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Dynamics of a New Rumor Propagation Model with the Spread of Truth

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Abstract

A mathematical model described the propagation of information including rumor and truth presented and its properties investigated. We explored exists of the equilibria, local stability and global asymptotical stability, and obtained the propagation threshold of rumor spreading. Numerical simulation is shown to demonstrate our results. Uncertainty and sensitivity analysis shows the importance of the parameters in our model.

Keywords

Rumor Propagation, Counter-Rumor, Sensitivity Analysis, Local Bistable, Global Stability

1. Introduction

Rumors, as daily events have been occurred. With the development of new media such as Internet, mobile phone has broadened the public access to information, optimized the circulation of information, but also exacerbated the spread of rumors. In emergency communication, financial markets fluctuation and all kinds of contagions, rumors play an important role [1] [2] [3]. Therefore, understanding the propagation of rumors and how to control it effectively is a very meaningful topic.

The dynamic behavior of the rumor spread has a great similarity with the spread of infectious diseases. Epidemic models have been developed by many researchers [4]-[14]. The classical model of rumor spreading was introduced by Daley and Kendal [15] [16]. In their model, the population is subdivided into three groups, the ignorant who know nothing about the rumor, the spreader who have heard the rumor and spread it again, the Stifler, who have heard the rumor but lose interesting with it and have ceased to spread it. Since then, many scholars worked on improving the model [17] [18]. Rumor spreading is under a close relationship with the network topology. Zanette [19] [20] examined the rumor spreading dynamics on small-world networks and obtained a critical threshold of rumor spreading. Some scholars developed applications of the stochastic Maki-Thompson model on scale-free networks [21] [22] [23] [24]. In recent years, numerous researchers have been studying the influence of psychological factors, such as memory, suspicion, forgetting and other factors of the spread of rumors [25]-[31]. Kawachi [25] studied the effect of the Stifler's memory on the spread of rumors, and when he was remembered for the rumors, it was the first to judge the true or false of the rumor. Zhao [30] discussed the influence of forgetting and remembering on the final size of the rumor. He suggested that as the forgetting rate increases, the final size of the rumor decreases, reversely, the bigger remembering rate makes the final size of the rumor larger. Wang [31] considered two different spreaders to spread two rumors individually and found that the spreading of one rumor inhibits the spreading of another rumor. Although above models are on the rumor spreading research made a significant contribution, these models are not reflecting the impact on the government counter-rumor. In the era of big data, the relevant government departments can take a variety of channels to eliminate or limit the spread of rumors. For example, after the rumor, they can through SMS (Short Message Service), government radio, broadcast, official microblog and counter-rumor sites to the society announced the truth of the incident, to eliminate unnecessary suspicion, alleviate people's anxiety and panic caused by the rumor spread and eliminate or reduce the loss of rumors to us. The rumor model takes into account the counter-rumor conforms to the rumors spread dynamic in today's society. Research considering the mechanism of the government anti-rumor spreading model is more practical significance.

The remainder of this paper is organized as follows. In Section 2, we derived the model considering the influence of rumors, after which a detailed steady-state analysis is carried out in Section 3, and the threshold of the rumor propagation is obtained. In Section 4, numerical simulation, uncertainty and sensitivity analysis of the model are presented. Finally, Section 5 summarizes this work.

2. Model

We assume that there are two kinds of spreaders in the whole population which called spreader with rumor and spreader with truth, respectively. As shown in **Figure 1**, the total population is divided into four different compartments name ignorants, spreaders with rumor, spreaders with truth and recovers, noted with $I(t)$, $S_+(t)$, $S_-(t)$, $R(t)$ respectively. When an ignorant contacts a spreader with truth, the ignorant becomes the other side with rate β_+, otherwise, if he contacts a spread with rumor, the ignorant becomes the other party with rate β_-. When a spreader with truth contacts with spreader with rumor, they will become a

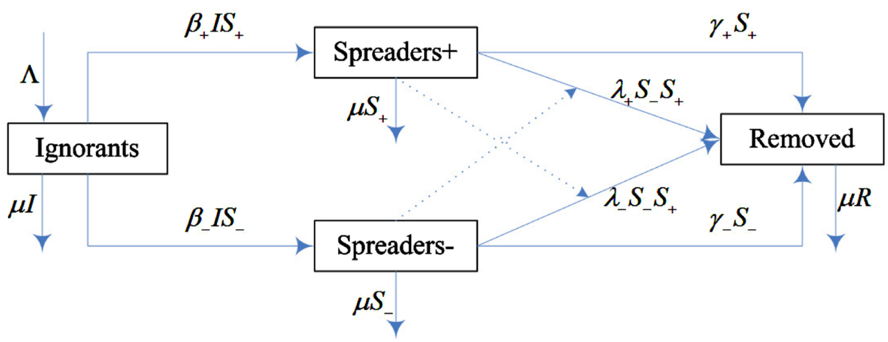


Figure 1. The flow diagram of the rumor propagation model.

removed with probability λ_+ and λ_-. The spreaders ($S_+ \setminus S_-$) spontaneously become the removed at a rate ($\gamma_+ \setminus \gamma_-$) for losing interesting.

