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Classification of Single Traveling Wave Solutions to the Generalized Kadomtsev-Petviashvili Equation without Dissipation Terms in *p* = 2

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ABSTRACT

By using the complete discrimination system for the polynomial method, the classification of single traveling wave solutions to the generalized Kadomtsev-Petviashvili equation without dissipation terms in $p=2$ is obtained.

Keywords: Complete Discrimination System for Polynomial; Traveling Wave Solution; Generalized Kadomtsev-Petviashvili Equation without Dissipation Terms

1. Introduction

In mathematics and physics, the Kadomtsev-Petviashvili (KP) equation is a partial differential equation to describe nonlinear wave motion. It can be used to model water waves of long wavelength with weakly nonlinear restoring forces and frequency dispersion [1]. A number of modified forms of the KP equation have been studied [2-6]. In [1,7], the generalized Kadomtsev-Petviashvili equation without dissipation terms was given by

$$\frac{\partial}{\partial x}\left(u_t + du^p u_x + bu^{2p} u_x + \delta u_{xxx}\right) + 3k^2 u_{yy} = 0, \quad (1)$$

where d,b,δ are constants, $\delta \neq 0$, $p>0$. Some of modified form of the KP equation can be written in the form of Equation (1).

Many reliable methods are used in the literature to examine the completely integrable nonlinear evolution equations. The Hirota bilinear method, the Bäcklund transformation method, the inverse scattering method, the Painlevé analysis, the simplified Hirotas method established by Hereman *et al.* [8], and others were effectively used in [1-13]. Liu proposed a complete discrimination system for polynomial method [10-13]. That is, by using of elementary integral method and complete discrimination system for polynomial, the single wave solutions can be classified for some nonlinear differential equations which can be directly reduced to integral forms.

In this paper, we consider the following generalized Kadomtsev-Petviashvili equation without dissipation terms in $p=2$:

$$\frac{\partial}{\partial x}\left(u_t + du^2 u_x + bu^4 u_x + \delta u_{xxx}\right) + 3k^2 u_{yy} = 0, \quad (2)$$

where d,b,δ are constants, $\delta \neq 0$. By using Liu's complete discrimination system for polynomial method, the classification of single traveling wave solutions to Equation (2) is obtained.

2. Classification of Solutions to Equation (2)

Take wave transformation

$$u(x,y,t) = u(\xi) \quad \text{and} \quad \xi = lx + my - \omega t$$

into Equation (1), the following nonlinear ordinary difference equation is given:

$$l\left(-\omega u' + dlu^2 u' + blu^4 u' + l^3 \delta u'''\right)' + 3k^2 m^2 u'' = 0, \quad (3)$$

Integrating Equation (3) once with respect to ξ, and setting the integral constant to zero yields:

$$l\left(-\omega u' + dlu^2 u' + blu^4 u' + l^3 \delta u'''\right) + 3k^2 m^2 u' = 0. \quad (4)$$

Integrating Equation (4) twice yields

$$(u')^2 = -\frac{b}{15l^2\delta}u^6 - \frac{d}{6l^2\delta}u^4 - \frac{3k^2m^2-\omega l}{l^4\delta}u^2 + a_1u + a_0, \tag{5}$$

where a_1, a_0 are arbitrary constants.

***Case* 2.1.** $a_1 = a_0 = 0$, we substitute the transformation

$$u(\xi) = v^{\frac{1}{2}}(\xi)$$

into Equation (5) yields

$$\int \frac{\mathrm{d}v}{v\sqrt{F(v)}} = \pm 2(\xi - \xi_0), \tag{6}$$

where

$$F(v) = -\frac{b}{15l^2\delta}v^2 - \frac{d}{6l^2\delta}v - \frac{3k^2m^2-\omega l}{l^4\delta} \tag{7}$$

Let

$$\Delta = \left(-\frac{d}{6l^2\delta}\right)^2 - \frac{4b\left(3k^2m^2-\omega l\right)}{15l^6\delta^2},$$

and Δ is the discriminant of the polynomial $F(v)$. According to the classification of the roots of $F(v)$, there are three cases to be discussed.

