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Interaction of Nonstationary Waves on Cylindrical Body

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Abstract

In this paper, the case of the interaction of a flat compression pulse with a layered cylindrical body in an infinite homogeneous and isotropic elastic medium is studied. The problem by the methods of integral Fourier transforms is solved. The inverse transform numerically by the Romberg method is calculated. With a time of toast and a decrease in momentum, the accuracy is not less than 2%. Taking into account the diffracted waves the results are obtained.

Keywords

Compression Impulses, Fourier Transform, Romberg Method, Heaviside Function, Reflections, Diffraction

1. Introduction

Various issues related to the interaction of bodies with a continuous medium (the creation of effective mathematical models is, theoretical and experimental methods for the study of non-stationary problems of dynamics) are described in monographs [1] [2] [3] [4] [5]. We have to deal with these questions when solving a wide variety of tasks. Their successful solution is associated with the further harmonic interaction of various sciences: aerodynamics, the theory of elasticity and plasticity, soil mechanics and underground structures, and others. The range of tasks for solving which is necessary to take into account the influence of the environment on the behavior of structures, structures and systems is continuously expanding: problems of pipe transport, defect scope, calculation of elements of nuclear reactors, seismic effects and others. Despite the great successes achieved recently in this area, many problems still remain unresolved. The problems of unsteady interaction of deformable bodies with elastic media and with the ground are especially poorly studied. In the future, it is necessary to pay more attention to the following issues: building more accurate schemes (models) of the interaction of waves (of varying intensity) and bodies with deformable barriers; development and creation of computing systems based on modern computers for solving applied dynamics problems.

The problems of the no stationary dynamics of a homogeneous isotropic linearly elastic medium in cylindrical coordinates are given in the work of C. Chree [6]. Some problems of the dynamics of elastic cylindrical bodies are given in [7] [8]. In [9] [10], using the Laplace transform in time, the problem of radial oscillations of a thick-walled sphere immersed in an infinite elastic medium was investigated by specifying the uniform unsteady pressure. The stress-strain state of a hollow elastic cylinder surrounded and filled with acoustic or elastic media, under the action of non-stationary loads applied on the side surfaces, was investigated in [11] [12].

Some issues related to the diffraction of no stationary waves on cavities and absolutely rigid obstacles are considered in the works of A.N. Guzz, V.D. Kubenko and M.A. Cherevko [13] and Y.H. Pao and C.C. Mowa [14]. Works devoted to these problems are partially cited in the reviews of A.G. Gorshkov [15]. A general approach to solving plane diffraction problems in elastic media, based on the method of boundary integral equations, was developed by G.D. Manos and D.E. Beskos [16], D.M. Cole, D.D. Kosloff and J.B. Minster [17].

The influence of various factors on the behavior of a smooth infinitely long thin cylindrical shell during the diffraction of a plane shock wave on it (a plane problem) was studied by many authors [18]-[23]. The interaction of a plane mobile shock wave with a thin-walled structure consisting of coaxial cylindrical shells was considered in [24] [25]. Recently, considerable attention has been paid to the problems of non-stationary dynamics associated with the calculation of engineering structures for the action of seismic loads. The works of K. Fujita [26] are devoted to determining the response of some types of structures to seismic effects (Harouma and G.W. Housnera [27]). The creation of universal algorithms for calculating piecewise-homogeneous cylindrical bodies under the influence of non-stationary loads is an actual unsolved problem.

2. Statement and Methods for Solving the Problem of the Interaction of Non-Stationary Waves with a Cylindrical Body with a Liquid

The problem of the action of non-stationary waves on layered cylindrical bodies with radius R_k is considered. The motion vector of the medium is connected with the potentials φ_N and $\boldsymbol{\psi}_k$ by means of the formulas

$$\boldsymbol{u}_k = grad\,\varphi_k + rot\left(\boldsymbol{\psi}_k\right)\ \left(k = 1, 2, \cdots, N\right).$$

Suppose that the elastic medium is in plane strain conditions in the plane. In polar coordinates r,θ, the basic ratios of the plane problem are

$$u_{rk}=\frac{\partial\varphi_k}{\partial r}+\frac{1}{r}\frac{\partial\psi_k}{\partial\theta},\ u_{\theta k}=\frac{1}{r}\frac{\partial\varphi_k}{\partial\theta}-\frac{\partial\psi_k}{\partial r}$$

$$\sigma_{rrk}=\frac{\lambda_k}{c_{1k}}\frac{\partial^2\varphi_k}{\partial t^2}+2\mu_k\left(\frac{\partial^2\varphi_k}{\partial r^2}+\frac{1}{r}\frac{\partial^2\psi_k}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial\psi_k}{\partial\theta}\right)$$