According to the dynamic interact as mentioned above, we established the S2IR rumor spreading model based on the above assumptions. The model is described as follows:

$$\begin{aligned} \dot{I} &= \Lambda - \beta_- S_- I - \beta_+ S_+ I - \mu_1 I \\ \dot{S}_+ &= \beta_+ S_+ I - \lambda_+ S_- S_+ - \gamma_+ S_+ - \mu_2 S_+ \\ \dot{S}_- &= \beta_- S_- I - \lambda_- S_- S_+ - \gamma_- S_- - \mu_3 S_- \\ \dot{R} &= \lambda_+ S_- S_+ + \lambda_- S_- S_+ - \gamma_- + \gamma_+ S_+ + \gamma_- S_- - \mu_4 R \end{aligned} \quad (1)$$

The parameters of the model descript as follows:

Λ, the constant recruitment rate of the population;

μ_1, μ_2, μ_3, the natural fade away rate to recover of the ignorants, spreaders with rumor and spreaders with truth respectively;

β_+, β_-, the force of infection from ignorants to spreaders of rumor, spreaders of truth respectively;

λ_+, the fade away rate to recover when a spreader of rumor disseminate the rumor to a spreader of truth;

λ_-, the fade away rate to recover when a spreader of truth disseminate the truth to a spreader of rumor;

γ_+, γ_-, the forgetting rate of spreader of rumor and spreader of truth, respectively.

The initial condition of (1) is given as $I(0) > 0, S_+(0) \ge 0, S_-(0) \ge 0, R(0) \ge 0$, for the fourth equation of system of (1) is not comprised in the first three equation. Then we can consider the simplified model as follows:

$$\begin{aligned} \dot{I} &= \Lambda - \beta_- S_- I - \beta_+ S_+ I - \mu_1 I \\ \dot{S}_+ &= \beta_+ S_+ I - \lambda_+ S_- S_+ - \gamma_+ S_+ - \mu_2 S_+ \\ \dot{S}_- &= \beta_- S_- I - \lambda_- S_- S_+ - \gamma_- S_- - \mu_3 S_- \end{aligned} \quad (2)$$

The initial condition of (2) is given as $I(0) > 0, S_+(0) \ge 0, S_-(0) \ge 0$. Note $\mu = \max\{\mu_1, \mu_2, \mu_3\}$ Then we can easily verify that the system of (2) is mathematically well posed in the positive invariant region $G = \{(I, S_+, S_-) \mid I > 0, S_+ \ge 0, S_- \ge 0, I + S_+ + S_- \le \Lambda/\mu\}$. And all the solutions with $I(0)>0, S_+(0)\ge 0, S_-(0)\ge 0$ approaches or enter the region *G*, so it is sufficient to consider solutions in *G*.

3. Equilibria and Stability

Noting that

$$R_0 = \max\{R_{01}, R_{02}\} \tag{3}$$

where $R_{01} = \frac{\Lambda\beta_+}{\mu_1(\gamma_+ + \mu_2)}$, $R_{02} = \frac{\Lambda\beta_-}{\mu_1(\gamma_- + \mu_3)}$, and

$$R^* = \min\{R_1^*, R_2^*\} \tag{4}$$

where $R_1^* = \frac{\Lambda\beta_+\lambda_- + \beta_+(\mu_2+\gamma_+)(\mu_3+\gamma_-)}{\beta_-(\mu_2+\gamma_+)^2 + \lambda_-\mu_1(\mu_2+\gamma_+)}$,

$R_2^* = \frac{\Lambda\beta_-\lambda_+ + \beta_-(\mu_3+\gamma_-)(\mu_2+\gamma_+)}{\beta_+(\mu_3+\gamma_-)^2 + \lambda_+\mu_1(\mu_3+\gamma_-)}$.

Letting the left-hand side of the differential equations of model (2) equal to zero yields the following equations

$$\begin{aligned} 0 &= \Lambda - \beta_- S_- I - \beta_+ S_+ I - \mu_1 I \\ 0 &= \beta_+ S_+ I - \lambda_+ S_- S_+ - \gamma_+ S_+ - \mu_2 S_+ \\ 0 &= \beta_- S_- I - \lambda_- S_- S_+ - \gamma_- S_- - \mu_3 S_- \end{aligned} \tag{5}$$

Obviously, the system exists a trivial equilibrium $E_0(I^0, 0, 0)$ where $I^0 = \Lambda/\mu_1$. If $R_{01} > 1$, let $S_- = 0$, we easily get the rumor-free equilibrium $E_1(I^1, S_+^1, 0)$ where $I^1 = (\gamma_+ + \mu_2)/\beta_+$, and $S_+^1 = \Lambda/(\gamma_+ + \mu_2) - \mu_1/\beta_+ = \frac{\mu_1}{\beta_+}(R_{01} - 1)$. Similarly, If $R_{02} > 1$, we get the counter-free equilibrium $E_2(I^2, 0, S_-^2)$ where $I^2 = (\gamma_- + \mu_3)/\beta_-$, and $S_-^2 = \Lambda/(\gamma_- + \mu_3) - \mu_1/\beta_- = \frac{\mu_1}{\beta_-}(R_{02} - 1)$.

In the following, we shall study the existence of the positive equilibrium $E^*(I^*, S_+^*, S_-^*)$ of system (1). From the second and the third equation of (5), we have $S_+^* = (\beta_- I^* - (\mu_3 + \gamma_-))/\lambda_-$, $S_-^* = (\beta_+ I^* - (\mu_2 + \gamma_+))/\lambda_+$.

