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On Lorentzian α-Sasakian Manifolds

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ABSTRACT

The object of the present paper is to study Lorentzian α-Sasakian manifolds satisfying certain conditions on the W_2-curvature tensor.

Keywords: Lorentzian α-Sasakian Manifold; W_2-Curvature Tensor; Einstein Manifold

1. Introduction

In 1970, Pokhariyal and Mishra [1] have introduced new curvature tensor called W_2-curvature tensor in a Riemannian manifold and studied their properties. Further, Pokhariyal [2] has studied some properties of this curvature tensor in a Sasakian manifold. Matsumoto, Ianus and Mihai [3], Ahmet Yildiz and U. C. De [4] and Venkatesha, C. S. Bagewadi, and K. T. Pradeep Kumar [5], have studied W_2-curvature tensor in P-Sasakian, Kenmotsu and Lorentzian para-Sasakian manifolds respectively.

In [6], S. Tanno classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold, the sectional curvature of a plane sections containing is a constant, say c. He showed that they can be divided into three classes:

1) Homogeneous normal contact Riemannian manifolds with $c > 0$;

2) Global Riemannian products of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature if $c = 0$ and;

3) A warped product space $\mathbb{R}\times_f\mathbb{C}$ if $c > 0$. It is known that the manifolds of class (1) are characterized by admitting a Sasakian structure.

In the Gray-Hervella classification of almost Hermitian manifolds [7], there appears a class, W_4, of Hermitian manifolds which are closely related to locally conformal Kaehler manifolds [8]. An almost contact metric structure on a manifold M is called a trans-Sasakian structure [9] if the product manifold $M\times\mathbb{R}$ belongs to the class W_4. The class $C_6\oplus C_5$ ([10,11]) coincides with the class of the trans-Sasakian structures of type (α, β). In fact, in [11], local nature of the two subclasses, namely, C_5 and C_6 structures of trans-Sasakian structures are characterized completely.

We note that trans-Sasakian structures of type (0, 0), $(0, \beta)$ and $(\alpha, 0)$ are cosymplectic [12], β-Kenmotsu [13] and α-Sasakian [13] respectively. In [14] it is proved that trans-Sasakian structures are generalized quasi-Sasakian. Thus, trans-Sasakian structures also provide a large class of generalized quasi-Sasakian structures.

An almost contact metric structure (ϕ,ξ,η,g) on M is called a trans-Sasakian structure [9] if $(M\times\mathbb{R},J,G)$ belongs to the class W_4 [7], where J is the almost complex structure on $M\times\mathbb{R}$ defined by

$$J\left(X, fd/\mathrm{d}t\right)=\left(\phi X-f,\eta(X)d/\mathrm{d}t\right),$$

for all vector fields X on M and smooth functions f on $M\times\mathbb{R}$, and G is the product metric on $M\times\mathbb{R}$. This may be expressed by the condition [15]

$$(\nabla_X\phi)Y=\alpha\left(g(X,Y)-\eta(Y)X\right)+\beta\left(g(\phi X,Y)-\eta(Y)\phi X\right),$$

for some smooth functions α and β on M, and we say that the trans-Sasakian structure is of type (α, β).

A trans-Sasakian structure of type (α, β) is α-Sasakian if $\beta = 0$ and α a nonzero constant [16]. If $\alpha = 1$, then α-Sasakian manifold is a Sasakian manifold.

2. Preliminaries

A differentiable manifold of dimension n is called Lorentzian α-Sasakian manifold if it admits a (1, 1)-tensor field ϕ, a contravariant vector field ξ, a covariant vector field η and Lorentzian metric g satisfy ([17-19])

$$\eta(\xi)=-1, \tag{2.1}$$

$$\phi^2=I+\eta\otimes\xi, \tag{2.2}$$

$$g(\phi X,\phi Y)=g(X,Y)+\eta(X)\eta(Y), \tag{2.3}$$

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$$g(X,\xi)=\eta(X), \tag{2.4}$$

$$\phi\xi=0, \eta(\phi X)=0, \tag{2.5}$$

for all $X,Y\in TM$.