***Case* 2.1.1.** $\Delta = 0$, when $\frac{b}{\delta} < 0$,

from Equation (6), we have

$$v = \frac{5d}{b\left(1 \mp \mathrm{e}^{\pm\frac{5d}{2b}\sqrt{-\frac{b}{15l^2\delta}}(\xi-\xi_0)}\right)}, \tag{8}$$

***Case* 2.1.2.** $\Delta > 0$, when

$$\frac{b}{\delta} < 0,$$

from Equation (6), we have

$$\pm 2\sqrt{-\frac{b}{15l^2\delta}}(\xi-\xi_0) = \frac{1}{\sqrt{\beta\gamma}}\ln\frac{\left[\sqrt{(-\gamma)(v-\beta)} - \sqrt{(-\beta)(v-\gamma)}\right]^2}{|v|}, \tag{9}$$

$$\pm 2\sqrt{-\frac{b}{15l^2\delta}}(\xi-\xi_0) = \frac{1}{\sqrt{\beta\gamma}}\ln\frac{\left[\sqrt{(\gamma)(v-\beta)} - \sqrt{(\beta)(v-\gamma)}\right]^2}{|v|}, \tag{10}$$

$$\pm 2\sqrt{-\frac{b}{15l^2\delta}}(\xi-\xi_0) = \frac{1}{\sqrt{-\beta\gamma}}\arcsin\frac{(-\gamma)(v-\beta) + (-\beta)(v-\gamma)}{|v||\beta-\gamma|}. \tag{11}$$

When

$$\frac{b}{\delta} > 0,$$

from Equation (6), we have

$$\pm 2\sqrt{\frac{b}{15l^2\delta}}(\xi-\xi_0) = \frac{1}{\sqrt{-\beta\gamma}}\ln\frac{\left[\sqrt{(-\gamma)(v+\beta)} - \sqrt{(\beta)(v-\gamma)}\right]^2}{|v|}, \tag{12}$$

$$\pm 2\sqrt{\frac{b}{15l^2\delta}}(\xi-\xi_0) = \frac{1}{\sqrt{-\beta\gamma}}\ln\frac{\left[\sqrt{(-\gamma)(-v+\beta)} - \sqrt{(-\beta)(v-\gamma)}\right]^2}{|v|}, \tag{13}$$

$$\pm 2\sqrt{\frac{b}{15l^2\delta}}(\xi-\xi_0) = \frac{1}{\sqrt{\beta\gamma}}\arcsin\frac{(-\gamma)(v+\beta) + \beta(v-\gamma)}{|v||\beta-\gamma|}. \tag{14}$$

where

$$\beta = \frac{5d + 30l^2\delta\sqrt{\Delta}}{-b}, \quad \gamma = \frac{5d - 30l^2\delta\sqrt{\Delta}}{-b}.$$

***Case* 2.1.3.** $\Delta < 0$. From Equation (6), we have

$$\mp\frac{6k^2m^2-2\omega l}{l^4\delta}\left(\xi-\xi_0\right)=\ln\left|\frac{\frac{3l^2\delta}{d\sqrt{-\frac{3k^2m^2-\omega l}{l^4\delta}}}v+\sqrt{\frac{3k^2m^2-\omega l}{l^4\delta}}-\sqrt{-\frac{b}{15l^2\delta}v^2-\frac{d}{6l^2\delta}v-\frac{3k^2m^2-\omega l}{l^4\delta}}}{v}\right|, \quad (15)$$

where

$$\frac{3k^2m^2-\omega l}{l^4\delta}<0.$$

***Case* 2.2.** $a_1=0, a_0\neq 0$. Substituting the transformation

$$u(\xi)=w^{\frac{1}{2}}(\xi)$$

into Equation (5) yields

$$\pm 2\sqrt{-\frac{\epsilon b}{15l^2\delta}}\left(\xi-\xi_0\right)=\int\frac{\mathrm{d}w}{\sqrt{\epsilon wF(w)}}, \quad (16)$$

where

$$F(w)=w^3+\frac{d}{2b}w^2+\frac{15\left(3k^2m^2-\omega l\right)}{bl^2}w-\frac{15l^2\delta a_0}{b} \quad (17)$$

