = 2$, $E = 1$, $d = 1$, $\alpha_1 = 1$ and $-10 \leq x, t \leq 10$.
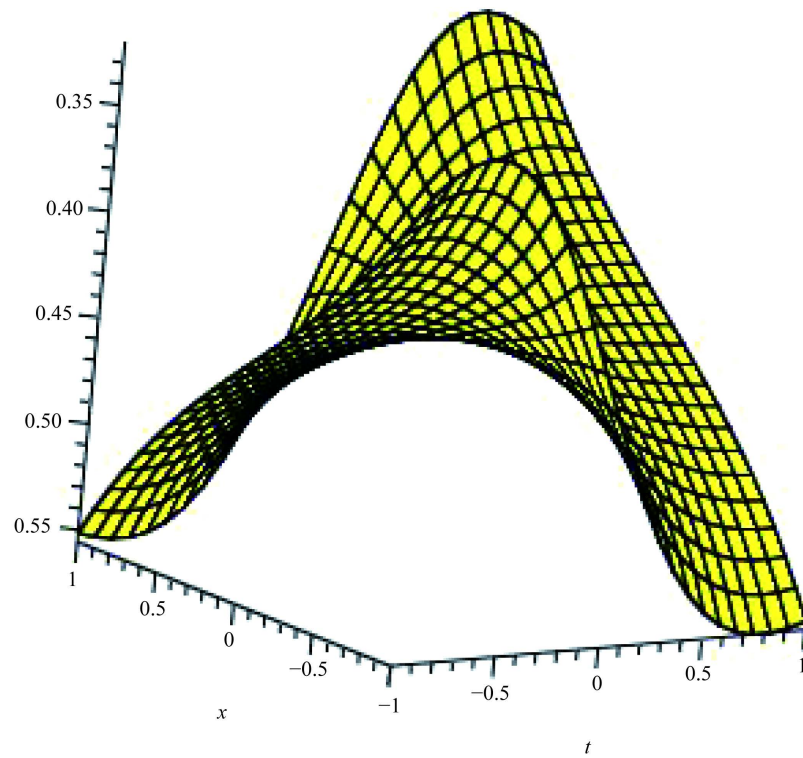
Figure 5. Singular periodic solution of $v_{1_4}(\eta)$ when $\alpha_1 = 1$, $d = 1$, $A = 2$, $B = 1$, $C = 4$, $E = 1$ and $-1 \leq x, t \leq 1$.
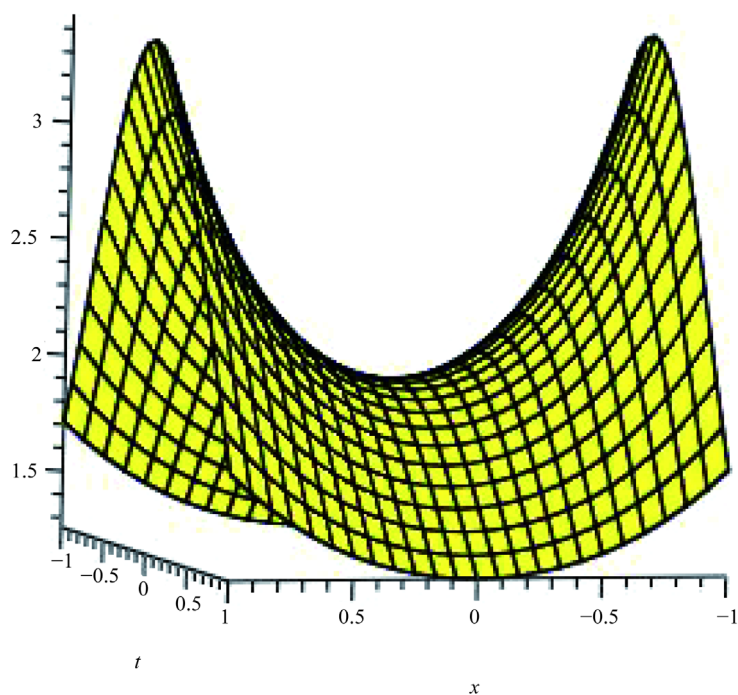
Figure 6. Periodic solutions of $v_{1_9}(\eta)$ when $\alpha_1 = 1$, $d = 1$, $A = 1$, $B = 0$, $C = 2$, $E = 2$ and $-1 \leq x, t \leq 1$.

