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A Knowledge Measure with Parameter of Intuitionistic Fuzzy Sets

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Abstract

A new knowledge measure with parameter of intuitionistic fuzzy sets (IFSs) is presented based on the membership degree and the non-membership degree of IFSs, which complies with the extended form of Szmidt-Kacprzyk axioms for intuitionistic fuzzy entropy. And a sufficient and necessary condition of order property in the Szmidt-Kacprzyk axioms is discussed. Additionally, some numerical examples are given to illustrate the applications of the proposed knowledge measure and some conventional entropies and knowledge measures of IFSs. The experimental results show that the results of the parametric model proposed in this paper are more accurate than those of most of the classic models.

Keywords

Intuitionistic Fuzzy Sets, Knowledge Measure, Entropy, Uncertainty, Order Property

1. Introduction

Entropy is a basic parameter that characterizes the state of matter launched by Shannon [1], which is used to characterize the degree of disorder in the system, the uncertainty of the system structure and movement, and the degree of irregularity. In 1965, Zadeh launched fuzzy sets (FS) [2], and Atanassov proposed intuitionistic fuzzy sets (IFS) in 1986 by introducing the degree of hesitation, which means that the research of intuitionistic fuzzy sets is more complex than that of fuzzy sets [3]. Fuzzy entropy is one of the most important methods to measure the degree of disorder in fuzzy sets, and knowledge measure is one of the most important bases for measuring the degree of order between fuzzy sets [4] [5] [6]. In 1972, De Luca and Termini put forward an axiom system of fuzzy entropy in terms of Shannon's entropy function [4] [6]; Yager proposed some fuzzy entropy formulas according to fuzzy distance measure [5]. Since then, many scholars began to use various methods to study fuzzy entropy and intuitionistic fuzzy entropy [6]-[11]. Because intuitionistic fuzzy sets are an extension of ordinary fuzzy sets, many scholars focus on entropy and knowledge measure of intuitionistic fuzzy sets [7]-[16]. In the past 20 years, based on the study of fuzzy sets, many scholars have proposed a variety of methods to calculate intuitionistic fuzzy entropy and knowledge measure [7]-[16]. Based on the axioms of intuitionistic fuzzy entropy [10] [11], Szmidt and Kacprzyk presented a standard judgment of intuitionistic fuzzy knowledge measure with a relatively wide application [12]: non-negative boundedness, symmetry and order. Some researchers also studied Szmidt and Kacprzyk's axiom system and introduced some classic knowledge measure formulas [13] [14] [15]. According to the order property, Guo put forward a new knowledge measure with order [13], Nguyen presented a model from a classic distance measure [14], and Das *et al.* proposed a new model based on a series of similarity measures [15]. However, most of the existing research only focused on knowledge measure based on membership degree and non-membership, lack of the research of the known extent for information amount. In order to make full use of intuitionistic fuzzy information to construct information measurement tools, this paper studies a fractional knowledge measure.

Taking into account the extensive application and its rationality of the axiom system by Szmidt and Kacprzyk [12], we first put forward a simple necessary & sufficient condition of order property in Section 2. And then, we comprehensively analyze the differences among some classic models of intuitionistic fuzzy knowledge measure. Hence, in Section 3, we bring about a new construction method consisting of the decision-making advantages and the known extent and theoretically prove that this knowledge measure satisfies all the conditions of Szmidt & Kacprzyk's axiom system. It is proved theoretically that the operators with the order condition in Szmidt & Kacprzyk axiom system will be better than those without the order condition. In Section 4, combined with the research results of De *et al.* [17], an experimental case construction and empirical test scheme are put forward. Experimental results show that the performance of the presented model with parameters is better than that of the majority of classical operators, and the operators with the order condition will be more accurate than those without the order condition.

2. Intuitionistic Fuzzy Sets

Definition 1 An fuzzy sets (FS) A in a finite set X is an object with the following form:

$$A=\{\langle x,\mu_A(x)\rangle \mid x\in X\},\ \mu_A(x)\in[0,1].$$

where $\mu_A(x): X\to[0,1]$, $x\to\mu_A(x)$.

$$A^C=\{\langle x,\mu_{A^C}(x)\rangle \mid x\in X\}=\{\langle x,1-\mu_A(x)\rangle \mid x\in X\},$$

where $\mu_{A^C}(x): X\to[0,1], x\to 1-\mu_A(x)$.

Definition 2 An intuitionistic fuzzy sets (IFS) A in a finite set X is an object with the following form:

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}$$

$$\mu_A(x): X\to[0,1], x\to\mu_A(x); \nu_A(x): X\to[0,1], x\to\nu_A(x),$$
$$\mu_A(x)+\nu_A(x)\in[0,1].$$

$\mu_A(x)$ and $\nu_A(x)$ are the degree of membership and non-membership, respectively. $\pi_A(x)=1-\mu_A(x)-\nu_A(x)\in[0,1]$, $\pi_A(x)$ is the degree of hesitancy.

Definition 3 Let A and B be two IFSs, then we have:

1) $A=B$ if and only if $\mu_A(x)=\mu_B(x), \nu_A(x)=\nu_B(x)$.

2) $A\subseteq B$ if $\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x)$.

3) $A^C=\{\langle x,\mu_{A^C}(x),\nu_{A^C}(x)\rangle \mid x\in X\}\{\langle x,\nu_A(x),\mu_A(x)\rangle \mid x\in X\}$.

3. Entropy and Knowledge Measure of IFS

Claudius's entropy is one of the important parameters in physics that characterize the state of matter. It is a measure of the degree of chaos in the physical sense and describes the disorder degree of matter in an isolated system. In 1948 Shannon first launched entropy into information theory in the "Mathematical Principles of Communication", which characterize the degree of disorder, and uncertainty and irregularity of system structure and motion [1]. After the creation of fuzzy sets, many scholars proposed a series of fuzzy entropies and its formulas, which are used to express fuzzy uncertainty. Since Atanassov proposed intuitionistic fuzzy sets [3], many scholars presented many intuitionistic fuzzy entropy formulas and knowledge measures [7]-[16]. Next some classic entropy formulas and their knowledge measures will be introduced.