Also a Lorentzian α-Sasakian manifold M is satisfying [18]

$$\text{(a) } \nabla_X=-\alpha\phi X, \text{(b) } (\nabla_X\eta)(Y)=-\alpha g(\phi X,Y), \tag{2.6}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g.

Further, on Lorentzian α-Sasakian manifold M the following relations hold:

$$\eta(R(X,Y)Z)=\alpha^2(g(Y,Z)\eta(X)-g(X,Z)\eta(Y)), \tag{2.7}$$

$$R(\xi,X)Y=\alpha^2(g(X,Y)\xi-\eta(Y)X), \tag{2.8}$$

$$R(X,Y)\xi=\alpha^2(\eta(Y)X-\eta(X)Y), \tag{2.9}$$

$$R(\xi,X)\xi=\alpha^2(\eta(X)\xi+X), \tag{2.10}$$

$$S(X,\xi)=(n-1)\alpha^2\eta(X), \tag{2.11}$$

$$S(\phi X,\phi Y)=S(X,Y)+(n-1)\alpha^2\eta(X)\eta(Y), \tag{2.12}$$

for all vector fields X, Y, Z where S is the Ricci tensor and Q is the Ricci operator given by $S(X,Y)=g(QX,Y)$.

An Lorentzian α-Sasakian manifold M is said to be Einstein if its Ricci tensor S is of the form

$$S(X,Y)=ag(X,Y), \tag{2.13}$$

for any vector fields X and Y, where α is a function on M.

In [1], Pokhariyal and Mishra have defined the curvature tensor W_2, given by

$$W_2(X,Y,U,V)=R(X,Y,U,V)+\frac{1}{n-1}[g(X,U)S(Y,V)-g(Y,U)S(X,V)], \tag{2.14}$$

where S is a Ricci tensor of type (0, 2).

Consider in an Lorentzian α-Sasakian manifold satisfying $W_2=0$ in (2.14), then we have

$$R(X,Y,U,V)=1/(n-1)([g(Y,U)S(X,V)-g(X,U)S(Y,V)]. \tag{2.15}$$

Putting $X=U=$ in (2.15) then using (2.8) and (2.11), we obtain

$$S(Y,V)=\alpha^2(n-1)g(Y,V). \tag{2.16}$$

Thus M is an Einstein manifold.

Theorem 2.1. If on a Lorentzian α-Sasakian manifold M, the condition $W_2=0$ holds, then M is an Einstein manifold.

Definition 2.1. An Lorentzian α-Sasakian manifold is called W_2-semisymmetric if it satisfies

$$R(X,Y)\cdot W_2=0, \tag{2.17}$$

where $R(X,Y)$ is to be considered as a derivation of the tensor algebra at each point of the manifold for tangent vectors X and Y.

In an Lorentzian α-Sasakian manifold the W_2-curvature tensor satisfies the condition

$$\eta(W_2(X,Y)Z)=0. \tag{2.18}$$

3. Lorentzian α-Sasakian Manifolds Satisfying $\tilde{P}(X,Y)\cdot W_2=0$

The pseudo projective curvature tensor $\tilde{P}$ is defined as [20]

$$\begin{aligned}\tilde{P}(X,Y)Z &=aR(X,Y)Z\\&+b[S(Y,Z)X-S(X,Z)Y]\\&-\frac{r}{n}\left[\frac{a}{n-1}+b\right][g(Y,Z)X-g(X,Z)Y].\end{aligned}$$

Using (2.8) and (2.11), Equation (3.1) reduces to

$$\begin{aligned}\tilde{P}(\xi,Y)Z=&h[g(Y,Z)-\eta(Z)Y]\\&+b[S(Y,Z)\xi-\alpha^2(n-1)\eta(Z)Y].\end{aligned}$$

where $h=\left(a\alpha^2-\frac{r}{n}\left[\frac{a}{n-1}+b\right]\right)$.