Substituting them in the first equation of (5), we have

$$F(I) = A(I^*)^2 + BI^* - C \tag{6}$$

where $A = \beta_+\beta_-(\lambda_+ + \lambda_-)$, $B = (\lambda_+ + \lambda_-)\mu_1 - \beta_+\lambda_+(\mu_3 + \gamma_-) - \beta_-\lambda_-(\mu_2 + \gamma_+)$, and $C = \Lambda\lambda_+\lambda_-$. Obviously, $A > 0, C > 0$ then exist a unique positive solution $I = I^*$, yield $F(I^*) = 0$. For the $S_+^* > 0$ and $S_-^* > 0$, we have $F((\mu_2 + \gamma_+)/\beta_+) < 0$ and $F((\mu_3 + \gamma_-)/\beta_-) < 0$

From (6) we get that $R_1^* > 1$, $R_2^* > 1$, namely $R^* > 0$. Thus, we obtain the following theorem.

Theorem 1. *System* (2) *has the following equilibrium*:

1) if $R_0 < 1$, system (2) has only the trivial equilibrium E_0;

2) if $R_{01} > 1$, $R_{02} < 1$, system (2) has a trivial equilibrium E_0 and a

truth-free equilibrium E_1;

3) if $R_{01} < 1$, $R_{02} > 1$, system (2) has a trivial equilibrium E_0 and a rumor-free equilibrium E_2;

4) if $R_{01} < 1$, $R_{02} > 1$, system (2) has a trivial equilibrium E_0, a truth-free equilibrium E_1 and a rumor-free equilibrium E_2;

5) if $R_{01} > 1$, $R_{02} > 1$, $R^* > 1$, system (2) has a trivial equilibrium E_0, a truth-free equilibrium E_1, a rumor-free equilibrium E_2 and coexist equilibrium E^*.

The general Jacobian of (2) is given by

$$J_{(2.2)} = \begin{vmatrix} -\beta_+ S_+ - \beta_- S_- - \mu_1 & -\beta_+ I & -\beta_- I \\ \beta_+ S_+ & \beta_+ I - \lambda_+ S_- - \gamma_+ - \mu_2 & -\lambda_+ S_+ \\ \beta_- S_- & -\lambda_- S_- & \beta_- I - \lambda_- S_+ - \gamma_- - \mu_3 \end{vmatrix} \tag{7}$$

Theorem 2. *The boundary equilibrium points of* (1) *have the following local stability properties*:

1) if $R_0 < 1$, $E_0(I_0,0,0)$ is a stable node, and if $R_0 > 1$, $E_0(I_0,0,0)$ is a saddle node.

2) $E_1(I_1, S_+^1, 0)$ is a stable if and only if $R_1^* > 1$.

3) $E_2(I_1, 0, S_-^2)$ is a stable if and only if $R_2^* > 1$.

4) $E^*(I^*, S_+^*, S_-^*)$ is always unstable if it exists.

Proof. 1) At E_0, we have

$$J_{(2.2)} \mid E_0 = \begin{vmatrix} -\mu_1 & -\frac{\Lambda\beta_+}{\mu_1} & -\frac{\Lambda\beta_-}{\mu_1} \\ 0 & \frac{\Lambda\beta_+}{\mu_1} - (\gamma_+ + \mu_2) & 0 \\ 0 & 0 & \frac{\Lambda\beta_-}{\mu_1} - (\gamma_- + \mu_3) \end{vmatrix} \tag{8}$$

The eigenvalues of the Equation (8) are $\lambda_1 = -\mu_1$,
$\lambda_2 = (\Lambda\beta_+ - \mu_1(\gamma_+ + \mu_2))/\mu_1 = (\gamma_+ + \mu_2)/(R_{01} - 1)$,
$\lambda_3 = (\Lambda\beta_- - \mu_1(\gamma_- + \mu_3))/\mu_1 = (\gamma_- + \mu_3)/(R_{02} - 1)$, and therefore the E_0 is an unstable node if $R_0 < 1$.

2) At E_1, the Jacobian of (2) is

$$J_{(2.2)} \mid E_1 = \begin{vmatrix} -\frac{\Lambda\beta_+}{\gamma_+ + \mu_2} & -(\gamma_+ + \mu_2) & -\frac{\beta_-(\gamma_+ + \mu_2)}{\beta_+} \\ \frac{\Lambda\beta_+}{\gamma_+ + \mu_2} - \mu_1 & 0 & -\frac{(\Lambda\beta_1 - \mu_1(\lambda_+ + \mu_2))\gamma_+}{\beta_+(\lambda_+ + \mu_2)} \\ 0 & 0 & \frac{(R_1^* - 1)(\beta_-(\lambda_+ + \mu_2) + \gamma_-\mu_1)}{\beta_+} \end{vmatrix} \tag{9}$$

The eigenvalues are $\lambda_1 = (1 - R_1^*)(\beta_+(\gamma_- + \mu_3))/\beta_2$,
$\lambda_{2,3} = -\frac{\mu_1\left(R_{01} \pm \sqrt{R_{01}^2 - (R_{01} - 1)(\lambda_+ + \mu_2)/\mu_1}\right.}{2}$, obviously they all have negative

real parts and E_1 is local stability if $R_1^* > 1$ and $R_{01} > 1$.

In a similar fashion, we can conclude that if $R_2^* > 1$ and $R_{02} > 1$. E_2 is local stability.