Fuzzy entropy is defined as follows [4] [5] [6]:

Definition 4 A fuzzy set A in the domain X, for each $x\in X$, $E(A)=f_E(\mu_A)$ is the entropy of A with the following properties:

(EP1) $E(A)=0 \Leftrightarrow \mu_A(x)=0$ or $\mu_A(x)=1$.

(EP2) $E(A)=1 \Leftrightarrow \mu_A(x)=0.5$.

(EP3) $E(A)=E(A^C)$.

(EP4) For another fuzzy set B, $E(B)=f_E(\mu_B)$ denotes the entropy of B, and then we have: If $\mu_B(x)\le 0.5$ and $A\subseteq B$, $E(A)\le E(B)$; If $\mu_B(x)\ge 0.5$ and $B\subseteq A$, $E(A)\le E(B)$.

where $\mu_A=\mu_A(x)$ and $\mu_B=\mu_B(x)$ are the degree of membership of fuzzy sets A and B, respectively. EP1 and EP2 denote the property of non-negative boundedness, EP3 is the property of symmetry, and EP4 is the property of order.

Intuitionistic fuzzy entropy is defined as follow [7]-[11]:

Definition 5 For IFS A in the domain X, $E(A)=f_E(\mu_A,\nu_A,\pi_A)$ is the entropy of A with the following properties for $x\in X$:

(EP1) $E(A)=0\Leftrightarrow\mu_A(x)=0\,\&\,\nu_A(x)=1$ or $\mu_A(x)=1\,\&\,\nu_A(x)=0$.

(EP2) $\mu_A(x)=\nu_A(x)=0\Rightarrow E(A)=1$.

(EP3) $E(A)=E(A^C)$.

(EP4) For another IFS B, $E(B)=f_E(\mu_B,\nu_B,\pi_B)$ denotes the entropy of B, and we have: If $\mu_B(x)\le\nu_B(x)$ and $A\subseteq B$, $E(A)\le E(B)$; If $\mu_B(x)\ge\nu_B(x)$ and $B\subseteq A$, $E(A)\le E(B)$.

Where μ_A,ν_A,π_A are the degree of membership, non-membership and hesitancy of IFS, respectively.

EP1 and EP2 are the property of non-negative boundedness, EP3 is the property of symmetry, and EP4 is the property of order.

In terms of EP4, we obtain the following necessary and sufficient conditions:

(EP4I)

If $\mu_A(x)\le\mu_B(x)\le\nu_B(x)\le\nu_A(x)$ or $\mu_A(x)\ge\mu_B(x)\ge\nu_B(x)\ge\nu_A(x)$, then we have $E(A)\le E(B)$.

Proof. $A\subseteq B\Leftrightarrow\begin{cases}\mu_A(x)\le\mu_B(x)\\ \nu_B(x)\le\nu_A(x)\end{cases}$, and $\mu_B(x)\le\nu_B(x)$, then $\mu_A(x)\le\mu_B(x)\le\nu_B(x)\le\nu_A(x)$.

And if $\mu_A(x)\le\mu_B(x)\le\nu_B(x)\le\nu_A(x)$, then we have:

$$\mu_B(x)\le\nu_B(x)\,\&\,\mu_A(x)\le\mu_B(x)\,\&\,\nu_B(x)\le\nu_A(x),$$

and then we get $\mu_B(x)\le\nu_B(x)$ and $A\subseteq B$.

Similarly, $B\subseteq A\Leftrightarrow\begin{cases}\mu_A(x)\ge\mu_B(x)\\ \nu_B(x)\ge\nu_A(x)\end{cases}$, and $\mu_B(x)\ge\nu_B(x)$, then

$$\mu_A(x)\ge\mu_B(x)\ge\nu_B(x)\ge\nu_A(x).$$

And if $\mu_A(x)\ge\mu_B(x)\ge\nu_B(x)\ge\nu_A(x)$, then we have: $\mu_B(x)\ge\nu_B(x)\,\&\,\mu_A(x)\ge\mu_B(x)\,\&\,\nu_B(x)\ge\nu_A(x)$, and then we have $\mu_B(x)\ge\nu_B(x)$ and $B\subseteq A$.

EP4 is equivalent to EP4I, thus we obtain $E(A)\le E(B)$.

According to Definition 5, intuitionistic fuzzy knowledge measure can be defined as follows [12] [13]:

Definition 6 For IFS A, $K(A)=f_K(\mu_A,\nu_A,\pi_A)$ is an intuitionistic fuzzy knowledge measure of A if $K(A)$ have the following properties:

(KP1) $\mu_A(x)=\nu_A(x)=0\Rightarrow K(A)=0$.

(KP2) $K(A)=1\Leftrightarrow\mu_A(x)=0\,\&\,\nu_A(x)=1$ or $\mu_A(x)=1\,\&\,\nu_A(x)=0$.

(KP3) $K(A)=K(A^C)$.

(KP4) If B is also an IFS, and $K(B)=f_K(\mu_B,\nu_B,\pi_B)$ is the intuitionistic fuzzy knowledge measure of B, then we have: If $\mu_B(x)\le\nu_B(x)$ and $A\subseteq B$,

$K(A) \geq K(B)$; If $\mu_B(x) \geq \nu_B(x)$ and $B \subseteq A$, $K(A) \geq K(B)$.

KP1 and KP2 are the property of non-negative boundedness, KP3 is the property of symmetry, and KP4 is the property of order.

In terms of EP4, we obtain the following necessary and sufficient conditions KP4I:

(KP4I)

If $\mu_A(x) \leq \mu_B(x) \leq \nu_B(x) \leq \nu_A(x)$ or $\mu_A(x) \geq \mu_B(x) \geq \nu_B(x) \geq \nu_A(x)$, then we have $K(A) \geq K(B)$.

Obviously, KP4 I means that for $|\mu_A(x) - \nu_A(x)| \geq |\mu_B(x) - \nu_B(x)|$, we infer $K(A) \geq K(B)$. Hence, knowledge measure $K(A)$ can be considered to be a positive relation to $|\mu_A(x) - \nu_A(x)|$.