$$\sigma_{\theta\theta k}=\rho_k\frac{\partial^2\varphi_k}{\partial t^2}-2\mu_k\left(\frac{\partial^2\varphi_k}{\partial r^2}+\frac{1}{r}\frac{\partial^2\psi_k}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial\psi_k}{\partial\theta}\right)$$

$$\sigma_{r\theta k}=\rho_k\frac{\partial^2\psi_k}{\partial t^2}+2\mu_k\left(\frac{1}{r}\frac{\partial^2\varphi_k}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial\varphi_k}{\partial\theta}-\frac{\partial^2\psi_k}{\partial r^2}\right)$$

here λ_k and μ_k the Lame elastic constants of the k-th layer; ρ_k—density of the material of the k-th layer; $\sigma_{rrk},\sigma_{\theta\theta k},\sigma_{r\theta k}$—components of the stress tensor of the k-th layer.

Non-stationary stress waves $\sigma_{xx}^{(i)}$ and $\sigma_{xy}^{(i)}$, whose front is parallel to the longitudinal axis of the cylinder, fall on a layered cylinder (**Figure 1**).

The basic equations of the theory of elasticity for this problem of plane strain in displacement potentials are reduced to the following:

$$\nabla^2\varphi_j=\frac{1}{c_{pj}^2}\frac{\partial^2\varphi_j^{\kappa}}{\partial t^2};\quad (j=1,2,\cdots,N)$$
$$\nabla^2\psi_j=\frac{1}{c_{\rho j}^2}\frac{\partial^2\psi_j}{\partial t^2}. \tag{1}$$

where U_j and ψ_j are the displacement potentials of the j-th layer, c_{pj} and $c_{\beta j}$—are the phase velocities of the extension and shear waves of the j-th layer.

Suppose that time t is counted from the moment when the incident pulse touches the surface of the external ($N-1$)-th cylinder at point $r=r_N$, $\theta=0$. Until that moment, peace remains. In accordance with the foregoing, the task of finding the field of diffracted waves and the stress-strain state caused by the incident pulse [17]

$$\sigma_{xx}^{(i)}=\sigma_0H(\hat{t}),$$
$$\sigma_{xy}^{(i)}=\sigma_0\frac{\nu_N}{1-\nu_N}H(\hat{t}),\ \hat{t}=t-(x+r_N)/C_{PN} \tag{2}$$

σ_0—the amplitude of the incident waves; $H(\hat{t})$—the unit Heaviside function, reduces to solving differential Equations (1). Boundary conditions on the contact of two cylindrical surfaces should be equal to displacement and tension

$$r=a_\kappa:\quad \sigma_{rr\kappa}=\sigma_{rr(\kappa+1)};\quad \sigma_{r\theta\kappa}=\sigma_{r\theta(\kappa+1)};\quad \sigma_{rz\kappa}=\sigma_{rz(\kappa+1)};$$
$$u_\kappa=u_{\kappa+1};\quad \vartheta_\kappa=\vartheta_{\kappa+1};\ w_\kappa=w_{\kappa+1}.$$

At infinity ($r\to\infty$), the perturbations must die out. If φ_N and ψ_N—diverging waves, then

$$\varphi_N\to0,\psi_N\to0\ \text{ at }\ \sqrt{x^2+y^2+z^2}\to\infty.$$

The problem is solved under the following zero initial conditions [19]:

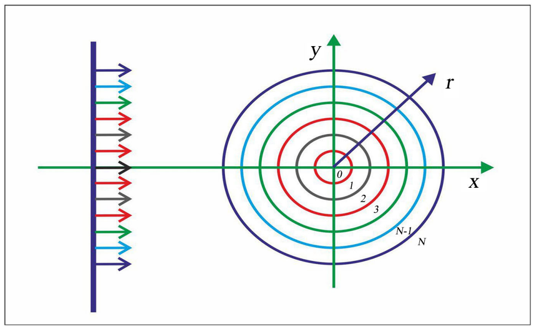

Figure 1. The effect of non-stationary waves on a layered body.

$$\left.\frac{\partial\phi_j}{\partial r}+\frac{1}{r}\frac{\partial\psi_j}{\partial\theta}\right|_{t=0}=\frac{\partial}{\partial t}\left.\left(\frac{\partial\phi_j}{\partial r}+\frac{1}{r}\frac{\partial\psi_j}{\partial\theta}\right)\right|_{t=0}=0,$$
$$\left.\frac{1}{r}\frac{\partial\psi_j}{\partial\theta}-\frac{\partial\phi_j}{\partial r}=\frac{\partial}{\partial t}\left(\frac{1}{r}\frac{\partial\psi_j}{\partial\theta}-\frac{\partial\phi_j}{\partial r}\right)\right|_{t=0}=0, \tag{3}$$

where $j=1,2,\cdots,N$; Nt = j—is the number of cylindrical layers; $j=N$—environment.