For the positive equilibrium E^*,

$$\begin{vmatrix} \frac{\Lambda^*}{I} & -\beta_+ I^* & -\beta_- I^* \\ \frac{\beta_- I^* - (\lambda_- + \mu_3)\beta_+}{\beta_-} & 0 & -\frac{(\beta_- I^* - (\lambda_- + \mu_3))\gamma_+}{\beta_-} \\ \frac{(\beta_+ I^* - (\lambda_+ + \mu_2))\beta_-}{\beta_+} & -\frac{(\beta_+ I^* - (\lambda_+ + \mu_2))\gamma_-}{\beta_+} & 0 \end{vmatrix} \quad (10)$$

The characteristic polynomial of (10) can be calculated as follows is

$$\lambda^3 + \frac{\Lambda}{I^*}\lambda^2 + \left(\beta_+^2 S_+^* I^* + \beta_-^2 S_-^* I^* - \lambda_+ \lambda_- S_+^* S_-^*\right)\lambda - f = 0 \quad (11)$$

which $f = \left(\beta_+ \beta_- (\lambda_+ + \lambda_-) S_+^* S_-^*\right) + \frac{\Lambda S_+^* S_-^* \lambda_+ \lambda_-}{I^*}$, the eigenvalues satisfied $\lambda_1 \lambda_2 \lambda_3 = f > 0$, It can be obtained by the Routh-Hurwitz criterion: It exists at least one eigenvalue of the polynomial equation has positive real part, then E^* is always unstable.□

Next, we will consider the global stability of quiet equilibrium E_0 of system (2).

Theorem 3. *The quiet equilibrium* $E_0 = (I_0, 0, 0)$ *is globally asymptotically stable provided that* $R_0 < 1$.

Proof. Define the Lyapunov function

$$V(I, S_+, S_-) = V_1(I, S_+, S_-) + V_2(I, S_+, S_-) \quad (12)$$

where $V_1(I, S_+, S_-) = I - I_0 - I_0 \ln \frac{I}{I_0}$, $V_2(I, S_+, S_-) = S_+ + S_-$, then differentiating V_1 with respect to t along solutions of system (2) gives

$$\begin{aligned} \frac{dV_1}{dt} &= \left(1 - \frac{I_0}{I}\right)(\Lambda - \beta_- S_- I - \beta_+ S_+ I - \mu_1 I) \\ &= \left(1 - \frac{I_0}{I}\right)(-\beta_- S_- I - \beta_+ S_+ I) + \mu_1 I_0 \left(1 - \frac{I_0}{I}\right)\left(1 - \frac{I}{I_0}\right) \\ &\le \left(1 - \frac{I_0}{I}\right)(-\beta_- S_- I - \beta_+ S_+ I) \\ &= -\beta_- S_- I - \beta_+ S_+ I + \beta_- S_- I_0 + \beta_+ S_+ I_0 \end{aligned} \quad (13)$$

$$\frac{dV_2}{dt} = \beta_+ S_+ I - \lambda_+ S_- S_+ - \gamma_+ S_+ - \mu_2 S_+ + \beta_- S_- I - \lambda_- S_- S_+ - \gamma_- S_- - \mu_3 S_- \quad (14)$$

Then, we have

$$\begin{aligned} \frac{dV}{dt} &= \frac{dV_1}{dt} + \frac{dV_2}{dt} \\ &\le \beta_+ S_+ I_0 - \lambda_+ S_- S_+ - (\gamma_+ + \mu_2) S_+ + \beta_- S_- I_0 - \lambda_- S_- S_+ - (\gamma_- - \mu_3) S_- \\ &= (\gamma_+ + \mu_2)(R_{01} - 1) S_+ + (\gamma_- + \mu_3)(R_{02} - 1) S_- - (\lambda_+ + \lambda_-) S_- S_+ \\ &\le (\gamma_+ + \mu_2)(R_0 - 1) S_+ + (\gamma_- + \mu_3)(R_0 - 1) S_- - (\lambda_+ + \lambda_-) S_- S_+ \end{aligned} \quad (15)$$

Therefore, when $R_0 < 1$, $\frac{\mathrm{d}V}{\mathrm{d}t} \le 0$, and the equality holds only for $I = I_0$, $S_+ = 0$ and $S_- = 0$, According to the LaSalle invariant principle, we have $\lim_{t\to\infty} I(t) = I_0$, $\lim_{t\to\infty} S_+(t) = 0$ and $\lim_{t\to\infty} S_-(t) = 0$. Thus, the proof is completed.□

4. Numerical Studies

4.1. Numerical Simulation

We have formulated a S2IR rumor-spread model. We have established some threshold conditions for the steady state. To illustrate the analytical results, we do some numerical simulations. From Table 1, we can set $\mu_1 = 0.3$, $\mu_2 = 0.3$, $\mu_3 = 0.3$, $\gamma_+ = 0.4$, $\gamma_- = 0.4$, $\lambda_+ = 0.38$, $\lambda_- = 0.1$, then system (2) becomes

$$\begin{aligned} \dot{I} &= \Lambda - \beta_- S_- I - \beta_+ S_+ I - 0.3I \\ \dot{S}_+ &= \beta_+ S_+ I - 0.38 S_- S_+ - 0.4 S_+ - 0.3 S_+ \\ \dot{S}_- &= \beta_- S_- I - 0.1 S_- S_+ - 0.4 S_- - 0.3 S_- \end{aligned} \tag{16}$$

Let $\Lambda = 0.2$, $\beta_+ = 0.705$, $\beta_- = 0.725$, then $R_0 = 0.6905 < 1$. By theorem 1.(a), theorem 2.(a) and theorem 3, system (16) has only the trivial equilibrium $E_0(0.6667,0,0)$ and it is globally asymptotically stable (see Figure 2), it means that the message(rumor or truth) will eliminate.