From the concept of entropy and knowledge measure above, we can define the knowledge measure of IFS A by:

$$K(A) = 1 - E(A) \tag{1}$$

Some intuitionistic fuzzy knowledge measure formulas can be defined according to some classic intuitionistic fuzzy entropy formulas as follows:

$$E_{BB}(A) = \frac{1}{n}\sum_{i=1}^{n} \pi_A(x_i) \tag{2}$$

$$E_{SKB}(A) = \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{\min\{\mu_A(x_i), \nu_A(x_i)\} + \pi_A(x_i)}{\max\{\mu_A(x_i), \nu_A(x_i)\} + \pi_A(x_i)} + \pi_A(x_i)\right) \tag{3}$$

$$E_G(A) = \frac{1}{2n}\sum_{i=1}^{n}\left(1 - |\mu_A(x_i) - \nu_A(x_i)|\right)\left(1 + \pi_A(x_i)\right) \tag{4}$$

$$E_{HC}^{\alpha}(A) = \begin{cases} \sum_{i=1}^{n} \dfrac{1 - \mu_A(x_i)^{\alpha} - \nu_A(x_i)^{\alpha} - \pi_A(x_i)^{\alpha}}{(\alpha - 1)n}, \alpha \neq 1 (\alpha > 0) \\ -\dfrac{1}{n}\sum_{i=1}^{n}\left(\mu_A(x_i)\log(\mu_A(x_i)) + \nu_A(x_i)\log(\nu_A(x_i))\right. \\ \left. + \pi_A(x_i)\log(\pi_A(x_i))\right), \alpha = 1 \end{cases} \tag{5}$$

$$E_R^{\beta}(A) = \frac{1}{n}\sum_{i=1}^{n} \frac{\log\left(\mu_A(x_i)^{\beta} + \nu_A(x_i)^{\beta} + \pi_A(x_i)^{\beta}\right)}{1 - \beta}, 0 < \beta < 1 \tag{6}$$

$$K_{BB}(A) = 1 - \frac{1}{n}\sum_{i=1}^{n} \pi_A(x_i) \tag{7}$$

$$\begin{aligned} K_{SKB}(A) &= S_{DGM1}(U, V) \\ &= 1 - \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{\min\{\mu_A(x_i), \nu_A(x_i)\} + \pi_A(x_i)}{\max\{\mu_A(x_i), \nu_A(x_i)\} + \pi_A(x_i)} + \pi_A(x_i)\right) \end{aligned} \tag{8}$$

$$K_G(A) = 1 - \frac{1}{2n}\sum_{i=1}^{n}\left(1 - |\mu_A(x_i) - \nu_A(x_i)|\right)\left(1 + \pi_A(x_i)\right) \tag{9}$$

$$K_{HC}^{\alpha}(A)=1-E_{HC}^{\alpha}(A)$$
$$=\begin{cases}1-\sum_{i=1}^{n}\frac{1-\mu_A(x_i)^{\alpha}-\nu_A(x_i)^{\alpha}-\pi_A(x_i)^{\alpha}}{(\alpha-1)n},\alpha\neq 1(\alpha>0)\\ 1+\frac{1}{n}\sum_{i=1}^{n}\big(\mu_A(x_i)\log(\mu_A(x_i))+\nu_A(x_i)\log(\nu_A(x_i))\\ +\pi_A(x_i)\log(\pi_A(x_i))\big),\alpha=1\end{cases} \tag{10}$$

$$K_R^{\beta}(A)=1-E_R^{\beta}(A)=1-\frac{1}{n}\sum_{i=1}^{n}\frac{\log\left(\mu_A(x_i)^{\beta}+\nu_A(x_i)^{\beta}+\pi_A(x_i)^{\beta}\right)}{1-\beta},0<\beta<1 \tag{11}$$

$$K_N(A)=\frac{1}{n}\sum_{i=1}^{n}\sqrt{\frac{\mu_A(x_i)^2+\nu_A(x_i)^2+(1-\pi_A(x_i))^2}{2}} \tag{12}$$

$$K_{SK}(A)=S_{DGM2}(U,V)=\frac{1}{n}\sum_{i=1}^{n}\max\left\{|\mu_A(x_i)|^p,|\nu_A(x_i)|^p,(1-\pi_A(x_i))^p\right\} \tag{13}$$

In 1996, Bustince and Burillo proposed an entropy formula $E_{BB}(A)$ [7]. In 2014, Szmidt and Kacprzyk introduced an improved knowledge measure formula $K_{SKB}(A)$, which is derived from the entropy $E_{SKB}(A)$ [12]. In terms of the Szmidt and Kacprzyk's axiom system [10] [11] [12], in 2016 Guo put forward $K_G(A)$, which is the basis of $E_G(A)$ [13]. Moreover, Huang and Yang presented $E_{HC}^{\alpha}(A)$ and $E_R^{\beta}(A)$ in 2006 [9]. In [14], Nguyen introduced a knowledge measure formula $K_N(A)$ according to distance measure proposed by Szmidt and Kacprzyk [18], and Szmidt and Kacprzyk presented a knowledge measure $K_{SK}(A)$ from similarity measure [16]. Both $K_N(A)$ and $K_{SK}(A)$ are proved to be equivalent to knowledge measure $S_{DGM}(U,V)$ introduced by Das, Guha, and Mesiar [15].

It is easy to prove that the classic knowledge measure formulas above meet the property of non-negative boundedness and symmetry. For the property of order, we have the following Lemma 2.

Lemma 2 $K_G(A)$ meet the property of order KP4I, while $K_{BB}(A),K_{SKB}(A),K_{HC}^{\alpha}(A),K_R^{\beta}(A),K_N(A),K_{SK}(A)$ and $S(U,V)$ don't meet KP4I.