Now consider in a Lorentzian α-Sasakian manifold

$$\tilde{P}(X,Y)\cdot W_2=0.$$

This condition implies that

$$\begin{aligned}&\tilde{P}(X,Y)W_2(U,V)Z-W_2(\tilde{P}(X,Y)U,V)Z\\&-W_2(U,\tilde{P}(X,Y)V)Z-W_2(U,V)\tilde{P}(X,Y)Z=0.\end{aligned} \tag{3.3}$$

Put $X=\xi$ in (3.3) and then taking the inner product with ξ, we obtain

$$\begin{aligned}&g(\tilde{P}(\xi,Y)W_2(U,V)Z,\xi)\\&-g(W_2(\tilde{P}(\xi,Y)U,V)Z,\xi)\\&-g(W_2(U,\tilde{P}(\xi,Y)V)Z,\xi)\\&-g(W_2(U,V)\tilde{P}(\xi,Y)Z,\xi)=0.\end{aligned} \tag{3.4}$$

Using (3.2) in (3.4), we obtain

$$\begin{aligned}&h\Big[-g\big(Y,W_2(U,V)Z\big)-g(Y,U)\eta\big(W_2(\xi,V)Z\big)\\&-g(Y,V)\eta\big(W_2(U,\xi)Z\big)-g(Y,Z)\eta\big(W_2(U,V)\xi\big)\\&-\eta(Y)\eta\big(W_2(U,V)Z\big)+\eta(U)\eta\big(W_2(Y,V)Z\big)\\&+\eta(V)\eta\big(W_2(U,V)Z\big)+\eta(Z)\eta\big(W_2(U,V)Y\big)\Big]\\&-b\Big[S\big(Y,W_2(U,V)Z\big)+S(Y,U)\eta\big(W_2(\xi,V)Z\big)\\&+S(Y,V)\eta\big(W_2(U,\xi)Z\big)+S(Y,Z)\eta\big(W_2(U,V)\xi\big)\\&+\alpha^2(n-1)\eta(Y)\eta\big(W_2(U,V)Z\big)\\&+\alpha^2(n-1)\eta(U)\eta\big(W_2((Y,V)Z\big)\\&+\alpha^2(n-1)\eta(V)\eta\big(W_2(U,V)Z\big)\\&+\alpha^2(n-1)\eta(Z)\eta\big(W_2(U,V)Y\big)\Big]=0.\end{aligned}\tag{3.5}$$

By using (2.18) in (3.5), we get

$$h\Big[g\big(Y,W_2(U,V)Z\big)\Big]+b\Big[S\big(Y,W_2(U,V)Z\big)\Big]=0.\tag{3.6}$$

Taking $U=Z=\xi$ in (3.6) and using (2.14) and (2.10), we have

$$\begin{aligned}&\frac{b}{n-1}S(QY,V)-\left(b\alpha^2-\frac{h}{n-1}\right)S(Y,V)\\&-h\alpha^2 g(V,Y)=0.\end{aligned}\tag{3.7}$$

If $b = 0$, we get

$$h\left\{\frac{1}{n-1}S(Y,V)-\alpha^2 g(Y,V)\right\}=0.$$

Then, either $h = 0$ (or)

$$S(Y,V)=\alpha^2(n-1)g(Y,V).$$

If $b \neq 0$, then we get

$$\begin{aligned}S(QY,V)=&\left(\alpha^2(n-1)-\frac{h}{b}\right)S(Y,V)\\&+\frac{h}{b}\alpha^2(n-1)g(V,Y).\end{aligned}\tag{3.8}$$

Thus, we can state the following:

Theorem 3.2. If M is an Lorentzian α-Sasakian manifold satisfying the condition $\tilde{P}(X,Y)\cdot W_2=0$ Then:

- If $b = 0$, then either $h = 0$ on M, or M is an Einstein manifold;
- If $b \neq 0$, then the Equation (3.8) holds on M.