It is required to determine the dynamic stress-strain state of the cylinder and its environment caused by the incident voltage pulse (2).

3. Solution Methods

To solve the plane problem, the integral Laplace transform (or Fourier transform) over time t is often used. When applying the integral Laplace transform for a function $f(t)$ that is integral in the sense of Lebegue on any open interval $0<t<T$, is expressed by the formula

$$f^L(s)=\int_0^\infty e^{-st}f(t)\,dt=L\left[f(t)\right]$$

The function $f^L(s)$ is usually called the image (transform ant), the function of the $f(t)$—original. The inversion of the Laplace transform is determined by the formula

$$f(t)=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}e^{st}f^L(s)\,ds=L^{-1}\left[f^L(s)\right],$$

where the integral is taken along the path to the right of the singularities of the integrand. Using the Laplace transform problem, the interaction of non-stationary waves with a layered cylindrical body is a time-consuming task. Under the integral function is complex and has a complex form. Therefore, to find the exact expression of the original and bring to the numerical calculation is almost impossible. This method is applied in the work of V.D. Kubenko [1] for the problem of interaction of non-stationary waves of the cavity and obtained some particular solutions. Therefore, to solve this problem, the Fourier integral transform is used [28].

Integral Fourier transform. The stress field caused by the forces (2) satisfies the wave Equation (1), *i.e.* every cylindrical layer satisfies it. To solve the above problem, apply the t-integral Fourier transform with respect to time

$$\phi^F(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \phi(\Omega)\ell^{-i\xi\Omega}\mathrm{d}\Omega; \quad \phi(\Omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \phi^F(\xi)\ell^{i\xi\Omega}\mathrm{d}\xi \tag{4}$$

Using zero initial conditions, we obtain the depicted problem

$$\begin{aligned} &\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi_j^F}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi_j^F}{\partial \theta^2} + \frac{\Omega^2}{C_{P_j}^2}\bar{\phi}_j^F = 0, \\ &\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \bar{\psi}_j^F}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \bar{\psi}_j^F}{\partial \theta^2} + \frac{\Omega^2}{C_{P_j}^2}\bar{\psi}_j^F = 0, \end{aligned} \tag{5}$$

where Ω —Fourier transforms parameter; φ_j^F, ψ_j^F —image of the Fourier transform of functions $\varphi_j(t)$ and $\psi_j(t)$ respectively. Then the solution of Equations (4) and (5) will be

$$\begin{aligned} &\phi_j^F(r,\theta,\Omega) = \bar{\phi}_j^F(r,\Omega)\cos n\theta; \\ &\psi_j^F(r,\theta,\Omega) = \bar{\psi}_j^F(r,\Omega)\sin n\theta \end{aligned} \tag{6}$$

here

$$\begin{aligned} &\bar{\phi}_j^F(r,\Omega) \\ &= \begin{cases} A_n H_n^{(1)}(\Omega r/C_{PN}), \; r \ge r_N, \\ A_{nj} H_n^{(1)}(\Omega r/C_{Pj}) + B_{nj} H^{(2)}(\Omega r/C_{Pj}), \; r_0 \le 2 \le r_N \; (j = 1,2,\cdots,N-1), \\ A_{n0} I_n(\Omega r/C_{SN}), \; 0 \le r \le r_0; \end{cases} \end{aligned} \tag{7}$$

$$\bar{\psi}_j^F(r,\Omega) = \begin{cases} C_{nJ} H_n^{(1)}(\Omega r/C_{Sj}) + L_{nJ} H_n^{(2)}(\Omega r/C_{Sj}), \; r_0 \le r \le r_N, \\ C_n H_n^{(1)}(\Omega r/C_{SN}), \quad r \ge r_N, \\ C_{n0} I_n(\Omega r/C_{S0}), \; r_n \le r \le r. \end{cases} \tag{8}$$

Coefficients $A_{n0}, A_{nj}, A_{nN}, B_{nj}, C_{nj}, C_{nN}$ —determined from the boundary conditions (7)-(8), which are placed on the contact of two cylindrical surfaces. Boundary conditions at $r = R_n$ taking into account the incident waves (1) take the form

a) $\sigma_{rrN}^F + \sigma_{rrN}^{(i)F} = \sigma_{rr(N-1)}^F$,
b) $\sigma_{r\theta N}^F + \sigma_{r\theta N}^{(i)F} = \sigma_{r\theta(N-1)}^F$,
v) $u_{rN}^F + u_{rN}^{(i)F} = u_{r(N-1)}^F$,
g) $u_{\theta N}^F + u_{\theta N}^{(i)F} = u_{\theta(N-1)}^F$,

where

a) $\sigma_{rrN}^{(i)F}(\Omega) = \sigma_{01}^{(P)} \sum_{n=0}^{\infty} (-1)^n \in_n I_n(\Omega r/C_{PN}) \cos n\theta$;
b) $\sigma_{rrN}^F(\Omega) = \sigma_{rrN}^F(\cos^2\theta + \in_N \sin^2\theta)$;
v) $\sigma_{r\theta N}^F = -\sigma_{rr}^F[(1 - E_N)/2]\sin 2\theta$;
g) $u_{rN}^F = u_{rN}^F \cos\theta$;
d) $u_{\theta N}^F = u_{\theta N}^F \sin\theta$;
$\sigma_{01}^{(P)} = \sigma_0 \mathrm{e}^{-N\Omega/C_{PN}}$.