Let $\Lambda = 0.312$, $\beta_+ = 0.856$, $\beta_- = 0.656$. We have $R_{01} = 1.272 > 1$, $R_{02} = 0.974 < 1$, $R_1^* = 1.303 > 1$, $R_2^* = 0.7996 < 1$, It presents the system (16) has have two equilibria, $E_0(1.04,0,0)$ and $E_1(0.8178,0.095,0)$ where E_0 is unstable but E_1 is stable according to theorem 1.(b) and theorem 2.(a)-2.(b) (see Figure 3(a)). Let $\Lambda = 0.323$, $\beta_+ = 0.6215$, $\beta_- = 0.8545$, then system $R_{01} = 0.956 < 1$, $R_{02} = 1.314 > 1$, $R_1^* = 0.738 < 1$, $R_2^* = 1.362 > 1$. It shows that the system has

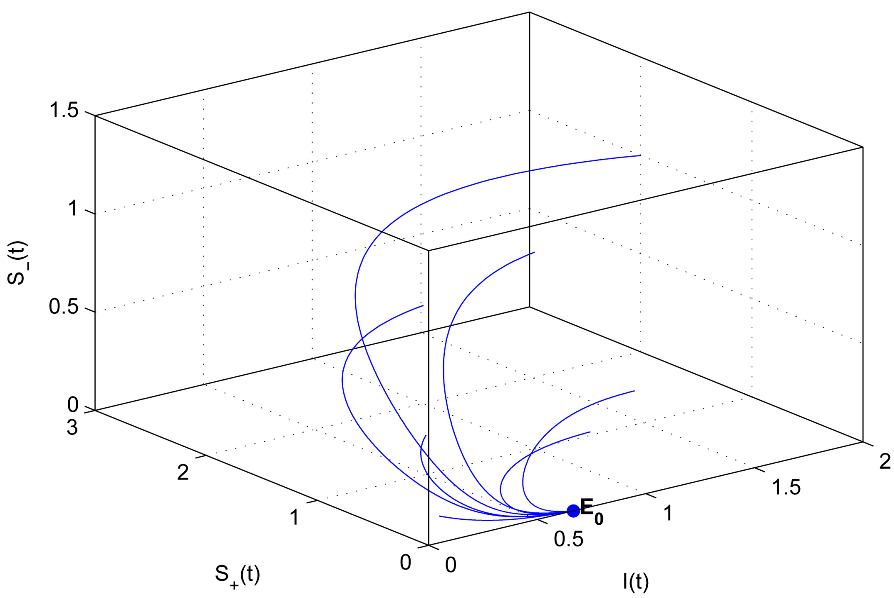
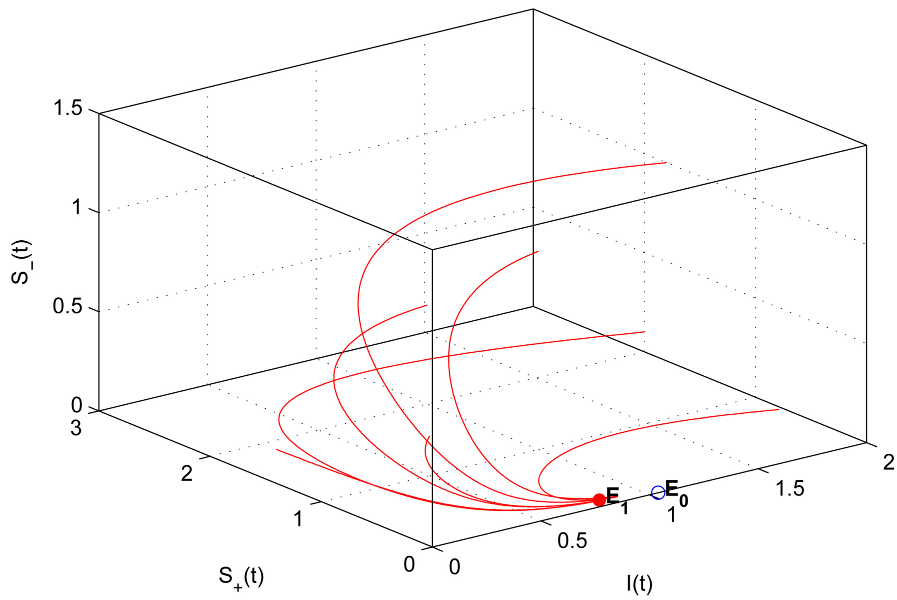
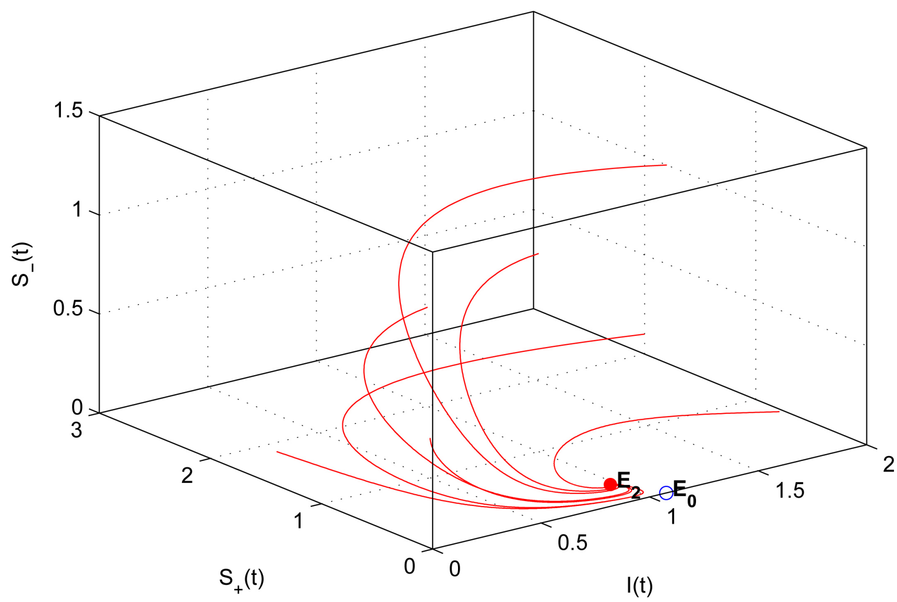