4. Lorentzian α-Sasakian Manifold Satisfying $\tilde{Z}(X,Y)\cdot W_2=0$

The concircular curvature tensor Z is defined as [21]

$$\begin{aligned}\tilde{Z}(X,Y)Z=&R(X,Y)Z\\&-\frac{r}{n(n-1)}\big[g(Y,Z)X-g(X,Z)Y\big].\end{aligned}\tag{4.1}$$

Using (2.8) and (2.11), Equation (4.1) reduces to

$$\tilde{Z}(\xi,Y)Z=\left[\alpha^2-\frac{r}{n(n-1)}\right]\big[g(Y,Z)\xi-\eta(Z)Y\big].\tag{4.2}$$

Now consider in a Lorentzian α-Sasakian manifold

$$\tilde{Z}(X,Y)\cdot W_2=0.$$

This condition implies that

$$\begin{aligned}&\tilde{Z}(X,Y)W_2(U,V)Z-W_2\big(\tilde{Z}(X,Y)U,V\big)Z\\&-W_2\big(U,\tilde{Z}(X,Y)V\big)Z-W_2(U,V)\tilde{Z}(X,Y)Z=0.\end{aligned}\tag{4.3}$$

Put $X=\xi$ in (4.3) and then taking the inner product with ξ, we obtain

$$\begin{aligned}&g\big(\tilde{Z}(\xi,Y)W_2(U,V)Z,\xi\big)\\&-g\big(W_2\big(\tilde{Z}(\xi,Y)U,V\big)Z,\xi\big)\\&-g\big(W_2\big(U,\tilde{Z}(\xi,Y)V\big)Z,\xi\big)\\&-g\big(W_2(U,V)\tilde{Z}(\xi,Y)Z,\xi\big)=0.\end{aligned}\tag{4.4}$$

Using (4.2) in (4.4), we obtain

$$\begin{aligned}&\left[\alpha^2-\frac{r}{n(n-1)}\right]\Big[-g\big(Y,W_2(U,V)Z\big)\\&-g(Y,U)\eta\big(W_2(\xi,V)Z\big)-g(Y,V)\eta\big(W_2(U,\xi)Z\big)\\&-g(Y,Z)\eta\big(W_2(U,V)\xi\big)-\eta(Y)\eta\big(W_2(U,V)Z\big)\\&+\eta(U)\eta\big(W_2(V,Y)Z\big)+\eta(V)\eta\big(W_2(U,Y)Z\big)\Big]\\&+\eta(Z)\eta\big(W_2(U,V)Y\big)\Big]=0.\end{aligned}\tag{4.5}$$

By using (2.18) in (4.5), we get

$$\left[\alpha^2-\frac{r}{n(n-1)}\right]\Big[g\big(Y,W_2(U,V)Z\big)\Big]=0.\tag{4.6}$$

Again from (4.2) we have $\alpha^2-\frac{r}{n(n-1)}\neq 0.$ And so

$$W_2(U,V,Z,Y)=0.\tag{4.7}$$

In view of (2.14) and (4.7), it follows that

$$\begin{aligned}&R(U,V,Z,Y)\\&=\frac{1}{n-1}\big[g(V,Z)S(U,Y)-g(U,Z)S(V,Y)\big].\end{aligned}\tag{4.8}$$

Contracting (4.8), we have

$$S(V,Z)=(n-1)g(V,Z).\tag{4.9}$$

Therefore M is an Einstein manifold.

Theorem 4.3. If on a Lorentzian α-Sasakian manifold M, the condition $\tilde{Z}(X,Y)\cdot W_2=0.$ holds, then M is an

Einstein manifold.

5. Lorentzian α-Sasakian Manifolds Satisfying $N(X,Y)\cdot W_2=0$

The conhormonic curvature tensor N is defined as

$$N(X,Y)Z=R(X,Y)Z-\frac{1}{n-2}\big[S(Y,Z)X-S(X,Z)+g(Y,Z)QX-g(X,Z)QY\big]. \tag{5.1}$$

Using (2.8) and (2.11), Equation (5.1) reduces to

$$N(\xi,Y)Z=\frac{-\alpha^2}{n-2}\big[g(Y,Z)\xi-\eta(Z)Y\big]-\frac{1}{n-2}\big[S(Y,Z)\xi-\eta(Z)QY\big]. \tag{5.2}$$