Substituting (5) and (6) into the boundary conditions (7) and (8), we obtain a system of complex algebraic equations with $(4j+3)$ unknowns in the form

$$[Z]\{g\}=\{P\}, \tag{9}$$

$$[Z]=\left(\begin{array}{cc|cc} [Z_1] & & & 0 \\ & [Z_2] & & \\ \hline & & [Z_{(N-1)}] & \\ 0 & & & [Z_N] \end{array}\right)$$

$[Z_j]$—4 × 4 matrix, the elements of which are of the nth order first and second kind Bessel and Henkel functions; $\{g\}$—Vector columns of unknown coefficients; $\{P\}-\{0,0,\cdots,0,P_{1N},P_{2N},P_{3N},P_{4N}\}^{\mathrm{T}}$—vector columns characterizing the falling loads, where $P_{1N},P_{2N},P_{3N},P_{4N}$ corresponds to $\sigma_{rrN}^{(i)F},\sigma_{r\theta N}^{(i)F},u_{rN}^{(i)F},u_{\theta N}^{(i)F}$. Let the stepped waves interact with a cylindrical hole when $r=r_0$ and a stress-free hole $\left(\sigma_{rr}\big|_{r=a}=\sigma_{r\theta}\big|_{r=a}=0\right)$. The only voltage that does not vanish at $r=r_0$, is the ring voltage $\sigma_{\theta\theta n}/\sigma_0$. Applying the Fourier transform to the equation of motion and the boundary conditions [5], we obtain the expression for ring stresses at $\sigma_{rr}=\sigma_0 H(t)\cos nt$, $\sigma_{r\theta}=\tau_0 H(t)\sin\theta$:

$$\sigma_{\theta\theta n}^{*}=\frac{\sigma_{\theta\theta n}(r_{01}\theta,t)}{\sigma}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\Delta_1(r_0\Omega)\mathrm{e}^{i\Omega t}}{\Omega_1[\Delta_2\Delta_3+\Delta_4\Delta_5]}\mathrm{d}\Omega, \tag{10}$$

$$\Delta_1(r_{01}\Omega)=(\Delta_3+\tau_0 E)\left[2\Omega H_{n-1}^{(1)}(\Omega)-\left((2n^2+2n)+\Omega^2\right)H_n^{(1)}(\Omega)\right]$$
$$+[\tau_0\Delta_2-\Delta_4]\left[2n(n+1)H_n^{(1)}\left((C_{P1}/C_{S1})\Omega\right)+\frac{2C_P n\Omega}{C_{S1}}H_{n-1}^{(1)}\left(\frac{C_P}{C_S}\Omega\right)\right].$$

Expression $\Delta_k(k=1,2,3,4,5)$ given in [20]. The improper integral (10) is solved numerically using the developed algorithms [21]. Practically, the calculation (10) on a computer can be carried out as follows. Since infinite numerical integration is unthinkable, the integral (10) is replaced by

$$\sigma_{\theta\theta n}^{*}=\frac{1}{2\pi}\int_{\omega_a}^{\omega_b}\frac{\Delta_1(r_{01}\Omega_1)}{\Omega_1[\Delta_2\Delta_3+\Delta_4\Delta_5]}\mathrm{e}^{-i\Omega t}\mathrm{d}\Omega. \tag{11}$$

Values of the limits of integration ω_a,ω_b are selected depending on the type of incident pulse. Numerical values of spectral density $\sigma_{rr}^{(i)F}(\Omega)$ from (9) of the final incident pulse; only in a small frequency range is significantly different from zero. Limits of integration ω_a,ω_b should be selected in accordance with this range and taking into account the required accuracy. At the same time, the question remains open as to what error the neglect of the contribution of integrals of the type (10), within the limits of integration of $-\infty$ to ω_a and from ω_b to ∞. The numerical summation of the infinite sum (10) is, of course, also impossible. However, it was shown in [22] that for sufficiently large n (the n-order of the Bessel and Henkel functions), we can construct an asymptotic representation of the general term of this sum. As a result, it becomes possible to either estimate the error of the transition from an infinite to a finite sum, or approximate summation of an infinite sum. In view of the above, we keep in (10) an infinite sum. The calculation by the considered method is reduced to the construction of two calculation algorithms: coefficients $Z_{ke}(\Omega)(k,e=1,2)$ (11) and integral (10). The first and second algorithms do not depend on the type of mathematical model of the object.