Proof: According to KP4I, $K_G(A)$ meet the property of order.

$$K_G=1-\frac{(1-|\mu-\nu|)(1+\pi)}{2}=\begin{cases}\frac{\mu^2-\mu-\nu^2+3\nu}{2},\mu\le\nu,\\ \frac{-\mu^2+3\mu+\nu^2-\nu}{2},\mu\ge\nu.\end{cases}$$

For $\mu_A(x)\le\mu_B(x)\le\nu_B(x)\le\nu_A(x)$, according to $0\le\mu_A(x)+\nu_A(x)\le 1$, $0\le\mu_B(x)+\nu_B(x)\le 1$, we have:

$$\mu_B(x)\le\nu_B(x)\,\&\,\mu_B(x)+\nu_B(x)=1-\pi_B(x)\le 1$$
$$\Rightarrow 2\mu_B(x)\le\mu_B(x)+\nu_B(x)\le 1\Rightarrow\mu_B(x)\le\frac{1}{2}.$$

Therefore, we have $0\le\mu_A(x)\le\mu_B(x)\le 0.5$.

Similarly, We also have:

When $\mu_A(x) \ge \mu_B(x) \ge \nu_B(x) \ge \nu_A(x)$, $0 \le \nu_A(x) \le \nu_B(x) \le 0.5$. Thus we get:

$$K_G(A) - K_G(B)$$
$$= \begin{cases} \frac{1}{2}\left[(\mu_A - \mu_B)(\mu_A + \mu_B - 1) + (\nu_A - \nu_B)(3 - \nu_A - \nu_B)\right] \ge 0, \mu \le \nu, \\ \frac{1}{2}\left[(\mu_A - \mu_B)(3 - \mu_A - \mu_B) + (\nu_A - \nu_B)(\nu_A + \nu_B - 1)\right] \ge 0, \mu \ge \nu. \end{cases}$$

Thus we obtain: For $\mu_A(x) \le \mu_B(x) \le \nu_B(x) \le \nu_A(x)$, $K_G(A) \ge K_G(B)$.

Similarly, we also have:

For $\mu_A(x) \ge \mu_B(x) \ge \nu_B(x) \ge \nu_A(x)$, $K_G(A) \ge K_G(B)$.

Therefore, $K_G(A)$ meet the property of order.

According to KP4I, $K_{BB}(A), K_{SKB}(A), K_{HC}^{\alpha}(A)$ and $K_R^{\beta}(A)$ don't meet KP4I.

For $\mu_A(x) \le \mu_B(x) \le \nu_B(x) \le \nu_A(x)$, $K_{BB}(A) = 1 - \pi_A = \mu_A + \nu_A$, $K_{BB}(B) = 1 - \pi_B = \mu_B + \nu_B$. We cannot have $K_{BB}(A) \ge K_{BB}(B)$.

$$K_{HC}^{\alpha}(A) = \begin{cases} \dfrac{\mu_A^{\alpha} + \nu_A^{\alpha} + \pi_A^{\alpha}}{\alpha - 1}, \alpha \ne 1 (\alpha > 0) \\ 1 + \mu_A \log \mu_A + \nu_A \log \nu_A + \pi_A \log \pi_A, \alpha = 1 \end{cases}$$
$$= \begin{cases} \dfrac{\mu_A^{\alpha} + \nu_A^{\alpha} + \pi_A^{\alpha}}{\alpha - 1}, \alpha \ne 1 (\alpha > 0) \\ 1 + \log\left((\mu_A)^{\mu_A} (\nu_A)^{\nu_A} (\pi_A)^{\pi_A}\right), \alpha = 1 \end{cases}$$

$$K_{HC}^{\alpha}(B) = \begin{cases} \dfrac{\mu_B^{\alpha} + \nu_B^{\alpha} + \pi_B^{\alpha}}{\alpha - 1}, \alpha \ne 1 (\alpha > 0) \\ 1 + \mu_B \log \mu_B + \nu_B \log \nu_B + \pi_B \log \pi_B, \alpha = 1 \end{cases}$$
$$= \begin{cases} \dfrac{\mu_B^{\alpha} + \nu_B^{\alpha} + \pi_B^{\alpha}}{\alpha - 1}, \alpha \ne 1 (\alpha > 0) \\ 1 + \log\left((\mu_B)^{\mu_B} (\nu_B)^{\nu_B} (\pi_B)^{\pi_B}\right), \alpha = 1 \end{cases}$$

Obviously, we cannot get $K_{HC}^{\alpha}(A) \ge K_{HC}^{\alpha}(B)$.

$$K_R^{\beta}(A) = 1 - \frac{\log\left((\mu_A)^{\beta} + (\nu_A)^{\beta} + (\pi_A)^{\beta}\right)}{1 - \beta}, 0 < \beta < 1;$$

$$K_R^{\beta}(B) = 1 - \frac{\log\left((\mu_B)^{\beta} + (\nu_B)^{\beta} + (\pi_B)^{\beta}\right)}{1 - \beta}, 0 < \beta < 1;$$

We cannot obtain $K_R^{\beta}(A) \ge K_R^{\beta}(B)$ too.

$$K_{SKB}(A) = 1 - \frac{1}{2}\left(\frac{\min\{\mu_A, \nu_A\} + \pi_A}{\max\{\mu_A, \nu_A\} + \pi_A} + \pi_A\right) = \frac{1}{2}\left(\pi_A + \frac{1 - \nu_A}{1 - \mu_A}\right), \mu_A \le \nu_A,$$

$$K_{SKB}(B) = 1 - \frac{1}{2}\left(\frac{\min\{\mu_B, \nu_B\} + \pi_B}{\max\{\mu_B, \nu_B\} + \pi_B} + \pi_B\right) = \frac{1}{2}\left(\pi_B + \frac{1 - \nu_B}{1 - \mu_B}\right), \mu_B \le \nu_B,$$

Obviously, $K_{SKB}(A) \ge K_{SKB}(B)$ cannot be determined. Hence, K_{SKB} does not meet the property KP4I.

For $\mu_A(x) \le \mu_B(x) \le \nu_B(x) \le \nu_A(x)$, we have

$$\mu_A(x)^p \le \mu_B(x)^p, \nu_B(x)^p \le \nu_A(x)^p,$$

and we cannot have $\pi_A(x)^p \le \pi_B(x)^p \;\&\; \pi_B(x)^p \le \pi_A(x)^p$. Hence, We cannot have $K_N(A) \ge K_N(B)$ and $K_{SK}(A) \ge K_{SK}(B)$, Hence, K_N, K_{SK} and $S_{DGM}(U,V)$ do not meet the property KP4I.