If

$$\frac{b}{\delta}>0,$$

we take

$$\epsilon=1;\text{ if }\ \frac{b}{\delta}<0,$$

we take $\epsilon=-1$. The complete discrimination system for the third order polynomial $F(w)$ is given as follows:

$$\Delta=\frac{4860l^4\delta a_0b^3-3d^2l^2+4d^2l^2b-\left(24300l^2b\delta a_0+720b^2\right)\left(3k^2m^2-\omega l\right)}{12b^3l^2},$$

$$D_1=\frac{180bK^2m^2-60b\omega l-d^2l^2}{4b^2l^2}, \quad (18)$$

According to the classification of the roots of $F(w)$, there are four cases to be discussed.

***Case* 2.2.1.** $\Delta=0, D_1<0$. Then

$$F(w)=(w-\alpha)^2(w-\beta),$$

where α,β are real constants, $\alpha\neq\beta$, and $\beta>0$. If $\epsilon=1$, when $\alpha>\beta$ and $w>\beta$, or when $\alpha<0$ and $w<0$, from Equation (16), we have

$$\pm 2\sqrt{-\frac{b}{15l^2\delta}}\alpha(\alpha-\beta)\left(\xi-\xi_0\right)=\ln\frac{\left[\sqrt{\alpha(w-\beta)}-\sqrt{w(\alpha-\beta)}\right]^2}{|w-\alpha|}, \quad (19)$$

when $\alpha>\beta$, and $w<0$, or $\alpha<w<\beta$, we have

$$\pm 2\sqrt{-\frac{b}{15l^2\delta}}\alpha(\alpha-\beta)\left(\xi-\xi_0\right)=\ln\frac{\left[\sqrt{-\alpha(w-\alpha)}-\sqrt{w(\beta-\alpha)}\right]^2}{|w-\alpha|}, \quad (20)$$

when $\beta>\alpha>0$, we have

$$\pm 2\sqrt{-\frac{b}{15l^2\delta}}\alpha(\beta-\alpha)\left(\xi-\xi_0\right)=\arcsin\frac{\alpha(w-\beta)+w(\alpha-\beta)}{|\beta(w-\alpha)|}. \quad (21)$$

If $\epsilon=-1$, when $\alpha>\beta$ and $w>\beta$, or when $\alpha<0$ and $w<0$, from Equation (16), we have

$$\pm 2\sqrt{-\frac{b}{15l^2\delta}}\alpha(\beta-\alpha)\left(\xi-\xi_0\right)=\ln\frac{\left[\sqrt{\alpha(-w+\beta)}-\sqrt{w(\beta-\alpha)}\right]^2}{|w-\beta|}, \quad (22)$$

when $\alpha>\beta$, and $w<0$, or $\alpha<0$, and $w<\beta$, we have

$$\pm 2\sqrt{-\frac{b}{15l^2\delta}}\alpha(\beta-\alpha)(\xi-\xi_0)=\ln\frac{\left[\sqrt{-\alpha(-w+\beta)}-\sqrt{w(\alpha-\beta)}\right]^2}{|w-\beta|}, \tag{23}$$

when $\beta>\alpha>0$, we have

$$\pm 2\sqrt{-\frac{b}{15l^2\delta}}\alpha(\alpha-\beta)(\xi-\xi_0)=\arcsin\frac{\alpha(-w+\beta)+w(\beta-\alpha)}{|\beta(w-\alpha)|}. \tag{24}$$

***Case* 2.2.2.** $\Delta=0, D_1=0$. Then

$$F(w)=(w-\alpha)^3,$$

where α is a real constant. If $\epsilon=1$, when $w>\alpha$, and $w>0$, or $w<\alpha$, and $w<0$, we have

$$w=\frac{15\alpha l^2\delta}{-b\alpha^2(\xi-\xi_0)^2-15l^2\delta}+\alpha, \tag{25}$$