Now consider in a Lorentzian α-Sasakian manifold

$$N(X,Y)\cdot W_2=0.$$

This condition implies that

$$N(X,Y)W_2(U,V)Z-W_2(N(X,Y)U,V)Z-W_2(U,N(X,Y)V)Z-W_2(U,V)N(X,Y)Z=0. \tag{5.3}$$

Put $X=\xi$ in (5.3) and then taking the inner product with ξ, we obtain

$$\begin{aligned}&g(N(\xi,Y)W_2(U,V)Z,\xi)\\&-g(W_2(N(\xi,Y)U,V)Z,\xi)\\&-g(W_2(U,N(\xi,Y)V)Z,\xi)\\&-g(W_2(U,V)N(\xi,Y)Z,\xi)=0.\end{aligned} \tag{5.4}$$

Using (5.2) in (5.4), we obtain

$$\begin{aligned}&\frac{-\alpha^2}{n-2}\big[-g(Y,W_2(U,V)Z)-g(Y,U)\eta(W_2(\xi,V)Z)\\&-g(Y,V)\eta(W_2(U,\xi)Z)-g(Y,Z)\eta(W_2(U,V)\xi)\\&-\eta(Y)\eta(W_2(U,V)Z)+\eta(U)\eta(W_2(Y,V)Z)\\&+\eta(V)\eta(W_2(U,Y)Z)+\eta(Z)\eta(W_2(U,V)Y)\big]\\&-\frac{1}{n-2}\big[-S(Y,W_2(U,V)Z)-S(Y,U)\eta(W_2(\xi,V)Z)\\&-S(Y,V)\eta(W_2(U,\xi)Z)-S(Y,Z)\eta(W_2(U,V)\xi)\\&-\eta(QY)\eta(W_2(U,V)Z)+\eta(U)\eta(W_2(QY,V)Z)\\&+\eta(V)\eta(W_2(U,QY)Z)+\eta(Z)\eta(W_2(U,V)QY)\big]=0.\end{aligned} \tag{5.5}$$

By using (2.18) in (5.5), we get

$$\left\{\frac{\alpha^2}{(n-2)}\right\}g(Y,W_2(U,V)Z)+\frac{1}{n-2}S(Y,W_2(U,V)Z)=0. \tag{5.6}$$

Taking $U=Z=\xi$ in (5.6) and then using (2.14) and (2.10), we have

$$S(QY,V)=\alpha^2(n-2)S(Y,V)+\alpha^4(n-1)g(Y,V). \tag{5.7}$$

Thus, we can state the following:

Theorem 5.4. If on a Lorentzian α-Sasakian manifold M, the condition $N(X,Y)\cdot W_2=0$ holds, then Equation (5.7) is satisfied on M.

6. Lorentzian α-Sasakian Manifolds Satisfying $\tilde{C}(X,Y)\cdot W_2=0$

The quasi-conformal curvature tensor $\tilde{C}$ is defined as

$$\tilde{C}(X,Y)Z=aR(X,Y)Z+b\big[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY\big]-\frac{r}{n}\left[\frac{a}{n-1}+2b\right]\big[g(Y,Z)X-g(X,Z)Y\big]. \tag{6.1}$$

Using (2.8) and (2.11), Equation (6.1) reduces to

$$\tilde{C}(\xi,Y)Z=k\big[g(Y,Z)\xi-\eta(Z)Y\big]+b\big[S(Y,Z)\xi-\eta(Z)QY\big], \tag{6.2}$$

where $k=\alpha^2(a+b(n-1))-\frac{r}{n}\left(\frac{a}{n-1}+2b\right)$.