4. Intuitionistic Fuzzy Knowledge Measure Model with Parameter

According to KP4I, knowledge measure $K(A)$ can be considered to be a positive relation to $|\mu_A(x) - \nu_A(x)|$. In addition, when $|\mu_A(x) - \nu_A(x)|$ is a constant, due to the same difference between membership and non-membership, the greater the minimum value of the degree of membership and non-membership is, the greater the maximum value of the degree of membership and non-membership will be, the higher the degree of known information will be, and hence the larger the knowledge measure value should be under the same difference between membership and non-membership. Thus, the knowledge measure should be positively correlated to $\dfrac{|\mu_A(x) - \nu_A(x)|}{1 - \min(\mu_A, \nu_A)}$.

Based on the definition of knowledge measure of IFSs and the analysis above, a model can be achieved:

$$K_p(A) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{|\mu_A(x_i) - \nu_A(x_i)|}{1 - \min(\mu_A(x_i), \nu_A(x_i))}\right)^p, p > 0. \tag{14}$$

$K_p(A)$ is proved to me*et al*l four properties of Definition 6.

Proof: For each $0 < p$ and for each $A \in$ IFSs, obviously, $0 \le K_p(A) \le 1$.

$(KP1)$ $\mu_A(x) = \nu_A(x) = 0 \Rightarrow K_p(A) = 0$.

$(KP2)$ $K_p(A) = 1 \Leftrightarrow \mu_A(x) = 0 \;\&\; \nu_A(x) = 1$ or $\mu_A(x) = 1 \;\&\; \nu_A(x) = 0$.

$(KP3)$ $K_p(A) = \dfrac{|\mu_A - \nu_A|^p}{(1 - \min(\mu_A, \nu_A))^p} = \dfrac{|\nu_A - \mu_A|^p}{(1 - \min(\nu_A, \mu_A))^p} = K_p(A^C)$.

(KP4I) For $\mu_A(x) \le \mu_B(x) \le \nu_B(x) \le \nu_A(x)$,

$$K_p(A) = \left(\frac{|\mu_A - \nu_A|}{1 - \min(\mu_A, \nu_A)}\right)^p = \frac{(\nu_A - \mu_A)^p}{(1 - \mu_A)^p}, K_p(B) = \frac{(\nu_B - \mu_B)^p}{(1 - \mu_B)^p}.$$

$$\because \mu_A(x) \le \mu_B(x) \le \nu_B(x) \le \nu_A(x)$$

$$\Rightarrow (\nu_A - \nu_B) \ge 0, (\mu_B - \mu_A) \ge 0$$

$$\Rightarrow (\nu_A - \nu_B)(1 - \mu_B) \ge 0, (\mu_B - \mu_A)(1 - \nu_B) \ge 0$$

$$\Rightarrow (\nu_A - \nu_B)(1 - \mu_B) + (\mu_B - \mu_A)(1 - \nu_B) \ge 0$$

$$\Rightarrow (\nu_A - \nu_B) + (\mu_B - \mu_A) + \mu_A\nu_B - \mu_B\nu_A \ge 0$$

$$\Rightarrow (\nu_A - \mu_A) - (\nu_B - \mu_B) + \mu_A\nu_B - \mu_B\nu_A \ge 0$$

$$\Rightarrow (\nu_A-\mu_A)(1-\mu_B)-(\nu_B-\mu_B)(1-\mu_A)\ge 0$$
$$\Rightarrow \frac{(\nu_A-\mu_A)(1-\mu_B)-(\nu_B-\mu_B)(1-\mu_A)}{(1-\mu_A)(1-\mu_B)}\ge 0$$
$$\Rightarrow \frac{\nu_A-\mu_A}{1-\mu_A}-\frac{\nu_B-\mu_B}{1-\mu_B}\ge 0 \Rightarrow \frac{\nu_A-\mu_A}{1-\mu_A}\ge\frac{\nu_B-\mu_B}{1-\mu_B}$$
$$\Rightarrow \frac{(\nu_A-\mu_A)^p}{(1-\mu_A)^p}\ge\frac{(\nu_B-\mu_B)^p}{(1-\mu_B)^p}$$
$$\Rightarrow K_p(A)\ge K_p(B)$$

Similarly, we also have:

For $\mu_A(x)\le\mu_B(x)\le\nu_B(x)\le\nu_A(x)$, $K_p(A)\ge K_p(B)$.

Hence, for each $p>0$, $K_p(A)$ is a knowledge measure of IFSs A.

4.1. Comparison between $K_p(A)$ and $K_G(A)$

According to Lemma 2 and the analysis above, $K_G(A)$ and $K_p(A)$ meet the property of order, while $K_{BB}(A), K_{SKB}(A), K_{HC}^{\alpha}(A), K_R^{\beta}(A), K_N(A), K_{SK}(A)$ and $S(U,V)$ don't meet KP4I. Hence, we compare $K_p(A)$ with $K_G(A)$ as follows.

$$K_p(A)=\left(\frac{|\mu_A-\nu_A|}{1-\min(\mu_A,\nu_A)}\right)^p \tag{15}$$

$K_p(A)$ is affected by the difference between membership and non-membership degree with the positive correlation, which is the same as $K_G(A)$. Meanwhile, when $|\mu_A-\nu_A|$, the difference between membership and non-membership degree, is a constant, $K_p(A)$ is also affected with the positive correlation by the minimum of membership and non-membership degree, while $K_G(A)$ the negative correlation.

$$K_G(A)=1-\frac{(1-|\mu_A-\nu_A|)(1+\pi_A)}{2}=1-\frac{(1-|\mu_A-\nu_A|)(2-\mu_A-\nu_A)}{2}.$$

From practical significance, $K_p(A)$ will be more reasonable than $K_G(A)$. If $|\mu-\nu|=d$ is a constant, then the greater the minimum value of membership and non-membership, the greater the amount of knowledge. For example, if $\mu_A-\nu_A=\mu_B-\nu_B=d\ge 0$ and $\min(\mu_A,\nu_A)=\nu_A\ge\min(\mu_B,\nu_B)=\nu_B\ge 0$, then we have $\mu_A\ge\mu_B, \nu_A\ge\nu_B, \mu_A\ge\nu_A, \mu_B\ge\nu_B, \pi_A\le\pi_B$, and hence we know that the known extent of A is more than that of B under the same difference between membership and non-membership degree. Thus, it means that $K(A)\ge K(B)$, which is the same as $K_p(A)$ and different from $K_G(A)$.