If $\epsilon=-1$, when $w>\alpha$, and $w<0$, or $w<\alpha$, and $w>0$, we have

$$w=\frac{15\alpha l^2\delta}{b\alpha^2(\xi-\xi_0)^2-15l^2\delta}+\alpha, \tag{26}$$

***Case* 2.2.3.** $\Delta>0, D_1<0$. Then

$$F(w)=(w-\alpha)(w-\beta)(w-\gamma),$$

where α,β,γ are different real constants. If $\epsilon=1$, when $w>0$, or $w<\gamma$, we have

$$w=\frac{-\gamma\alpha\,\mathrm{sn}^2\left(\sqrt{\frac{b}{15l^2\delta}\beta(\alpha-\beta)}(\xi-\xi_0),n\right)}{-\gamma\,\mathrm{sn}^2\left(\sqrt{\frac{b}{15l^2\delta}\beta(\alpha-\gamma)}(\xi-\xi_0),n\right)-(\alpha-\gamma)} \tag{27}$$

$$w=\frac{-\gamma(\alpha-\beta)\mathrm{sn}^2\left(\sqrt{\frac{b}{15l^2\delta}\beta(\alpha-\beta)}(\xi-\xi_0),n\right)-\beta(\alpha-\gamma)}{-\gamma\,\mathrm{sn}^2\left(\sqrt{\frac{b}{15l^2\delta}\beta(\alpha-\gamma)}(\xi-\xi_0),n\right)-(\alpha-\gamma)} \tag{28}$$

where

$$n^2=\frac{\gamma(\alpha-\beta)}{\beta(\alpha-\gamma)}.$$

If $\epsilon=-1$, when $0>w>\alpha$, and $\gamma<w<\beta$, we have

$$w=\frac{-\beta\alpha\,\mathrm{sn}^2\left(\sqrt{\frac{b}{15l^2\delta}\beta(\alpha-\beta)}(\xi-\xi_0),n\right)+\alpha\beta}{-\alpha\,\mathrm{sn}^2\left(\sqrt{\frac{b}{15l^2\delta}\beta(\alpha-\gamma)}(\xi-\xi_0),n\right)+\beta} \tag{29}$$

$$w=\frac{-\gamma\beta}{(\beta-\gamma)\mathrm{sn}^2\left(\sqrt{\frac{b}{15l^2\delta}\beta(\alpha-\gamma)}(\xi-\xi_0),n\right)-\beta} \tag{30}$$

where

$$n^2=\frac{\alpha(\beta-\gamma)}{\beta(\alpha-\gamma)}.$$

***Case* 2.2.4.** $\Delta<0$,

$$F(w)=(w-\alpha)\left[(w-\beta)^2+\gamma^2\right],$$

where α,β,γ are all real constants, and $\alpha>0,\beta>0$, and $\gamma>0$. we have

$$w=\frac{a\,\mathrm{cn}\left(\frac{\sqrt{\pm\frac{2b\gamma m_2\alpha}{15l^2\delta}}}{m_1m_2}(\xi-\xi_0),m_1\right)+b_1}{c\,\mathrm{cn}\left(\frac{\sqrt{\pm\frac{2b\gamma m_2\alpha}{15l^2\delta}}}{m_1m_2}(\xi-\xi_0),m_1\right)+d} \tag{31}$$

where

$$a=\frac{1}{2}\alpha(c-d),b_1=\frac{1}{2}\alpha(d-c),c=\alpha-\beta-\frac{\gamma}{m_2},$$

$$d=-\beta-\gamma m_2,$$

$$E=\frac{\gamma^2-\beta(\alpha-\beta)}{\gamma\alpha}, \quad .$$

$$m_2=E\pm\sqrt{E^2+1},m_1^2=\frac{1}{1+m_2^2}$$

***Case* 2.3.** $a_1\neq 0, a_0=0$. The Equation (5) becomes

$$\pm(\xi-\xi_0)=\int\frac{\mathrm{d}u}{\sqrt{-\frac{b}{15l^2\delta}uF(u)}}, \tag{32}$$

where $F(u)=u^5+qu^3+ru+s$, and

$$q=\frac{d}{2b},\quad r=\frac{15(3k^2m^2-\omega l)}{bl^2},\quad s=-\frac{15l^2\delta a_1}{b}.$$