4. Calculation Algorithm

Magnitude $\sigma_{\theta\theta n}/\sigma_0$ from (11) is calculated on a computer as follows. All numeric parameters required for calculations are specified. The following notation is introduced: $x_1=\Omega$, $x_2=n_1\Omega$, where $n_1=C_{P1}/C_{S1}$; $\Omega=\omega\alpha/C_{P1}$. For two values $x_k(k=1,2)$ Bessel function is determined $I_n(\xi)$ и $N_n(\xi)(n=1,2,\cdots,10)$. These arrays are calculated by the formula

$$u_n(\xi)=\frac{2(n-1)}{\xi}u_{n-1}(\xi)-u_{n-2}(\xi),\ u_n(\xi)=I_n(\xi),N_n(\xi) \tag{12}$$

As shown in [23], the absolute value of the Bessel function decreases rapidly with increasing index, starting from the moment when the index exceeds the argument. In this case, the direct use of formula (12) does not lead to the goal. Nevertheless, the calculation by (12) is possible, if by the recurrence formula

$$\overline{I}_n(\xi)=\frac{2(n-1)}{\xi}\overline{I}_{n+1}(\xi)-\overline{I}_{n+2}(\xi) \tag{13}$$

in the direction of decreasing index (from $n=N$ to $n=0$), an auxiliary function is calculated $\overline{I}_n(\xi)$. To calculate the integral (11) of the integrand function

$$\chi_1(r_0,\Omega,t)=\left(\Delta_1(r_0,\Omega_1)/\Omega_1(\Delta_2\Delta_3+\Delta_4\Delta_5)\right)\mathrm{e}^{i\Omega t}$$

can be integrated numerically by writing it in the form

$$\chi_1(r_0,\Omega,t)=x_1(r_0,\Omega,t)-ix_2(r_0,\Omega,t).$$

The falling pulse $\sigma_{xx}^{(i)}(\Omega)$ [23] is described by the expression

$$\sigma_{xx}^{(i)}(\Omega)=f_1(\Omega,t)-if_2(\Omega,t),$$

where $f_1(\Omega,t),f_2(\Omega,t)$—real functions. Using Euler's formula for $\ell xp(i\Omega t)$, dividing (18) into real and imaginary (19) parts, after some transformations we get

$$\sigma_{\theta\theta n}^*=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[x_1(\Omega,t)-ix_2(\Omega,t)\right]\mathrm{d}\Omega \tag{14}$$

Dividing the integral (14) into two terms

$$\sigma_{\theta\theta n}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0}\left[x_1(\Omega,t)-ix_2(\Omega,t)\right]\mathrm{d}\Omega+\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}\left[x_1(\Omega,t)-ix_2(\Omega,t)\right]\mathrm{d}\Omega. \tag{15}$$

And replacing the variable in the first integral Ω on $-\Omega$, will have

$$\sigma_{\theta\theta n}=\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}\left[x_1(\Omega,t)-x_1(-\Omega,t)\right]-i\left[x_2(\Omega,t)-x_2(-\Omega,t)\right]\mathrm{d}\Omega. \tag{16}$$

Since (16) is the inverse Fourier transform and contains the real value in the left-hand side [24], the relation

$$x_1(\Omega,t) = -x_1(-\Omega,t); x_2(\Omega,t) = -x_2(-\Omega,t). \tag{17}$$

Considering it, from (17) we finally get

$$\sigma^*_{\theta\theta n} = \frac{\sqrt{2}}{\pi}\int_{\omega_a}^{\omega_b}\left[x_1(\Omega,t) + ix_2(\Omega,t)\right]\mathrm{d}\Omega.$$

The value of integral (17) can be found numerically using the Romberg method [9] [10]. The basic algorithm of this method is given in the first chapter. When calculating the integral using the Romberg method, one has to repeatedly calculate the integrand. The inverse Fourier transform for some image, the original of which is known in advance, showed that with an integration step length of 0.01, the error of the procedure does not exceed 0.3% - 0.5%.