4.2. Analysis of Parameter *p* for $K_p(A)$

Let $K_1(A)=\frac{1}{n}\sum_{i=1}^{n}\frac{|\mu_A(x_i)-\nu_A(x_i)|}{1-\min(\mu_A(x_i),\nu_A(x_i))}$.

Obviously, $K_p(A)$ is a power function of $K_1(A)$, and we get

$K_p(A)=(K_1(A))^p$. According to the nature of the power function, we obtain:

1) When $p=1$, $K_p(A)=K_1(A)$ is linear function of $\frac{|\mu_A-\nu_A|}{1-\min(\mu_A,\nu_A)}$.

2) When $0<p<1$, $K_p(A)$ is a convex function on the defined domain $K_1(A)\in[0,1]$, which means that if $K_1(A)$ is close to 0, the amount of information $K_p(A)$ decreases rapidly; when $K_1(A)$ approaches 1, the amount of information $K_p(A)$ increases slowly.

3) Contrary to 2), when $p>1$, $K_p(A)$ is a concave function on the defined domain $K_1(A)\in[0,1]$, which means that if $K_1(A)$ is close to 0, the amount of information $K_p(A)$ decreases slowly; when $K_1(A)$ approaches 1, the amount of information $K_p(A)$ increases sharply.

According to the analysis above, in practical applications, people can find suitable information measurement models based on parameter adjustments.

5. Experimental Example and Result Analysis

Example 1. Consider five IFSs,

$$D_1=\{\langle x,0,0.5\rangle\},D_2=\{\langle x,0.1,0.5\rangle\},D_3=\{\langle x,0.2,0.5\rangle\},$$
$$D_4=\{\langle x,0.3,0.5\rangle\},D_5=\{\langle x,0.4,0.5\rangle\}.$$

It is clear that

$$\begin{aligned}&\mu_{D_1}(x)=0<\mu_{D_2}(x)<\mu_{D_3}(x)<\mu_{D_4}(x)<\mu_{D_5}(x)\\&=\nu_{D_5}(x)=\nu_{D_4}(x)=\nu_{D_3}(x)=\nu_{D_2}(x)=\nu_{D_1}(x)=0.5\\&\Rightarrow K_{D_1}(x)>K_{D_2}(x)>K_{D_3}(x)>K_{D_4}(x)>K_{D_5}(x).\end{aligned}$$

From Equations (7)-(14), we obtain Table 1. The evaluation index *Accuracy*

Table 1. Comparison of experimental results of $K(D_i)$, $(i=1,2,3,4,5)$.

Knowledge measure	Table Column Head					*Accuracy*	Right or Wrong
	D_1	D_2	D_3	D_4	D_5		
K_G	0.25	0.16	0.09	0.04	0.01	100%	Right
K_{SKB}	**0.5**	**0.522**	**0.538**	0.543	0.533	**40%**	**Wrong**
K_{BB}	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0%**	**Wrong**
K_{HY}^1	0.699	0.590	0.553	**0.553**	**0.590**	**60%**	**Wrong**
$K_r^{0.5}$	0.699	0.562	0.538	**0.538**	**0.562**	**60%**	**Wrong**
K_N	0.5	0.557	0.624	0.7	0.781	100%	Right
K_{SK}^2	0.25	0.36	0.49	0.64	0.81	100%	Right
$K_{0.5}$	0.707	0.667	0.612	0.535	0.408	100%	Right
K_1	0.5	0.444	0.375	0.286	0.167	100%	Right
K_2	0.25	0.198	0.141	0.082	0.028	100%	Right
K_3	0.125	0.088	0.053	0.023	0.005	100%	Right

Note. Each bold data means the wrong prediction result and the corresponding method.

can be defined as follows:

$$\text{Accuracy} = \frac{\text{Number}\left(\text{Entropies with Right Order in } D_i\right)}{\text{Number}\left(D_i\right)} \quad (16)$$

Results show that for the knowledge measures with the order property, such as K_G and K_p, the order of their results is completely correct, while the order of the results for the knowledge measures without the order property, such as K_{SKB}, K_{BB}, K_{HY}^1, $K_r^{0.5}$, K_N, K_{SK} and $S(U, V)$, do not meet the property KP4I. According to **Table 1**, K_G and K_p will be better than the others. Hence, we conclude that K_G and K_p are better than the others.

A type of classic intuitionistic fuzzy sets A_m are used to compare and analyze the difference of results among the proposed $K_p(A)$ and all those traditional knowledge measure formulas [18].

Example 2. Let $A = \left\{\left\langle x, \mu_A(x), \nu_A(x)\right\rangle | x \in X\right\}$ be an IFS in X. For any positive real number m, De *et al.* define the IFS A_m as follows [18]:

$$A_m = \left\{\left\langle x, \left(\mu_A(x)\right)^m, 1-\left(1-\nu_A(x)\right)^m\right\rangle | x \in X\right\}.$$

Obviously, we have $0 \le s \le t$, $0 \le \left(\mu_A(x)\right)^t \le \left(\mu_A(x)\right)^s \le 1$, $0 \le 1-\left(1-\nu_A(x)\right)^s \le 1-\left(1-\nu_A(x)\right)^t \le 1$, and a series of IFSs for contrast experiments can be constructed.

Using the operation above, they defined the concentration and dilation of A as follows:

Concentration: $CON(A) = A_2 = \left\{\left\langle x, \left(\mu_A(x)\right)^2, 1-\left(1-\nu_A(x)\right)^2\right\rangle | x \in X\right\}$.