The complete discrimination system for the fifth order polynomial $F(u)$ is given as follows:

$$D_2=-q, D_3=40rq-12q^3,$$

$$D_4=12q^4r-88r^2q^2+125qs^2+160r^3,$$

$$D_5=2000qs^2r^2-900rs^2q^3+16q^4r^3$$
$$+108q^5s^2-128r^4q^2+256r^5+3125s^4,$$
$$E_2=160r^2q^3-48rq^5+625s^2q^2,$$

$$F_2=-8rq. \tag{33}$$

According to the classification of the roots of $F(u)$, there are seven cases to be discussed.

Case **2.3.1.** $D_5=0, D_4=0, D_3>0, E_2\neq 0$, then

$$F(u)=(u-\alpha)^2(u-\beta)^2(u-\gamma),$$

$\alpha,\beta,$ and γ are real numbers, $\alpha\neq\beta\neq\gamma\neq 0.$ From Equation (32), we have

$$\pm(\alpha-\beta)(\xi-\xi_0)=\frac{1}{\sqrt{-\frac{b}{15l^2\delta}\alpha(\alpha-\gamma)}}\ln\frac{\left(\sqrt{-\frac{\varepsilon_1 b}{15l^2\delta}u(\alpha-\gamma)}-\sqrt{-\frac{\varepsilon_1 b}{15l^2\delta}\alpha(u-\gamma)}\right)^2}{|u-\alpha|}$$
$$-\frac{1}{\sqrt{-\frac{b}{15l^2\delta}\beta(\beta-\gamma)}}\arcsin\frac{-\frac{b}{15l^2\delta}u(\beta-\gamma)-\frac{b}{15l^2\delta}\beta(u-\gamma)}{\left|-\frac{b}{15l^2\delta}\gamma(u-\beta)\right|}, \tag{34}$$

$$\pm(\alpha-\beta)(\xi-\xi_0)=\frac{1}{\sqrt{-\frac{b}{15l^2\delta}\alpha(\alpha-\gamma)}}\ln\frac{\left(\sqrt{-\frac{\varepsilon_1 b}{15l^2\delta}u(\alpha-\gamma)}-\sqrt{-\frac{\varepsilon_1 b}{15l^2\delta}\alpha(u-\gamma)}\right)^2}{|u-\alpha|}$$
$$-\frac{1}{\sqrt{-\frac{b}{15l^2\delta}\beta(\beta-\gamma)}}\ln\frac{\left(\sqrt{-\frac{\varepsilon_2 b}{15l^2\delta}u(\beta-\gamma)}-\sqrt{-\frac{\varepsilon_2 b}{15l^2\delta}\beta(u-\gamma)}\right)^2}{|u-\alpha|}, \tag{35}$$

$$\pm(\alpha-\beta)(\xi-\xi_0)=\frac{1}{\sqrt{-\frac{b}{15l^2\delta}\alpha(\alpha-\gamma)}}\arcsin\frac{-\frac{b}{15l^2\delta}u(\alpha-\gamma)-\frac{b}{15l^2\delta}\alpha(u-\gamma)}{\left|-\frac{b}{15l^2\delta}\gamma(u-\alpha)\right|}$$
$$-\frac{1}{\sqrt{-\frac{b}{15l^2\delta}\beta(\beta-\gamma)}}\ln\frac{\left(\sqrt{-\frac{\varepsilon_2 b}{15l^2\delta}u(\beta-\gamma)}-\sqrt{-\frac{\varepsilon_2 b}{15l^2\delta}\beta(u-\gamma)}\right)^2}{|u-\beta|}, \tag{36}$$