Numerical results are presented for the ring voltage at $r = r_0$, caused by the incident flat shock wave with a stepped distribution of voltage over time. Numerical results were obtained for $\nu = 0.25$: $C_{S1}/C_{P1} = 0.5$; $\theta = 0^\circ$ и 90°. To determine the integral (17) of the boundary of the integral ω_a and ω_b have chosen $\left[10^{-4} - N\right], N = 1,2,3,4,5$, a step $h = 0.1, 0.01, 0.001$. At $N = 5$ and $N = 6$ the value of the ring voltage differs from the previous one by the fifth decimal place. Change $\sigma^*_{\theta\theta}$ depending on the τ at various $n = 0,1,2,3,4,5$ shown in **Figure 2**, **Figure 3** and **Figure 4**. The results of our numerical calculations were compared with known results [20]. The values obtained differ by approximately 30% at n = 0.1: the maximum ring stress at $h = 0.01$ and $\theta = 90$ is 2.962/3.0; and on work [11] [12]—3.28/3.0 ($\tau \approx 4.71$).

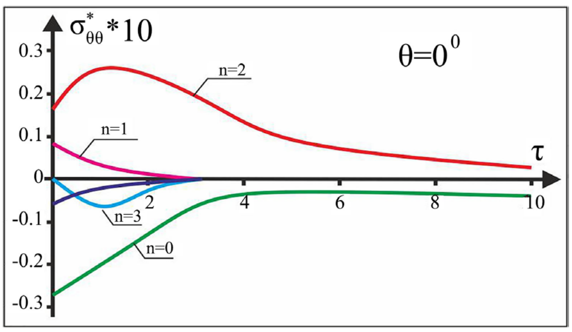

Figure 2. The dependence of ring stresses on time, with different *n*.

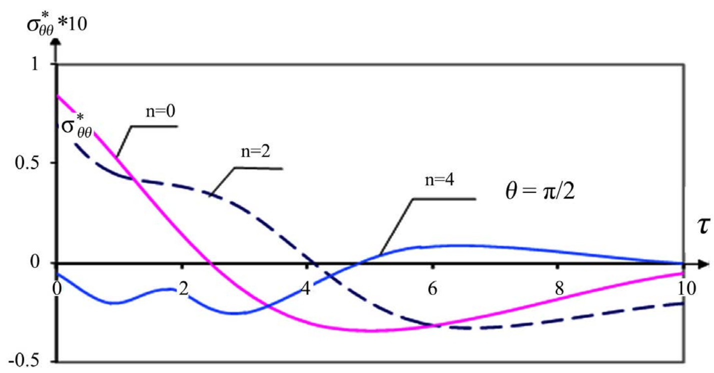

Figure 3. The dependence of ring stresses on time, with different *n*.

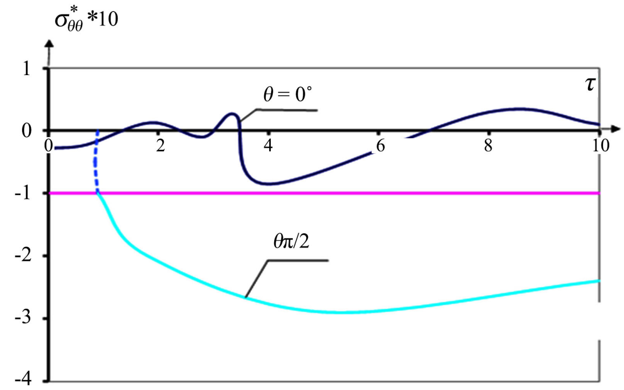

Figure 4. The dependence of the ring voltage on time τ.

5. Diffraction of Non-Stationary Waves on a Cylindrical Body

Let the inner boundary $(r = r_0)$ free from voltage, and on contact with the environment, the condition of equality of displacements and stresses (7) [25] [26]. After the Fourier transform, we obtain the cylindrical Bessel Equations (13) and (16), the solution of which has the form (7) and (8). In our problem there will be six arbitrary constants, which are determined from the boundary conditions (8). Here are some of them:

$$\sigma_{rr_2} = 2\mu_2 r^{-2}\sum_{k=1}^{2}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\left[C_{nk}\varepsilon_{1n}^{(k)} + D_{nk}\varepsilon_{2n}^{(k)}\right]\mathrm{e}^{i\Omega\tau}\mathrm{d}\Omega,$$

$$\sigma_{\theta\theta_2} = 2\mu_2 r^{-2}\sum_{k=1}^{2}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\left[C_{nk}\varepsilon_{3n}^{(k)} + D_{nk}\varepsilon_{4n}^{(k)}\right]\mathrm{e}^{i\Omega\tau}\mathrm{d}\Omega,$$

$$\sigma_{r\theta_2} = 2\mu_2 r^{-2}\sum_{k=1}^{2}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\left[C_{nk}\varepsilon_{5n}^{(k)} + D_{nk}\varepsilon_{6n}^{(k)}\right]\mathrm{e}^{i\Omega\tau}\mathrm{d}\Omega,$$

$$\sigma_{rr_1} = 2\mu_1 r^{-2}\sum_{k=1}^{2}\sum_{n=1}^{\infty}\int_{-\infty}^{+\infty}\left[A_n\delta_n^{(1)} + B_n\delta_n^{(2)}\right]\mathrm{e}^{i\Omega\tau}\mathrm{d}\Omega,$$