Dilation: $DIL(A) = A_{1/2} = \left\{\left\langle x, \left(\mu_A(x)\right)^{1/2}, 1-\left(1-\nu_A(x)\right)^{1/2}\right\rangle | x \in X\right\}$.

Like fuzzy sets, $CON(A)$ and $DIL(A)$ can be treated as "Very (A)" and "More or less (A)", respectively.

In the next, we consider an IFS A in $X = \{6, 7, 8, 9, 10\}$ defined in reference [9] [13] [17] as follows:

Taking into account the characteristics of the value of language variables, De *et al.* define IFS $A_{0.5}, A, A_2, A_3, A_4$ in X to be "More or Less Large", "Large", "Very Large", "Quite Very Large", "Very Very Large". In the same way, $B_{0.5}, B, B_2, B_3, B_4$ and $C_{0.5}, C, C_2, C_3, C_4$ can be defined [17].

$$A = \left\{\langle 6, 0.1, 0.8\rangle, \langle 7, 0.3, 0.5\rangle, \langle 8, 0.6, 0.2\rangle, \langle 9, 0.9, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\right\},$$

$$A_{0.5} = \left\{\langle 6, 0.316, 0.553\rangle, \langle 7, 0.548, 0.293\rangle, \langle 8, 0.775, 0.106\rangle, \langle 9, 0.949, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\right\},$$

$$A_2 = \left\{\langle 6, 0.01, 0.96\rangle, \langle 7, 0.09, 0.75\rangle, \langle 8, 0.36, 0.36\rangle, \langle 9, 0.81, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\right\},$$

$$A_3 = \left\{\langle 6, 0.001, 0.992\rangle, \langle 7, 0.027, 0.875\rangle, \langle 8, 0.216, 0.488\rangle, \langle 9, 0.729, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\right\},$$

$$A_4 = \{\langle 6, 0.000, 0.998\rangle, \langle 7, 0.008, 0.938\rangle, \langle 8, 0.13, 0.59\rangle, \langle 9, 0.656, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\}.$$

$$B = \{\langle 6, 0.1, 0.8\rangle, \langle 7, 0.3, 0.5\rangle, \langle 8, 0.5, 0.4\rangle, \langle 9, 0.9, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\},$$

$$B_{0.5} = \{\langle 6, 0.316, 0.553\rangle, \langle 7, 0.548, 0.293\rangle, \langle 8, 0.707, 0.225\rangle, \langle 9, 0.949, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\},$$

$$B_2 = \{\langle 6, 0.01, 0.96\rangle, \langle 7, 0.09, 0.75\rangle, \langle 8, 0.25, 0.64\rangle, \langle 9, 0.81, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\},$$

$$B_3 = \{\langle 6, 0.001, 0.992\rangle, \langle 7, 0.027, 0.875\rangle, \langle 8, 0.125, 0.784\rangle, \langle 9, 0.729, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\},$$

$$B_4 = \{\langle 6, 0.000, 0.998\rangle, \langle 7, 0.008, 0.938\rangle, \langle 8, 0.062, 0.870\rangle, \langle 9, 0.656, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\}.$$

$$C = \{\langle 6, 0.1, 0.8\rangle, \langle 7, 0.3, 0.5\rangle, \langle 8, 0.5, 0.5\rangle, \langle 9, 0.9, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\},$$

$$C_{0.5} = \{\langle 6, 0.316, 0.553\rangle, \langle 7, 0.548, 0.293\rangle, \langle 8, 0.707, 0.293\rangle, \langle 9, 0.949, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\},$$

$$C_2 = \{\langle 6, 0.01, 0.96\rangle, \langle 7, 0.09, 0.75\rangle, \langle 8, 0.25, 0.75\rangle, \langle 9, 0.81, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\},$$

$$C_3 = \{\langle 6, 0.001, 0.992\rangle, \langle 7, 0.027, 0.875\rangle, \langle 8, 0.125, 0.875\rangle, \langle 9, 0.729, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\},$$

$$C_4 = \{\langle 6, 0.000, 0.998\rangle, \langle 7, 0.008, 0.938\rangle, \langle 8, 0.062, 0.938\rangle, \langle 9, 0.656, 0.0\rangle, \langle 10, 1.0, 0.0\rangle\}.$$

From the data above, for each $x \in X = \{6, 7, 8, 9, 10\}$, we obtain:

$$\mu_{A^{0.5}}(x) \geq \mu_A(x) \geq \mu_{A^2}(x) \geq \mu_{A^3}(x) \geq \mu_{A^4}(x),$$
$$\nu_{A^{0.5}}(x) \leq \nu_A(x) \leq \nu_{A^2}(x) \leq \nu_{A^3}(x) \leq \nu_{A^4}(x);$$
$$\mu_{B^{0.5}}(x) \geq \mu_B(x) \geq \mu_{B^2}(x) \geq \mu_{B^3}(x) \geq \mu_{B^4}(x),$$
$$\nu_{B^{0.5}}(x) \leq \nu_B(x) \leq \nu_{B^2}(x) \leq \nu_{B^3}(x) \leq \nu_{B^4}(x);$$
$$\mu_{C^{0.5}}(x) \geq \mu_C(x) \geq \mu_{C^2}(x) \geq \mu_{C^3}(x) \geq \mu_{C^4}(x),$$
$$\nu_{C^{0.5}}(x) \leq \nu_C(x) \leq \nu_{C^2}(x) \leq \nu_{C^3}(x) \leq \nu_{C^4}(x).$$

According to the definition of knowledge measure of IFSs, obviously we get:

$$\text{Knowledge}\left(A^{0.5}\right) < \text{Knowledge}(A) < \text{Knowledge}\left(A^2\right) < \text{Knowledge}\left(A^3\right) < \text{Knowledge}\left(A^4\right);$$

$$\text{Knowledge}\left(B^{0.5}\right) < \text{Knowledge}(B) < \text{Knowledge}\left(B^2\right) < \text{Knowledge}\left(B^3\right) < \text{Knowledge}\left(B^4\right);$$

$$\text{Knowledge}\left(C^{0.5}\right) < \text{Knowledge}\left(C\right) < \text{Knowledge}\left(C^{2}\right) < \text{Knowledge}\left(C^{3}\right) < \text{Knowledge}\left(C^{4}\right).$$

The results are shown in the following Tables 2-4.