Figure 2. The trajectories, equilibria and stability of system (4.1) with $\Lambda = 0.2, \beta_+ = 0.705, \beta_- = 0.725$.



(a)

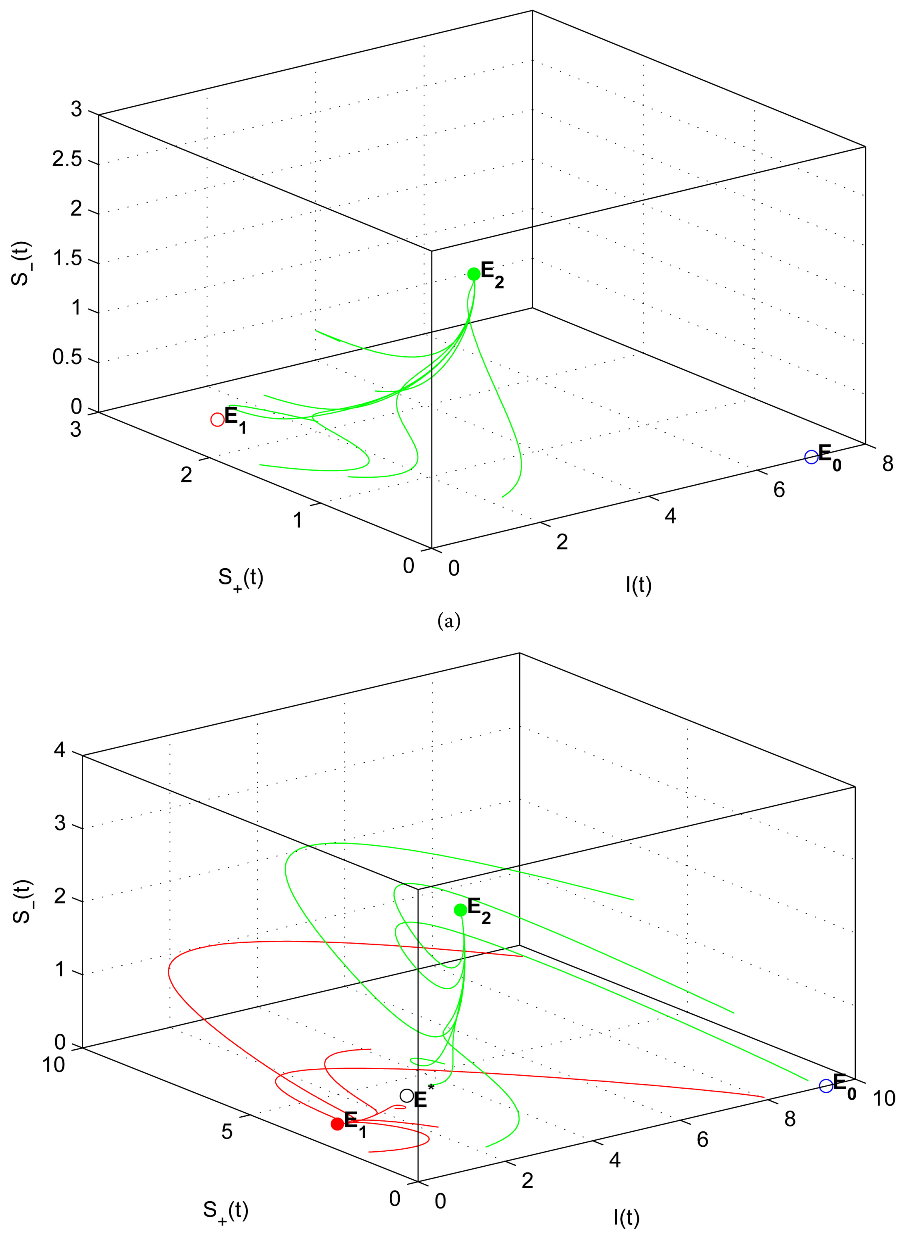


(b)

Figure 3. the trajectories, equilibria and stability of system (16) with (a) $\Lambda = 0.323, \beta_+ = 0.6215, \beta_- = 0.8545$, (b) $\Lambda = 0.323, \beta_+ = 0.6215, \beta_- = 0.8545$.

two equilibria, $E_0(1.077,0,0)$ and $E_2(0.819,0,0.11)$ where E_0 is unstable but E_2 is stable according to theorem 1.c and theorem 2.a, 2.c (see **Figure 3(b)**).