where C_{nk}, D_{nk}, A_n, B_n —arbitrary constants: $C_{nk} = \sigma_{kn}^{(c)}/\Delta_n$, $D_{nk} = \sigma_{kn}^{(D)}/\Delta_n$, $A_n = \delta_n^{(A)}/\Delta_n$, $B_n = \sigma_n^{(B)}/\Delta_n$; $\sigma_{kn}^{(k)}$ and Δ_n —square complex matrices (6 × 6). The remaining elements of the stress tensor are written similarly (17)

$$\begin{aligned}
&C_{nk} = \operatorname{Re}C_{nk} + i\operatorname{Im}C_{nk},\ D_{nk} = \operatorname{Re}D_{nk} + i\operatorname{Im}D_{nk},\\
&A_n = \operatorname{Re}A_n + i\operatorname{Im}A_n,\ B_n = \operatorname{Re}B_n + i\operatorname{Im}B_n,\\
&\delta_n^{(e)} = \operatorname{Re}\delta_n^{(e)} + i\operatorname{Im}\delta_n^{(e)},\ e = 1,2,\ \varepsilon_{mn}^{(k)} = \operatorname{Re}\varepsilon_{mn}^{(k)} + i\operatorname{Im}\varepsilon_{mn}^{(k)},\\
&\mathrm{e}^{i\Omega t} = \cos\Omega t + i\sin\Omega t,\ m = 1,2,3,4,5
\end{aligned}\tag{18}$$

Substituting (18) into (17), after some transformations, we obtain the stress tensor

$$\sigma_{ji} = \sum_{k=1}^{2}\sum_{n=0}^{\infty}\int_{\omega_a}^{\omega_b}\operatorname{Re}\sigma'_{ij}\mathrm{d}\Omega. \tag{19}$$

All these procedures are stored in the memory of the machine. A universal algorithm for calculating integrals of type (19) has been developed. The results of the calculations are shown in **Figure 5** with

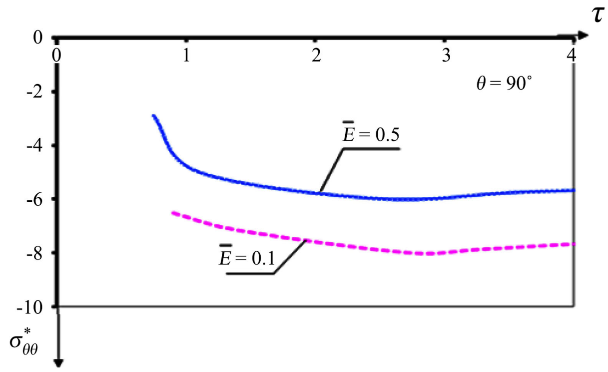

Figure 5. The dependence of the ring voltage of time τ.

$$\theta = 90^\circ \left(\nu_1 = 0.2; \nu_2 = 0.25; r_0/r_1 = 0.5; E_1/E_2 = 0.1; \eta = 0.1\right)$$

The obtained data are compared with known results [25] [26]. When integrating the limit $\omega_a = 10^{-4}, \omega_b = 4, h = 10^{-2}$ the results of my calculation are different from the data on ≈20%. Similar results were obtained for cylindrical shells in an elastic medium. The equation of motion of cylindrical shells has the form [27], and the circumferential stress $\sigma^*_{\theta\theta}$ in the shell but here $C_{n2} = D_{n2} = 0$. Change in peripheral voltage $\sigma^*_{\theta\theta}\left(\theta = 90^\circ, r = r_0\right)$ depending on the τ shown in **Figure 6**, where 1 is the results of [28], 2 are mine for given $\left(h/r = 0.04; h = \left(r_1 - r_0\right)/2\right)$. Similar results were obtained in [28], but the authors believe that $\in_1 = h^2/12R^2 = 0$, those. They take into account the bending moment. In the case of elastic cylindrical bodies, the determination of the stress-strain state of an object and its environment under the action of non-stationary waves is based on building a sequence of incident pulses from stationary components, where each pulse is a change in time of unsteady voltage in the incident wave.

Figure 7 shows the change in circumferential voltage $\sigma^*_{\theta\theta}$ $\left(\theta = 90^\circ, r = r_0, r = r_0 + \left(r_1 - r_0\right)/2, r = r_1\right)$, depending on the τ.

The difference between the stresses on the outer and inner surfaces reaches ≈15% - 20%, and the difference between the stresses on the middle and inner surfaces ≈10% $\left(r_0/r_1 = 0.5\right)$. Calculations show that when $\tau = 12\alpha/C_{P_1}$ the results of this study are approaching the exact static value $\sigma^*_{\theta\theta} = 8.13$. The dependence of the circumferential voltage on τ presented in **Figure 8**. It is seen that the maximum stress and displacement significantly depend on $\bar{\eta}$ and $\bar{E}$.