5. Conclusion

The new generalized (G'/G)-expansion method presented in this paper has been successfully implemented to construct many new and more general exact solutions of the new coupled Konno-Oono equation. The method offers solutions with free parameters that might be important to explain some complex physical phenomena. Comparing the currently proposed method with other methods, such as (G'/G)-expansion method, the Exp-function method and the modified simple equation method, we might conclude that the exact solutions to Equation (12) can be investigated using these methods with the help of the symbolic computation software such as Matlab, Mathematica and Maple to facilitate the complicated algebraic computations. This study shows that the new generalized (G'/G)-expansion method is quite efficient and practically well suited to be used in finding exact solutions of NLEEs. Also, we observe that the new generalized (G'/G)-expansion method is straightforward and can be applied to many other nonlinear evolution equations.

Acknowledgements

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Appendix: Khan and Akbar's Solutions [43]

We bring to the reader's attention that Equation (16) regarding $v(x,t)$ above is the same as Equation (10) in Khan and Akbar [43], where the authors manged established exact solutions of the new coupled Konno-Oono equation by using the modified simple equation method which are as follows (see Equation (23) for $u(x,t)$, in [43]):

$$u(\xi) = \pm I\sqrt{d} \times \left(1 + 2A\cosh\left(\frac{\sqrt{d}}{\omega}(x - \omega t)\right)\right) - \sinh\left(\frac{\sqrt{d}}{\omega}(x - \omega t)\right)$$
$$\times \left((LB - A)\cosh\left(\frac{\sqrt{d}}{\omega}(x - \omega t)\right)\right) + (LB + A)\sinh\left(\frac{\sqrt{d}}{\omega}(x - \omega t)\right)^{-1} \tag{A.0}$$

We can freely choose the constants A and B. Therefore, setting $A = LB$, Equation (A.0) reduces to

$$u_{1,2}(x,t) = \pm I\sqrt{d}\coth\left(\frac{\sqrt{d}}{\omega}(x - \omega t)\right), \qquad \text{for } d > 0. \tag{A.1}$$

$$v_1(x,t) = -\frac{d}{\omega}\operatorname{cosech}^2\left(\frac{\sqrt{d}}{\omega}(x - \omega t)\right), \qquad \text{for } d > 0. \tag{A.2}$$

Again, Setting $A = -LB$, Equation (A.0) reduces to

$$u_{3,4}(x,t) = \pm I\sqrt{d}\tanh\left(\frac{\sqrt{d}}{\omega}(x - \omega t)\right), \qquad \text{for } d > 0. \tag{A.3}$$

$$v_2(x,t) = \frac{d}{\omega}\operatorname{sech}^2\left(\frac{\sqrt{d}}{\omega}(x - \omega t)\right), \qquad \text{for } d > 0. \tag{A.4}$$

If $d < 0$, using hyperbolic function identities, from (A.0), we get the following periodic traveling wave solutions:

$$u_{5,6}(x,t) = \pm\sqrt{d}\cot\left(\frac{\sqrt{-d}}{\omega}(x - \omega t)\right), \tag{A.5}$$

$$v_3(x,t) = \frac{d}{\omega}\operatorname{cosec}^2\left(\frac{\sqrt{-d}}{\omega}(x - \omega t)\right), \tag{A.6}$$

$$u_{7,8}(x,t) = \pm\sqrt{d}\tan\left(\frac{\sqrt{-d}}{\omega}(x - \omega t)\right), \tag{A.7}$$

$$v_4(x,t) = \frac{d}{\omega}\sec^2\left(\frac{\sqrt{-d}}{\omega}(x - \omega t)\right), \tag{A.8}$$