Let $\Lambda = 2.1$, $\beta_+ = 0.6$, $\beta_- = 0.9$, then $R_0 = 9 > 0$, $R_1^* = 0.9091 < 1$, $R_2^* = 3.101 > 1$. By theorem 1.(d), system (16) has three equilibria $E_0(7,0,0)$, $E_1(1.167,2.5,0)$ and $E_2(0.778,0,2.667)$ and E_0, E_1 is unstable, but E_2 is stable (see **Figure 4(a)**). Let $\Lambda = 2.8$, $\beta_+ = 0.76$, $\beta_- = 0.72$, then $R_0 = 10.13 > 0$, $R^* = R_1^* = 1.566 > 1$, $R_2^* = 2.474 > 1$. By theorem 1.(e), system (16) has three equilibria $E_0(9.33,0,0)$, $E_1(0.921,3.605,0)$, $E_2(0.972,0,3.583)$ and $E^*(1.244,1.956,0.646)$, which E_0 and E^* are not stable but E_1 and



(a)

(b)

Figure 4. The trajectories, equilibria and stability of system (16) with (a) $\Lambda = 2.1, \beta_+ = 0.6, \beta_- = 0.9$ (b) $\Lambda = 2.8, \beta_+ = 0.76, \beta_- = 0.72$.

E_2 are bistable. The phase portrait is represented in **Figure 4(b)**.

4.2. Uncertainty and Sensitivity Analysis

In this part, uncertainty and sensitivity analysis, based on Latin hypercube sampling (LHS) and partial rank correlation coefficients (PRCC) scheme, will explore the dependence of R_0, R_1^* and R_2^* due to the change of input parameters in the estimation of the uncertainty. Each input parameters is sampling 1000 times. A uniform distribution function was used and the variation ranges of all parameters are given in **Table 1**.

Table 1. Input parameter sample values for simulation.

Descriptions	Symbols	Values (range)	Reference
The constant recruitment rate of the population	Λ	1.5 (0.2, 2.8)	Assumed
The natural fade away rate of the ignorants	μ_1	0.5 (0.1, 0.9)	[29]
The natural fade away rate of spreaders with rumor	μ_2	0.5 (0.1, 0.9)	[29]
The natural fade away rate of spreaders with truth	μ_3	0.5 (0.1, 0.9)	[29]
The force of infection from ignorants to rumor-monger	β_+	0.8 (0.6, 1)	[30]
The force of infection from ignorants to truth-spreader	β_-	0.8 (0.6, 1)	[30]
The fade away rate of rumor-mongers	λ_+	0.2 (0.01, 0.39)	Assumed
The fade away rate of truth-spreaders	λ_-	0.2 (0.01, 0.39)	Assumed
The forgetting rate of spreaders of rumor	γ_+	0.3 (0.1, 0.5)	[30]
The forgetting rate of spreaders of truth	γ_-	0.3 (0.1, 0.5)	[30]

Table 2. Input parameter sample values for simulation.

Parameter	R_0		R_1^*		R_2^*	
	PRCC	p-value	PRCC	p-value	PRCC	p-value
Λ	0.0867	0.0064	0.2224	0	0.3448	0
μ_1	−0.0379	0.2348	−0.0601	0.0592	−0.1306	0
μ_2	0.0186	0.5602	−0.7155	0	0.4062	0
μ_3	−0.0751	0.0183	0.3442	0	−0.7685	0
β_+	−0.0378	0.2349	0.5305	0	−0.5457	0
β_-	0.0721	0.0235	−0.489	0	0.6155	0
λ_+	0	0	−0.1302	0	0.772	0
λ_-	0	0	0.7415	0	−0.1256	0.0001
γ_+	0.038	0.2324	−0.6316	0	0.3833	0
γ_-	−0.0695	0.0291	0.3296	0	−0.6966	0

The PRCC results which illustrate the dependence on each parameters of R_0, R_1, R_2 respectively. We considered $|\text{PRCC}| \geq 0.4$ as indicating that the high correlation between input parameters and output variables, $0.2 \leq |\text{PRCC}| < 0.4$ as moderate correlations and $|\text{PRCC}| < 0.2$ as no relations.

The results of the simulation of PRCCs are shown in **Table 2** and **Figure 5**. For R_0, the absolute values of PRCCs of each parameters fall below 0.2. It suggests that the impact of R_0 is the result of the interaction of each parameters and it is very difficult to decrease R_0 by regulating and controlling a few parameters. The parameters with the most positive impact on R_1 are the fade away rate of truth-spreaders λ_- and the force of infection of rumor-spreaders β_+, while the most negative impact on R_1 are the natural fade away of truth-spreaders μ_2, the forgetting rate of rumor-spreader γ_+ and the force of infection of truth-spreader β_-. It means that the five factors play a critical role on the process of rumor-spreading. The parameters that have the moderate influence with R_1 are the recruitment Λ, the natural fade away of truth-spreader μ_3 and the forgetting rate of truth-spreader γ_-. Compare with R_2, we find the parameters β_+, β_-, μ_2, μ_3, γ_+, γ_- have opposite influences on R_1 and R_2. This exhibitions the propagation have the nature of competitive exclusion between rumor with truth. The parameters of the fade away rate λ_+, λ_- have no relations with R_1, R_2 respectively, but have high positive relations with R_2, R_1 respectively. If both λ_+ and λ_- are large, it is possible that both R_1 and R_2 are greater than 1 and this will lead to the occurrence of bistable on E_1 and E_2.