Now consider in a Lorentzian α-Sasakian manifold

$$\tilde{C}(X,Y)\cdot W_2=0.$$

This condition implies that

$$\tilde{C}(X,Y)W_2(U,V)Z-W_2(\tilde{C}(X,Y)U,V)Z-W_2(U,\tilde{C}(X,Y)V)Z-W_2(U,V)\tilde{C}(X,Y)Z=0. \tag{6.3}$$

Put $X=\xi$ in (6.3) and then taking the inner product with ξ, we obtain

$$\begin{aligned}&g(\tilde{C}(\xi,Y)W_2(U,V)Z,\xi)\\&-g(W_2(\tilde{C}(\xi,Y)U,V)Z,\xi)\\&-g(W_2(U,\tilde{C}(\xi,Y)V)Z,\xi)\\&-g(W_2(U,V)\tilde{C}(\xi,Y)Z,\xi)=0.\end{aligned} \tag{6.4}$$

Using (6.2) in (6.4), we obtain

$$k\left[-g\left(Y,W_2\left(U,V\right)Z\right)-g\left(Y,U\right)\eta\left(W_2\left(\xi,V\right)Z\right)\right.$$
$$-g\left(Y,V\right)\eta\left(W_2\left(U,\xi\right)Z\right)-g\left(Y,Z\right)\eta\left(W_2\left(U,V\right)\xi\right)$$
$$-\eta\left(Y\right)\eta\left(W_2\left(U,V\right)Z\right)+\eta\left(U\right)\eta\left(W_2\left(Y,V\right)Z\right)$$
$$\left.+\eta\left(V\right)\eta\left(W_2\left(U,Y\right)Z\right)+\eta\left(Z\right)\eta\left(W_2\left(U,V\right)Y\right)\right]$$
$$-b\left[S\left(Y,W_2\left(U,V\right)Z\right)+S\left(Y,U\right)\eta\left(W_2\left(\xi,V\right)Z\right)\right.$$
$$+S\left(Y,V\right)\eta\left(W_2\left(U,\xi\right)Z\right)+S\left(Y,Z\right)\eta\left(W_2\left(U,V\right)\xi\right)$$
$$+\eta\left(QY\right)\eta\left(W_2\left(U,V\right)Z\right)-\eta\left(U\right)\eta\left(W_2\left(QY,V\right)Z\right)$$
$$\left.-\eta\left(V\right)\eta\left(W_2\left(U,QY\right)Z\right)-\eta\left(Z\right)\eta\left(W_2\left(U,V\right)QY\right)\right]=0. \quad (6.5)$$

By using (2.18) in (6.5), we get

$$kg\left(Y,W_2\left(U,V\right)Z\right)+bS\left(Y,W_2\left(U,V\right)Z\right)=0. \quad (6.6)$$

Taking $U=Z=\xi$ in (6.6) and then using (2.14) and (2.10), we have

$$\frac{b}{n-1}S\left(QY,V\right)-\left(b\alpha^2-\frac{k}{n-1}\right)S\left(Y,V\right)$$
$$-k\alpha^2 g\left(V,Y\right)=0. \quad (6.7)$$

If $b = 0$, we get

$$k\left\{\frac{1}{n-1}S\left(Y,V\right)-\alpha^2 g\left(Y,V\right)\right\}=0.$$

Then, either $k = 0$ (or)

$$S\left(Y,V\right)=\alpha^2\left(n-1\right)g\left(Y,V\right).$$

If $b \neq 0$, then we get

$$S\left(QY,V\right)=\left(\alpha^2\left(n-1\right)-\frac{k}{b}\right)S\left(Y,V\right)$$
$$+\left(\frac{k}{b}\alpha^2\left(n-1\right)\right)g\left(V,Y\right). \quad (6.8)$$

Thus, we can state the following:

Theorem 6.5. If M is an Lorentzian α-Sasakian manifold satisfying the condition $\tilde{C}\left(X,Y\right)\cdot W_2=0$, then we get:

- If $b = 0$, then either $k = 0$ on M, or M is an Einstein manifold;
- If $b \neq 0$ then the Equation (6.8) holds on M.

REFERENCES

[1] G. P. Pokhariyal and R. S. Mishra, "The Curvature Tensor and Their Relativistic Significance," *Yokohoma Mathematical Journal*, Vol. 18, 1970, pp. 105-108.

[2] G. P. Pokhariyal, "Study of a New Curvature Tensor in a Sasakian Manifold," *Tensor N.S.*, Vol. 36, No. 2, 1982, pp. 222-225.