$$\pm(\alpha-\beta)(\xi-\xi_0)=\frac{1}{\sqrt{-\frac{b}{15l^2\delta}\alpha(\alpha-\gamma)}}\arcsin\frac{-\frac{b}{15l^2\delta}u(\alpha-\gamma)-\frac{b}{15l^2\delta}\alpha(u-\gamma)}{\left|-\frac{b}{15l^2\delta}\gamma(u-\alpha)\right|}$$
$$-\frac{1}{\sqrt{-\frac{b}{15l^2\delta}\beta(\beta-\gamma)}}\arcsin\frac{-\frac{b}{15l^2\delta}u(\beta-\gamma)-\frac{b}{15l^2\delta}\beta(u-\gamma)}{\left|-\frac{b}{15l^2\delta}\gamma(u-\beta)\right|}, \tag{37}$$

where $\varepsilon_1=\pm1,\varepsilon_2=\pm1$.

Case **2.3.2.**

$$D_5=0,D_4=0,D_3=0,D_2\neq0,F_2\neq0,$$

$$F(u)=(u-\alpha)^3(u-\beta)^2,$$

α and β are real numbers, $\alpha\neq\beta$. From Equation (32), we have

$$\pm(\alpha-\beta)(\xi-\xi_0)=\frac{2}{\frac{b}{15l^2\delta}\alpha}\sqrt{\frac{-\frac{b}{15l^2\delta}u}{u-\alpha}}-\frac{1}{\sqrt{-\frac{b}{15l^2\delta}\beta(\beta-\gamma)}}\arcsin\frac{-\frac{b}{15l^2\delta}u(\beta-\alpha)-\frac{b}{15l^2\delta}\beta(u-\alpha)}{\left|-\frac{b}{15l^2\delta}\alpha(u-\beta)\right|},\quad(38)$$

$$\pm(\alpha-\beta)(\xi-\xi_0)=\frac{2}{\frac{b}{15l^2\delta}\alpha}\sqrt{\frac{-\frac{b}{15l^2\delta}u}{u-\alpha}}-\frac{1}{\sqrt{-\frac{b}{15l^2\delta}\beta(\beta-\alpha)}}\ln\frac{\left(\sqrt{-\frac{\varepsilon_2 b}{15l^2\delta}u(\beta-\alpha)}-\sqrt{-\frac{\varepsilon_2 b}{15l^2\delta}\beta(u-\alpha)}\right)^2}{|u-\beta|},\quad(39)$$

where $\varepsilon_2=\pm1$.

Case **2.3.3.**

$$D_5=0,D_4=0,D_3=0,D_2\neq0,F_2=0,$$

$$F(u)=(u-\alpha)^4(u-\beta),$$

α,β are real numbers, $\alpha\neq\beta\neq0$. From Equation (32), we have

$$\mp\frac{b}{15l^2\delta}(\alpha-\beta)(\xi-\xi_0)=-\frac{\sqrt{-\frac{b}{15l^2\delta}u(u-\beta)}}{u-\alpha}+\frac{\frac{b}{15l^2\delta}\left(\alpha-\frac{1}{2}\beta\right)}{\sqrt{-\frac{b}{15l^2\delta}\alpha(\alpha-\beta)}}\ln\frac{\left(\sqrt{-\frac{\varepsilon_2 b}{15l^2\delta}u(\alpha-\beta)}-\sqrt{-\frac{\varepsilon_2 b}{15l^2\delta}\alpha(u-\beta)}\right)^2}{|u-\alpha|},\quad(40)$$

$$\mp\frac{b}{15l^2\delta}(\alpha-\beta)(\xi-\xi_0)=-\frac{\sqrt{-\frac{b}{15l^2\delta}u(u-\beta)}}{u-\alpha}+\frac{\frac{b}{15l^2\delta}\left(\alpha-\frac{1}{2}\beta\right)}{\sqrt{-\frac{b}{15l^2\delta}\alpha(\alpha-\beta)}}\arcsin\frac{-\frac{b}{15l^2\delta}u(\alpha-\beta)-\frac{b}{15l^2\delta}\alpha(u-\beta)}{\left|-\frac{b}{15l^2\delta}\beta(u-\alpha)\right|},\quad(41)$$

where $\varepsilon_2=\pm1$.