Figures 6(a)-(c) shows the LHSs frequency distributions. The mean value for R_0, R_1 and R_2 are $M_{R_0} = 2.4214$, $M_{R_1} = 1.4797$, $M_{R_2} = 1.4793$ respectively. The standard deviation with R_0 is 0.6642 and greater than with R_1 and R_2 (0.3120, 0.3037 respectively) which indicates that the derived frequency distribution for R_0 is dispersed than the distributions for R_1 and R_2.

5. Conclusion

Rumor propagation has been investigated through different types of mathematical models. In our study, we consider a rumor propagation model with truth-spreading and determine the threshold which governs the dynamics of the system. On one hand, the propagation of rumor and truth is mutually

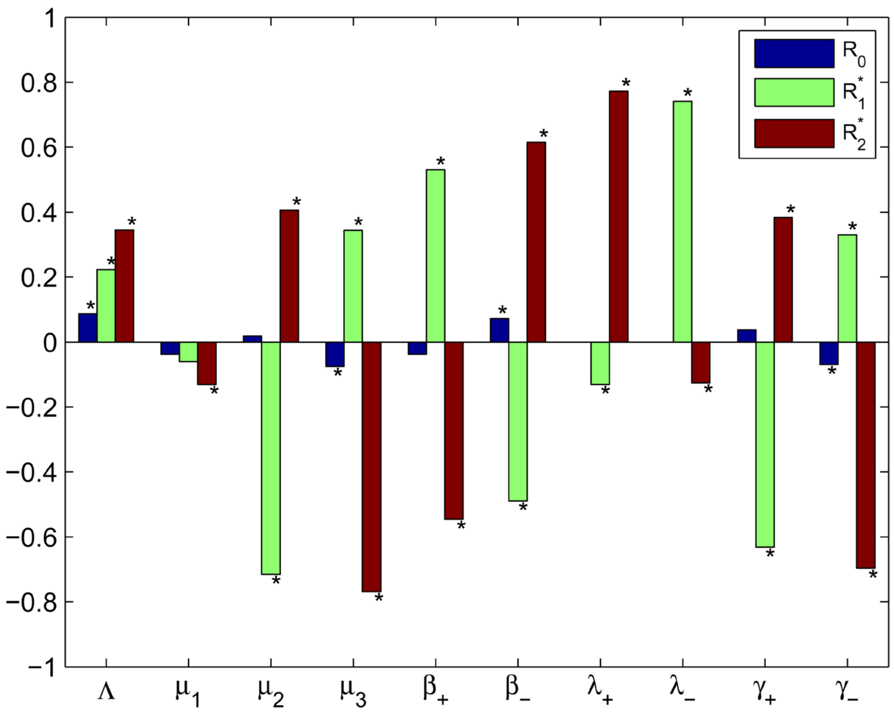


Figure 5. PRCCs illustrating the dependence of R_0, R_1^* and R_2^* for the rumor spread model on each parameter. Symbol * shows the PRCC value is not zero significantly ($P < 0.05$).

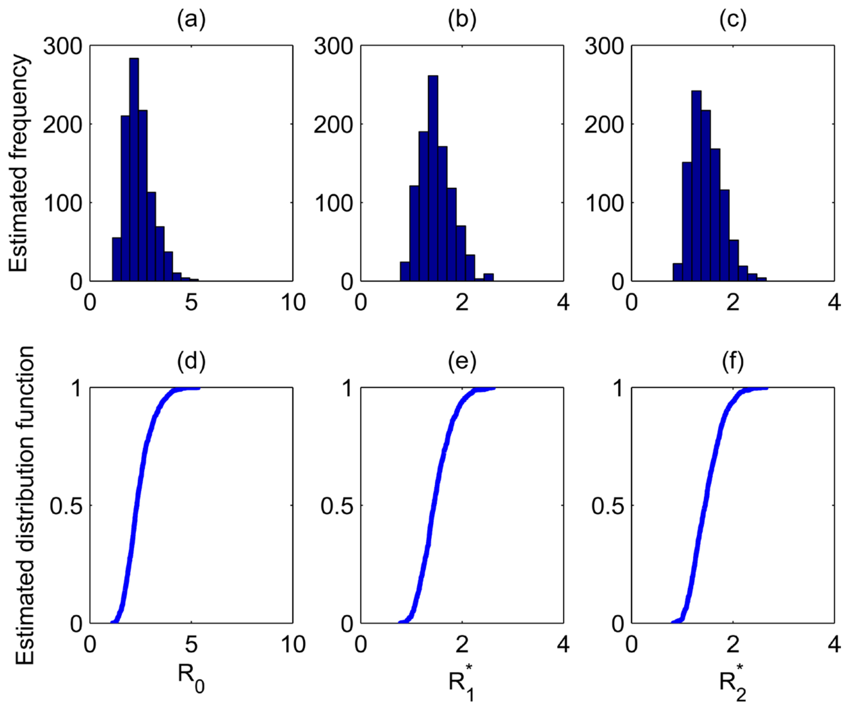


Figure 6. Uncertainty results based on Latin hypercube sampling. The top row consists of the frequency diagrams for (a) R_0, (b) R_1^* and (c) R_2^*. The following row shows estimates of CDFs for output variables ((d) R_0, (e) R_1^* and (f) R_2^*).

exclusive. Therefore, the appropriate increase in the spread of truth is conducive to the elimination of the spread of rumors. On the other hand, the spread of rumor and truth can coexist for a long time under certain conditions. At this point, the improvement of the spread of the truth is also conducive to the spread of rumors in a certain range. This requires the government and related organizations to further develop the corresponding work.

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