[3] K. Matsumoto, S. Ianus and I. Mihai, "On P-Sasakian manifolds Which Admit Certain Tensor Fields," *Publicationes Mathematicae Debrecen*, Vol. 33, 1986, pp. 61-65.

[4] A, Yildiz and U. C. De, "On a Type of Kenmotsu Manifolds," *Differential Geometry-Dynamical Systems*, Vol. 12, 2010, pp. 289-298.

[5] Venkatesha, C. S. Bagewadi and K. T. Pradeep Kumar, "Some Results on Lorentzian Para-Sasakian Manifolds," *ISRN Geometry*, Vol. 2011, Article ID 161523.

[6] S. Tanno, "The Automorphism Groups of Almost Contact Riemannian Manifolds," *Tohoku Mathematical Journal*, Vol. 21, No. 1, 1969, pp. 21-38. doi:10.2748/tmj/1178243031

[7] A. Gray and L. M. Hervella, "The Sixteen Classes of almost Hermitian Manifolds and Their Linear Invariants," *Annali di Matematica Pura ed Applicata*, Vol. 123, No. 4, 1980, pp. 35-58. doi:10.1007/BF01796539

[8] S. Dragomir and L. Ornea, "Locally Conformal Kaehler Geometry, Progress in Mathematics," Birkhauser Boston, Inc., Boston, 1998.

[9] J. A. Oubina, "New Classes of Contact Metric Structures," *Publicationes Mathematicae Debrecen*, Vol. 32, No. 4, 1985, pp. 187-193.

[10] J. C. Marrero, "The Local Structure of Trans-Sasakian Manifolds," *Annali di Matematica Pura ed Applicata*, Vol. 162, No. 4, 1992, pp. 77-86. doi:10.1007/BF01760000

[11] J. C. Marrero and D. Chinea, "On Trans-Sasakian Manifolds," *Proceedings of the XIVth Spanish-Portuguese Conference on Mathematics*, University La Laguna, Vol. I-III, 1990, pp. 655-659.

[12] D. E. Blair, "Contact Manifolds in Riemannian Geometry," *Lecture Notes in Mathematics*, Springer-Verlag, Berlin, 1976.

[13] K. Kenmotsu, "A Class of Almost Contact Riemannian Manifolds," *Tohoku Mathematical Journal*, Vol. 24, No. 1, 1972, pp. 93-103. doi:10.2748/tmj/1178241594

[14] M. M. Tripathi, "Trans-Sasakian Manifolds Are Generalized Quasi-Sasakian," *Nepali Mathematical Sciences Report*, Vol. 18, No. 2, 2000, pp. 11-14.

[15] D. E. Blair and J. A. Oubina, "Conformal and Related Changes of Metric on the Product of Two Almost Contact Metric Manifolds," *Publications Matematiques*, Vol. 34, 1990, pp. 99-207.

[16] D. Janssens and L. Vanhecke, "Almost Contact Structures and Curvature Tensors," *Kodai Mathematical Journal*, Vol. 4, No. 1, 1981, pp. 1-27. doi:10.2996/kmj/1138036310

[17] A. Yildiz and C. Murathan, "On Lorentzian _α-Sasakian Manifolds," *Kyungpook Mathematical Journal*, Vol. 45, No. 1, 2005, pp. 95-103.

[18] U. C. De and M. M. Tripathi, "Ricci Tensor in 3-Dimensional Trans-Sasakian Manifolds," *Kyungpook Mathematical Journal*, Vol. 43, 2003, pp. 247-255.

[19] K. Matsumoto and I. Mihai, "On a Certain Transformation in a Lorentzian Para-Sasakian Manifold," *Tensor N.S.*, Vol. 47, 1988, pp. 189-197.

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[20] K. Yano and M. Kon, "Structures on Manifolds," *Series in Pure Mathematics*, Vol. 3, World Scientific Publishing Co., Singapore, 1984.

[21] G. P. Pokhariyal and R. S. Mishra, "Curvature Tensor and Their Relativistic Significance II," *Yokohama Mathematical Journal*, Vol. 19, 1971, pp. 97-103.

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