6. Diffraction of Elastic Non-Stationary Waves in a Two-Layer Cylindrical Body

Let a non-stationary step load (1) fall on an elastic two-layer cylindrical body for $t > 0$. A hard contact condition is set at the borders of the contact. The stress tensor in each layer is written as

$$\sigma_{ij}^{(k)}\left(r_1\theta_1 t\right) = \frac{1}{\pi}\sum_{n=0}^{\infty}\int_{\omega_a}^{\omega_b} \operatorname{Re}\sigma_{nij}'^{(k)}\left(r_1\theta_1\Omega\right)\mathrm{d}\Omega, \quad k = 1,2,3. \tag{20}$$

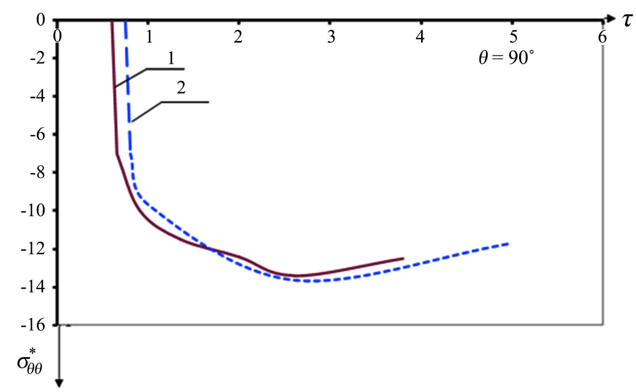

Figure 6. The dependence of the ring voltage of the middle surface of the layer on τ.

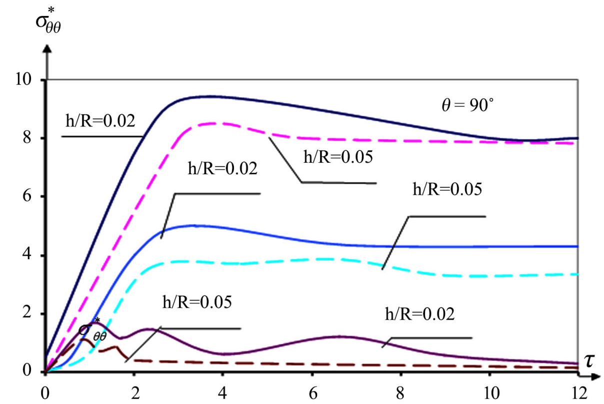

Figure 7. The dependence of the dimensionless ring stress on τ at various h/R.

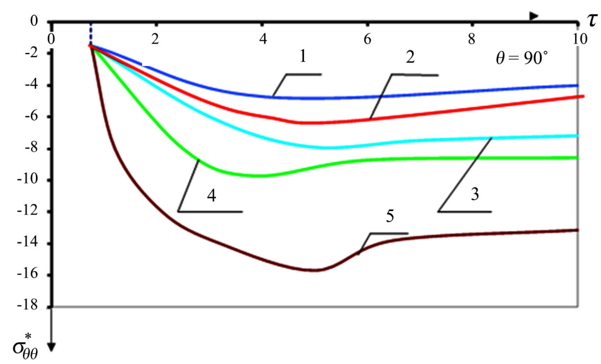

Figure 8. The dependence of the annular stress of the inner surface of the cylindrical layer on the time: 1—granite-concrete; 2—sandstone concrete; 3—soft concrete.

Stress tensor $\sigma_{ij}^{(k)}$ represents the functions of Bessel and Hankel of the first and second kind of the n-th order. Integral (20) is calculated according to the developed algorithm of the first chapter. The decision was limited to five members of the series (20), since the retention of the next members of the series has almost no effect on the results. For example, holding ten members (20) changes the voltage value by less than 2% - 3%. The following parameters were used in the calculations.: $r_0/r_2 = 0.2$; $r_1/r_2 = 0.6$; $\nu_1 = 0.2$; $\nu_2 = 0.25$; $\nu_3 = 0.2$; $E_1/E_2 = 0.3$; $E_3/E_2 = 0.1$; $\rho_1/\rho_2 = 0.3$; $\rho_3/\rho_2 = 0.1$.

7. Conclusions

1) In this paper, a method and algorithm are proposed for solving the problem of no stationary interaction of elastic waves on multilayer cylindrical bodies.

2) A new approach to solving dynamic problems of bodies interacting with the environment, based on the methods of Fourier and the Romberg method, is proposed.

3) It has been established that with the same loading characteristics in the material of the outer layer of a two-layer body, stress waves with the same parameters are formed at the initial moments of time.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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