Where the evaluation index *Accuracy* is defined as follows:

$$\text{Accuracy} = \frac{\text{Number}\left(\text{Entropies with Right Order in } A_i + B_i + C_i\right)}{\text{Number}\left(A_i + B_i + C_i\right)} \tag{17}$$

Table 2. Comparison of experimental results from A_k.

Knowledge measure	IFSs for comparison experiments					*Accuracy*	Number of wrong
	A_1	A_2	A_3	A_4			
K_G	0.785	0.788	0.805	0.854	1	93.3%	0
$\boldsymbol{K_{SKB}}$	0.794	**0.786**	**0.783**	0.827	1	73.3%	**2**
$\boldsymbol{K_{BB}}$	**0.908**	**0.880**	**0.868**	**0.866**	1	46.7%	**5**
K_{HY}^{1}	0.754	**0.744**	0.783	0.816	1	86.7%	**1**
$K_r^{0.5}$	0.713	**0.704**	0.737	0.773	1	86.7%	**1**
$\boldsymbol{K_N}$	**0.856**	0.835	0.839	0.847	1	80%	**1**
K_{SK}^{2}	**0.827**	**0.780**	0.764	0.765	1	60%	**2**
$K_{0.5}$	0.805	0.814	**0.746**	0.874	1	80%	**1**
K_1	0.681	0.693	0.699	0.788	1	86.7%	0
K_2	0.542	0.549	0.621	0.679	1	93.3%	0
K_3	0.472	**0.470**	0.559	0.613	0	93.3%	**1**

Note. Each bold data means the wrong prediction result and the corresponding method.

Table 3. Comparison of experimental results from B_k.

Knowledge measure	IFSs for comparison experiments					*Accuracy*	Number of wrong
	B_1	B_2	B_3	B_4	B_5		
K_G	0.767	**0.761**	0.865	0.911	0.926	93.3%	**1**
$\boldsymbol{K_{SKB}}$	0.787	**0.763**	0.852	0.888	0.899	73.3%	**1**
$\boldsymbol{K_{BB}}$	**0.918**	0.900	0.902	0.9066	0.90662	46.7%	**1**
K_{HY}^{1}	0.748	**0.745**	0.802	0.848	0.879	86.7%	**1**
$K_r^{0.5}$	0.710	**0.706**	0.748	0.791	0.826	86.7%	**1**
$\boldsymbol{K_N}$	**0.858**	0.847	0.874	0.893	0.900	80%	**1**
K_{SK}^{2}	**0.846**	**0.814**	0.819	0.831	0.838	60%	**2**
$K_{0.5}$	0.790	**0.755**	0.890	0.930	0.941	80%	**1**
K_1	0.655	**0.626**	0.803	0.869	0.891	86.7%	**1**
K_2	0.507	**0.505**	0.675	0.768	0.810	93.3%	**1**
K_3	0.437	0.445	0.587	0.691	0.748	93.3%	0

Note. Each bold data means the wrong prediction result and the corresponding method.

Table 4. Comparison of experimental results from C_k.

Knowledge measure	IFSs for comparison experiments					*Accuracy*	Number of wrong
	C_1	C_2	C_3	C_4	C_5		
K_G	0.481	0.488	0.672	0.683	0.722	93.3%	0
$\boldsymbol{K_{SKB}}$	0.790	**0.756**	0.878	0.907	0.913	73.3%	**1**
$\boldsymbol{K_{BB}}$	**0.932**	0.920	0.924	0.925	**0.920**	46.7%	**2**
K_{HY}^{1}	0.761	0.767	0.829	0.874	0.900	86.7%	0
$K_r^{0.5}$	0.499	0.489	0.312	0.218	0.164	86.7%	0
$\boldsymbol{K_N}$	**0.868**	0.864	0.895	0.911	0.913	80%	**1**
K_{SK}^{2}	**0.872**	0.852	0.861	0.866	**0.864**	60%	**2**
$K_{0.5}$	0.786	**0.673**	0.910	0.942	0.949	80%	**1**
K_1	0.648	**0.593**	0.832	0.890	0.905	86.7%	**1**
K_2	0.4985	0.4993	0.709	0.802	0.835	93.3%	0
K_3	0.429	0.445	0.619	0.731	0.783	93.3%	0

Note. Each bold data means the wrong prediction result and the corresponding method.

Based on the theoretical derivation, $K_G(A)$ and $K_p(A)$ satisfy the property of order KP4I, while $K_{BB}(A), K_{SKB}(A), K_{HC}^{\alpha}(A), K_{HC}^{1}(A), K_R^{\beta}(A), K_N(A)$, $K_{SK}(A)$ and $S(U,V)$ do not Satisfy this property. From the comparative analysis of the results in Tables 2-4, we found that the overall order accuracy of $K_p(A)$ is 89%, and that of $K_2(A) \& K_3(A) \& K_G(A)$ is 93.3%, owning the highest accuracy among all methods. Moreover, From Example 1 - 2, the order of all the results from $K_2(A)$ and $K_G(A)$ is exactly the same. And for the order of all the results from $K_3(A)$ and $K_G(A)$, there is only slight differences in Example 2 between them. Hence the overall performance of $K_p(A)$ is acceptable.

In conclusion, the above theoretical and experimental results show that the proposed parametric algorithm is simple and feasible, and it is an effective tool for knowledge measure. In future research, we will apply the constructed knowledge measure model to calculate the information volume of uncertain variables, study the information-based sorting and decision operators, and apply them to the research of management science decision making like references [19] [20], such as supply chain management and risk management.

6. Conclusion

On the basis of Szmidt & Kacprzyk's axiom system, a simple model of knowledge measure with parameters is presented. And we illustrate the validity of the measure tool from the theoretical and empirical evidence. At the same time, this paper also applies the proposed knowledge measure, along with some classical knowledge measure formulas of IFSs, from the theoretical and practical comparison, to verify a conclusion: In most knowledge measures of IFSs, the accuracy of those formulas satisfying the order property will be higher than that of those not satisfying.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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