Case **2.3.4.**

$$D_5=0,D_4=0,D_3<0,E_2\neq0,$$

$$F(u)=\left((u-\beta)^2+\gamma^2\right)^2(u-\alpha),\quad s>0.$$

Respectively, from Equation (32), we have

$$\pm(\xi-\xi_0)=\frac{\beta_1}{\alpha\rho^2}\left(\arctan\frac{\gamma}{u-\beta}+\arctan\frac{\gamma_2(u-\beta)+\delta_2}{\gamma_1(u-\beta)+\delta_1-\sqrt{-\frac{b}{15l^2\delta}u(u-\alpha)}}\right)+\frac{\beta_2}{2\alpha\rho^2}\ln\frac{(u-\beta)^2+\gamma^2}{\left(\gamma_1(u-\beta)+\delta_1\sqrt{-\frac{b}{15l^2\delta}u(u-\alpha)}\right)^2+\left(\gamma_2(u-\beta)+\delta_2\right)^2},\quad(42)$$

where

$$\rho^2=\sqrt{\frac{b^2\left(\gamma^2(2\beta-\alpha)^2+\left(\beta(\beta-\alpha)-\gamma^2\right)\right)^2}{225l^4\delta^2}},\ \beta_1=\pm\sqrt{\frac{\rho^2}{2}-\frac{b\left(\beta(\beta-\alpha)-\gamma^2\right)}{30l^2\delta}},$$
$$\beta_2=\pm\sqrt{\frac{\rho^2}{2}+\frac{b\left(\beta(\beta-\alpha)-\gamma^2\right)}{30l^2\delta}},\ \gamma_1=\frac{-b\gamma\beta_2+b\beta_1\left(\beta-\frac{\gamma}{2}\right)}{15\beta^2\rho^2\delta},\ \gamma_2=\frac{-b\gamma\beta_2-b\beta_1\left(\beta-\frac{\gamma}{2}\right)}{15\beta^2\rho^2\delta}, \tag{43}$$
$$\delta_1=\frac{-b\beta(\beta-\alpha)\beta_1-b\left(\beta-\frac{\gamma}{2}\right)\gamma\beta_2}{15l^2\delta\rho^2},\ \delta_2=\frac{b\beta(\beta-\alpha)\beta_2-b\left(\beta-\frac{\gamma}{2}\right)\gamma\beta_1}{15l^2\delta\rho^2},$$

where the signs of β_1 and β_2 must satisfy

$$\beta_1\beta_2=\gamma-\frac{b}{15l^2\delta}\left[\beta-\frac{\alpha}{2}\right].$$

***Case* 2.3.5.** $D_5=0, D_4>0$,

$F(u)=(u-\alpha)^2(u-\alpha_1)(u-\alpha_2)(u-\alpha_3), \alpha_1>\alpha>\alpha_3$.

Respectively, from Equation (32), we have

$$\pm(\xi-\xi_0)=\frac{2\delta_1}{(a-\alpha c)\gamma_1}\left[cF(\psi,k_1)+\frac{\delta_1}{b_1-\alpha d}\Pi\left(\psi,\frac{a-\alpha c}{b_1-\alpha d},k_1\right)\right], \tag{44}$$

where we renew to queue the orders of $\alpha_1,\alpha_2,\alpha_3$, and α_4, denote $\alpha_1>\alpha_2>\alpha_3>\alpha_4$.When

$$\frac{b}{\delta}>0, u\geq\alpha_1 \text{ or } u\leq\alpha_4$$

(other cases can be written similarly, they are omitted), the meaning of every parameter in Equation (44) are given as follows:

$$a=\alpha_2(\alpha_1-\alpha_4), b_1=-\alpha_1(\alpha_2-\alpha_4), c=\alpha_1-\alpha_4, d=\alpha_2-\alpha_4, \delta_1=(\alpha_1-\alpha_2)(\alpha_1-\alpha_4)(\alpha_2-\alpha_4),$$
$$\frac{\delta_1}{\gamma_1}=\frac{1}{\sqrt{\left|\frac{b}{\delta}(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)\right|}}, k_1^2=\frac{(\alpha_1-\alpha_4)(\alpha_2-\alpha_3)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}, F(\varphi,k_1)=\int_0^\varphi\frac{d\varphi}{\sqrt{1-k_1^2\sin^2\varphi}}, \tag{45}$$
$$\Pi(\varphi,n,k_1)=\int_0^\varphi\frac{d\varphi}{\left(1+n\sin^2\varphi\right)\sqrt{1-k_1^2\sin^2\varphi}}$$

***Case* 2.3.6.**

$D_5=0, D_4=0, D_3>0,$

$E_2=0, F(u)=(u-\alpha)^3(u-\beta)(u-\rho).$

where we renew to queue the orders of $\alpha,\beta,\rho,$ and 0, and denote $\alpha_1>\alpha_2>\alpha_3>\alpha_4$. When

$$\frac{b}{\delta}>0, u\geq\alpha_1, \text{ or } u\leq\alpha_4$$

(other cases can be written similarly, they are omitted), we have

$$\pm(\xi-\xi_0)=\frac{-2c}{k_1^2\gamma}\left[\left(c+k_1^2d\right)F(\varphi,k_1)-cE(\varphi,k_1)\right], \alpha_1=\alpha \tag{46}$$

$$\pm(\xi-\xi_0)=\frac{2d}{\gamma}\left[(c+d)F(\varphi,k_1)-dE(\varphi,k_1)-d\coth\left(\phi\sqrt{1-k_1^2\sin^2\phi}\right)\right], \alpha_2=\alpha. \tag{47}$$

The signs are the same as the ones in Equation (45), furthermore,

$$E(\varphi,k_1)=\int_0^{\varphi_i}\sqrt{1-k_1^2\sin^2\psi}\,d\psi. \tag{48}$$

***Case* 2.3.7.**

$D_5=0, D_4=0, D_3<0,$

$E_2=0, F(u)=(u-\alpha)^3\left((u-\beta)^2+\gamma^2\right)$

Now we renew to queue the orders of α and 0, and denote $\alpha_1 > \alpha_2$, we have

$$\pm(\xi-\xi_0) = \frac{cd}{\gamma}\left[F(\varphi,k_1)+\frac{c^2}{k_1\gamma}\arcsin(k_1\sin\varphi)\right],\alpha_1=\alpha. \tag{49}$$

$$\pm(\xi-\xi_0)=\frac{-d^2}{\gamma\sqrt{1-k_1^2}}\ln\frac{\sqrt{1-k_1^2\sin^2\varphi}+\sqrt{1-k_1^2}\sin\varphi}{\cos\varphi} -\frac{cd}{\gamma}F(\varphi,k_1),\alpha_2=\alpha. \tag{50}$$

where

$$c=\alpha_1-\beta-\frac{\gamma}{m_1},m_1=E\pm\sqrt{E^2+1}, \quad d=\alpha_1-\beta-\gamma m_1,E=\frac{\gamma^2+(\alpha_1-\beta)(\alpha_2-\beta)}{\gamma(\alpha_1-\alpha_2)}, \tag{51}$$

where the positive sign and negative sign for m_1 must satisfy

$$\frac{bm_1}{\delta}<0,$$

other signs are the same with the former.

From the description above, using elementary integral method and complete discrimination system for polynomial, we have obtained the solutions of Equations (6), (16) and (32) that can be expressed by elementary functions and elliptic functions. What's more, some solutions are explicit, but some solutions are implicit functions. So we can write concretely the exact traveling wave solutions of Equation (5) in some special cases. They are omitted for simplicity.

3. Conclusion

Using the complete discrimination system for polynomial method, we have obtained the classification of single traveling wave solutions to the generalized Kadomtsev-Petviashvili equation without dissipation terms in $p=2$. With the same method, some of other evolution equations can be dealt with.

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